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Naoto Kunitomo
Seisho Sato
Daisuke Kurisu

Separating Information Maximum Likelihood Method for High-Frequency Financial Data

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Naoto Kunitomo · Seisho Sato
Daisuke Kurisu

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Naoto Kunitomo
School of Political Science and Economics
Meiji University
Tokyo
Japan

Daisuke Kurisu
School of Engeneering
Tokyo Institute of Technology
Tokyo
Japan

Seisho Sato
Graduate School of Economics
The University of Tokyo
Bunkyo-ku, Tokyo
Japan

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Preface

In the last decade, considerable interest has been paid to the problem of estimating integrated volatility using financial high-frequency data. It is now possible to use copious high-frequency data including stock markets and foreign exchange markets. Although the statistical literature contains some discussion on estimating continuous stochastic processes, earlier studies often ignored the presence of micro-market noise when trying to estimate the volatility of the underlying stochastic processes. Because the micro-market noise is important variously to analyze high-frequency financial data both in economic theory and in statistical measurement, several new statistical estimation methods have been developed. The main purpose of this book is to develop a new statistical approach, which is called the separating information maximum likelihood (SIML) method, for estimating integrated volatility and integrated covariance by using high-frequency data in the presence of possible micro-market noise.

In April, 2007, I (Naoto Kunitomo) was invited to Osaka University as the first visiting Osaka Stock Exchange (OSE) Professor by Kazuhiko Nishina and Hideo Nagai. Back then, I assumed that I knew the basics about stochastic analysis and financial economics, the former having been invented by the Japanese mathematician Kiyoshi It*ô* and refined by subsequent mathematicians at Osaka and Kyoto. I had co-authored a book in Japanese on mathematical finance and financial derivatives (Kunitomo and Takahashi 2003), which was awarded as the 2003 Nikkei Prize. I asked some staff of OSE for access to their high-frequency data because the Nikkei-225 Futures index, one of OSE's major financial products, had been successful as the major financial derivatives actively traded in Japan since 1987.[1] Upon examining their data, I realized that my understanding on both stochastic analysis

[1]It is known in finance that Dôjima rice market at Osaka in the seventeenth century was the oldest organized futures market. The trade of modern Nikkei-225 Futures was started in 1987 as the first financial futures in Japan. Osaka Stock Exchange (OSE) and Tokyo Stock Exchange (TSE) were merged as Japan Exchange Group (JPX) in 2013.

and real financial data was too poor to allow meaningful analysis, and I decided to start investigating the problem of financial high-frequency data further.

We inclined to think that we have more accurate information on hidden parameter if we have finer observations. Apparently, it is not the case for the estimation problem of volatility and co-volatilities of financial prices when we use high-frequency financial data. After a while, it was fortunate for me to have a new idea to measure volatilities and co-volatilities from high-frequency financial data and began to write the first paper on the method that we termed the *separating information maximum likelihood* (SIML) method. I asked Seisho Sato, then at the Institute of Statistical Mathematics and now at the University of Tokyo, to do some simulations and data analysis, whereupon he kindly joined my research project. Since then, Sato and I have tried to investigate statistical estimation problem on the volatility and covariation by using high-frequency data. Our approach seems to be novel, and there have been several applications of the basic SIML method. At the last moment, Daisuke Kurisu, who is a graduate student at the University of Tokyo, joined our research project because he was interested in jumps in financial markets and the application of stochastic analysis of jumps. I would also like to mention Hiroumi Misaki, then a graduate student and now at Tsukuba University, who conducted some simulations in the early stage. Because 10 years have passed since we started the research project, it is a good time for us to summarize our research activities and results in a coherent way.

This book is a summary of our joint research project on the SIML method for financial applications. We hope that it gives many readers a better understanding of the high-frequency financial data problem and also that it will be a good starting point to investigate the unsolved related topics in future.

During the process of preparing manuscripts, we have received several comments from researchers including Hiroumi Misaki, Wataru Ôta, Yoshihiro Yajima, and Katsumi Shimotsu, and we thank them for their comments to parts of the earlier versions. This work was supported by JSPS Grant-in-Aid for Scientific Research No. 25245033 and 17H02513.

Tokyo, Japan Naoto Kunitomo
December 2017

Reference

Kunitomo, N., and A. Takahashi. (2003). *A Foundation of Mathematical Finance: An Application of Malliavin Calculus and Asymptotic Expansion*. Toyo-Keizai (in Japanese).

Contents

Chapter 1
Introduction

Abstract We introduce recent issues and research around volatility estimation based on high-frequency financial data. Previous studies often ignored the presence of micro-market noises, thereby obtaining misleading estimation results. In this book, we propose the separating information maximum likelihood (SIML) method.

Recently in the field of financial econometrics, considerable interest has been paid to the problem of estimating integrated volatility using high-frequency financial data. It is now possible to use copious high-frequency data in financial markets including stock markets and foreign exchange rates markets. Although the statistical literature contains some discussion on estimating continuous-time stochastic processes, earlier studies often ignored the presence of micro-market noise in financial markets when trying to estimate the volatility of the underlying stochastic process. Because micro-market noise is important variously to analyze high-frequency financial data both in economic theory and in statistical measurement, several new statistical estimation methods have been developed. For further discussion on the related topics, see Zhou (1998), Andersen et al. (2000), Gloter and Jacod (2001), Ait-Sahalia et al. (2005), Hayashi and Yoshida (2005), Zhang et al. (2005), Ubukata and Oya (2009), Barndorff-Nielsen et al. (2008), Christensen et al. (2009), Ait-Sahalia and Jacod (2014), and Camponovo et al. (2017) among others.

The main purpose of this book is to develop a new statistical method for estimating integrated volatility and integrated covariance by using high-frequency data in the presence of possible micro-market noise. Kunitomo and Sato (2008a, unpublished) originally proposed the separating information maximum likelihood (SIML) method. Subsequently, Kunitomo and Sato (2008b, 2010, 2011, 2013) investigated some further properties of the SIML method. The SIML estimator of integrated volatility and covariance for the underlying continuous (diffusion-type) process is represented as a specific quadratic form of returns. As we show in Chap. 3, the SIML estimator has reasonable asymptotic properties: It is consistent and asymptotically normal when the sample size is large and the integrated volatility is time-changing under general situations including some non-Gaussian processes and volatility models. When the integrated volatility is stochastic, we require the concept of stable convergence and there is a further technical problem involved, see Jacod and Shiryaev (2003), Jacod

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and Protter (2011). Gloter and Jacod (2001) developed the maximum likelihood (ML) estimation of a one-dimensional diffusion process with measurement errors and our method could be interpreted as a modification of their procedure. There have been related studies by Zhou (1998) and Barndorff-Nielsen et al. (2008). However, the SIML approach differs from their methods in certain aspects as well as embodying novel features.

The main motivation for our study is that it is usually difficult to handle the exact likelihood function and more importantly the ML estimation lacks robustness if the assumption of a Gaussian distribution does not hold in the underlying multivariate continuous stochastic process with micro-market noise. This aspect is quite important for analyzing multivariate high-frequency data in stock markets and the associated futures markets. Instead of calculating the full likelihood function, we try to separate the information on the signal and noise from the likelihood function and then use each type of information separately. This procedure simplifies the maximization of the likelihood function and makes the estimation procedure applicable to multivariate high-frequency data. We call our estimation method the separating information maximum likelihood (SIML) estimator because it represents an interesting extension of the standard ML estimation method. The main merit of SIML estimation is its simplicity and robustness against possible mis-specifications of the underlying stochastic processes. Then, it can be extended in several directions and can be used practically for multivariate (high-frequency) financial time series. Not only does the SIML estimator have desirable asymptotic properties in situations including some non-Gaussian processes and volatility models, it also has reasonable finite-sample properties. The SIML estimator is asymptotically robust in the sense that it is consistent when the noise terms are weakly dependent and endogenously correlated with the efficient market price process. The SIML method was developed in an empirical study of the multivariate high-frequency Nikkei-225 Futures data for risk hedging problem, and we noticed that in real applications, we must consider the micro-market structure and noise as illustrated in Chap. 4.

The plan of this book is as follows. In Chap. 2, we discuss on the underlying background to volatility and high-frequency econometrics. In Chap. 3, we introduce the basic model and the SIML estimation of integrated volatility and integrated covariances with micro-market noise as well as showing the asymptotic properties of the SIML estimator. In Chap. 4, we discuss the finite-sample properties of the SIML method and an application to the Nikkei-225 Futures data at the Osaka Securities Exchange (OSE). In Chap. 5, we detail the mathematical derivations of the asymptotic results given in Chap. 3. In Chaps. 6 and 7, we investigate the asymptotic robustness of SIML estimation when we have the round-off errors and price adjustment mechanisms. In Chap. 8, we propose the local SIML method, which is an extension of the basic SIML method introduced in Chap. 3. In Chap. 9, we consider the estimation of the quadratic variation of Ito's semi-martingales including jumps. Finally, in Chap. 10, we make some final comments and suggest possible extensions of the SIML method.

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