Guido Visconti

Fundamentals of Physics and Chemistry of the Atmosphere

Second Edition

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This book is dedicated to my family and to the late sisters, Anna and Rita, and brother, Sante.

No weather will be found in this book

Mark Twain, The American Claimant

Preface

This is the second edition of a book published about 15 years ago. Someone says that is better to write a new book rather than work on a second edition especially after such long time. Part of the problem is that the contract for a second edition was signed just before my hometown was hit by a 6.3 earthquake in 2009. In any case I think an honorable compromise was reached considering that the book is largely rewritten.

The first five chapters are an introduction to the general topics of atmospheric physics, and they deal with thermodynamics, radiation, dynamics with applications, and chemistry. Then the sixth chapter introduces to remote sensing. However, each one of these chapters contains one of the main novelties of this book, and that is the so-called examples. These either show some applications of the matter introduced in the chapter or represent a much more detailed explanation of the same topic. Sometimes the examples contain simple programs (MATLAB or FORTRAN) to solve problems.

The chapter on the origin and evolution of the atmosphere has been canceled because this theme has advanced so much (especially in connection with the exoplanets research) that it would require a textbook of its own.

Starting with Chap. 7 the book looks very similar to the previous edition but contains much more material. This chapter has a quite detailed treatment of the vorticity and its properties. Chapter 8 gives more details on the oceanic boundary layer and some introduction to the classical concepts of turbulence. Chapter 9 contains a complete new paragraph on clouds in planetary atmospheres. Atmospheric waves are treated in Chap. 10, and the examples contain a rather complete exposition about mountain waves including simple programs. Chapter 11 is very similar to the previous book with some additional information about the wave contribution to the general circulation. Chapter 12 is about theories on general circulation. Here we have rewritten the section on the Hadley circulation with an explicit calculation based on the work of Sobel and Schneider. Also in the examples the same problem is solved in the shallow water approximation.

Chapter 13 gives more detailed information about radiative transfer calculations that are necessary for the introduction to simple climate models. These are treated

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in Chap. 14 and 15. In particular Chap. 14 ends with examples that introduce the entropy approach to energy balance climate models, and as a preparatory step the same method is used to calculate the temperature profile of an elementary atmosphere. In Chap. 15 the section on the performance of GCM has been very much expanded. It includes the most recent development about metrics and the Bayesian point of view. This requires an elementary introduction to Bayesian statistics. Statistics also enter in some of the examples about the evaluation of the effect of uncertainty in model parameters.

The chemistry of the troposphere is the topic of Chap. 16. This is another chapter largely rewritten and expanded. The simple models for tropospheric ozone have been rewritten and used to evaluate the gas isopleth in an urban atmosphere. Chapter 17 about circulation of the middle atmosphere has not changed very much except for some additional examples on equatorial waves and the Holton model on quasi biennial oscillation. The same is true for the following chapter about stratospheric ozone chemistry. In this case the examples are about the calculation of loss rate of polar ozone and the explicit calculation of the effects of the catalytic cycle, a solved exercise proposed in the book by Andrews.

Another major difference with the first edition is Chap. 19 that has been extended to deal not only with chaos but also with nonlinear phenomena. As examples of nonlinear phenomena, the Stommel model for the thermohaline circulation is discussed in connection with climate theory, and there is an extensive treatment of the difference equations made so popular by Edward Lorenz. Then the delayed differential equations are discussed in connection with the ENSO and the aerosol-cloud problem seen as predator—prey problem as developed by Koren and Feingold. The interesting parts of this chapter are the examples with programs that solve most of the topics described in the chapter.

Finally a new chapter was added on the controversial theme of geoengineering. This is a huge field now, and we just discussed a few options on carbon capture and sequestration using what we had learned in Chap. 16 about the carbon cycle. Then we had an excuse to reintroduce the energy balance model as described by Kleidon and Renner. Also some additional requirements are treated in the examples concerning the radiative effects and the role of the aerosol in the cloud albedo (Twomey effect).

All these took a considerable effort (all the figures were drawn or redrawn by the author), and I asked many times during these years if the game was worth the candle especially when there is so much material around. After 14 years the field of atmospheric physics and chemistry has expanded impressively, and there are many excellent books published not considering the amount of material available on the Internet. Our intent is to take the reader through an approach that deepens some of the most unknown aspect of the field that many times are buried in "historical" forgotten paper that remains very instructive. A classical example is the problem of the breezes that most of the books limit to a couple of pages but contain many useful insights. Similar examples could be found in the tropospheric chemistry and nonlinear problems. Our ideas were to use very simple arguments wherever possible that could be translated in simple and comprehensible calculations.

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Other important reasons for a second edition are errors. Reading back the first edition, I found many mistakes, some of them the results of simple distraction but others had some deep and wrong roots. Most of them have been corrected, but nothing guarantees that we have not introduced new ones. On the other hand the gigantic amount of material referred before contains often errors that went beyond even the referees (this is the perverse side of the peer review).

As before the contribution of students and friends has been fundamental. Frank Marzano (who should have been a coauthor) suggested many features of this edition. Former students, now in the professional lineage, have very much contributed, in particular Gabriele Curci and Paolo Ruggieri. I have to give them all the credits they deserve to help out especially with the MATLAB scripts.

The support of my family must be acknowledged considering that we all went through the stress of a destructive earthquake from which neither the city nor the university have recovered yet. Their encouragement has been constant, and this is one of the reasons this work is dedicated to them.

L'Aquila, Italy Guido Visconti

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List of Symbols

α	Albedo, specific volume, dielectric polarizability
A	Wave action
A_{oz} , A_{wv}	Fraction of solar radiation absorbed by ozone and water vapor
A(u)	Absorptivity
$B_{ u}$	Planck function
β	Meridional gradient of Coriolis parameter, Bowen ratio, linewidth
	parameter
C	Circulation
χ	Mass mixing ratio, size parameter
C_p, C_v	Specific heat per unit mass at constant pressure and constant
	volume
c, c_p, c_g	Speed of light, phase velocity, group velocity
C_D	Drag coefficient
D	Molecular diffusion coefficient
D_e	Thickness of the Ekman layer
δ	Declination of the sun
$\varepsilon, \varepsilon_0$	Emissivity, dielectric constant
e , e_s	Water vapor pressure and saturation pressure
E,\mathbf{E}	Electric field, Eliassen and Palm flux, energy density, collision
	efficiency
E_I , E_k , E_p	Internal, kinetic, and potential energy of the atmosphere
f	Coriolis parameter, volume mixing ratio
ϕ	Latitude
F_A , F_{AO}	Atmosphere and atmosphere plus ocean heat flux
$F_{ u}$	Flux of radiation
ϕ	Phase of the wave
f(v)	Maxwell velocity distribution
Φ, Φ_e, Φ_l	Geopotential, escape flux, limiting flux
$F^+, F^-, F^\uparrow, F^\downarrow$	Upward and downward fluxes
F[y(x)]	Functional
g	Gravitational acceleration, asymmetry factor

xxii List of Symbols

G Gravitational constant, gain of the antenna, Green function

 G_F Feedback of climatic system Γ , Γ_d , Γ_s Lapse rate, dry and saturated G(y), H(z) Form factor for smoke plumes

 g_s Conductance

H Scale height, hour angle, sensible heat flux

hEnthalpy, hour angleIInfrared radiation I_{ν} Intensity of radiationkBoltzmann constant

K Eddy diffusion coefficient, kinetic energy

K(R, r) Coagulation coefficient

k, l Wave numbers in the x and y directions

J Photodissociation coefficient L Latent heat of condensation L_r Rossby deformation radius

 L_H Latent heat flux

λ Wavelength, escape parameter, climate sensitivity

m Complex refractive index

 m, m_v, m_d Mass per unit surface, mass, water vapor mass, mass of dry air

 $m\mu$ Cosine of zenith angle, dynamic viscosity

M Angular momentum

N Buoyancy frequency, density of dipoles

n Refractive indexν Frequency, viscosity

n(r) Size distribution for aerosols

N(z) Columnar density n(z) Number density

 Ω Angular velocity of the Earth, solid angle

ω Vorticity (vector)

ω Vertical velocity on pressure coordinates, frequency, single scat-

tering albedo

p Pressure

P Potential vorticity, irradiated power, potential energy Ψ_N, Ψ_D Angular momentum for the nebula and the disk

 ψ Streamfunction

 Ψ Montgomery streamfunction Π Ertel potential vorticity

 $P(\theta)$ Phase function P_0 Dipole moment

 P_t , P_r Transmitted and received power by a radar

q Heat, humidity, quasi-geostrophic potential vorticity $Q_{\rm ext}, Q_{\rm abs}, Q_{\rm sca}$ Extinction, absorption, and scattering efficiency

Q Heating rate

List of Symbols xxiii

ρ	Gas density
R , R_{ν}	Radius of the Earth or planet, gas constant, reflectivity, gas
	constant for water vapor
r_a	Aerodynamic resistance
R_a	Reflectivity of the atmosphere
R_e	Reynolds number
R_i	Richardson number
S	Poynting vector (module), band strength
S	Vector in the natural coordinate system
S	Entropy per unit mass
S_0	Solar constant
σ	Stefan Boltzmann constant, surface density, static stability param-
	eter
$\sigma_{x,y,z}$	Semidispersion for smoke plume
$\sigma_m(\theta)$	Cross section for molecular scattering
T	Temperature
τ	Time constant, optical thickness
Θ	Temperature
$ au_{ij}$	Viscous stresses
θ , θ_e , θ_w	Potential temperature and angle, equivalent potential temperature,
	wet bulb potential temperature
и	Internal energy
U	Most probable velocity, optical path
\bar{u}	Average zonal wind
u*	Friction velocity
u_g , v_g	Geostrophic wind component
u_{E}	Radiative zonal wind velocity
v, V	Velocity, gas volume
$\mathbf{V}_{\mathrm{g}},\mathbf{V}_{\mathrm{a}}$	Geostrophic wind (vector), ageostrophic wind
W , W_S	Mass mixing ratio, saturation mass mixing ratio
W	Water vapor amount
<i>x</i> , <i>y</i> , <i>z</i>	Coordinates
X	Mass streamfunction
Z	Geopotential height, radar reflectivity
ζ z*	Vertical component of relative vorticity
z	Log-pressure vertical coordinate

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Chapter 1

Fundamentals: Thermodynamics of the Atmosphere

In order to introduce even the most simple questions about atmospheric physics, we need to refresh some basic physics concept. We will start with thermodynamics and continue with radiation (Chap. 2) and very essential fluid dynamics (Chap. 3). This scheme will give us the possibility to compare some characteristics of the planetary atmospheres. We all have studied thermodynamics as a part of general physics and we may have wondered about the purpose of all those theorems and demonstrations. Are they of any utility, for example, for changing a tire on our car or talking to the plumber? Actually one of the most enlightening applications of thermodynamics is to study the atmosphere or, in general, complex systems. Of course we need to study more deeply real gases like water vapor because, in a sense, it is the fuel of the atmosphere. Also the atmosphere is actually a mix of different gases and one should know under which conditions this mix may be treated as a perfect or real gas. We will start from the most elementary concepts and then, as always, we will be very careful about the jargon.

1.1 Simple Laws

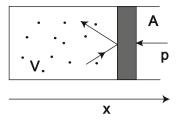
The applications we have in mind for those things learned in the early years are not many. It will be useful to introduce definitions more typical of meteorologists; for example, we start from the equation of a perfect gas

$$pV = \left(\frac{m_g}{M}\right)RT\tag{1.1}$$

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Fig. 1.1 The gas contained in a cylindrical box closed with a piston



where p is the pressure, V the volume, and m_g and M the mass of the gas and its molecular mass, respectively. R is the gas constant and T the temperature. Usually, Eq. (1.1) is simplified introducing the *specific volume* α and a new gas constant defined as R' = R/M. In the case of air, we consider a molecular mass obtained as a weight average of the two major gases composing the atmosphere (nitrogen and oxygen). We get $M = 0.78 \ 28 + 0.21 \ 32 = 28.9$ and the value of the new constant R' becomes $8314/28.9 = 286.7 \ J \ kg^{-1} \ K^{-1}$. Equation (1.1) becomes

$$p\alpha = RT \tag{1.2}$$

where R has been substituted for R'.

It is quite interesting to obtain the same law from the kinetic theory of gases that applies simple mechanics laws to a gas contained for simplicity in a box (Fig. 1.1).

Each molecule that strikes the piston will give up part of its momentum and will exert a force on the surface that will result in the pressure p. If the component of the velocity is v_x , then it will give up a momentum $2mv_x$. The number of molecules per second striking the surface A is the one contained in a cylinder with volume and is equal to nAv_x where n is the number density. The force is then

$$F = nAmv_x 2mv_x \tag{1.3}$$

The pressure is obtained by dividing by the surface A. We need also to consider that only half of the molecules are directed toward the piston and also we need to consider the average quadratic value for v_x . We then get

$$p = nm \langle v_x^2 \rangle \tag{1.4}$$

There is nothing special about the x direction so that we can easily assume that

$$\langle v_x^2 \rangle = \frac{1}{3} \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right) = \frac{\langle v^2 \rangle}{3}$$
 (1.5)

so that the pressure is simply

$$p = \left(\frac{2}{3}\right) n \left\langle \frac{mv^2}{2} \right\rangle \tag{1.6}$$

If we multiply this equation by the volume V, we get

$$pV = \left(\frac{2}{3}\right) N \left\langle \frac{mv^2}{2} \right\rangle \tag{1.7}$$

1.1 Simple Laws 3

where N is the total number of molecules in the volume. The temptation is too strong to compare (1.7) with (1.1) and get

$$\left(\frac{m_g}{M}\right)RT = \left(\frac{2}{3}\right)N\left\langle\frac{mv^2}{2}\right\rangle$$

And we can define the kinetic energy as

$$\left\langle \frac{mv^2}{2} \right\rangle = \frac{3}{2}kT \tag{1.8}$$

with k the Boltzmann constant $k = R/N_a$, where N_a is the Avogadro number and R the gas constant.

A very insightful application of (1.8) can be made to find how the pressure changes with altitude. First of all we write (1.2) in a different way introducing the Boltzmann constant k defined as $k = R/N_a$. It is easy to show that

$$p = nkT (1.9)$$

where n is the number of molecules for unit volume

We consider two parallel planes in the atmosphere separated by a distance dz, and then the force on each molecule times their density must be balanced by the change in pressure

$$Fndz = dp = kTdn$$

that is equivalent to

$$F = kT \frac{1}{n} \frac{dn}{dz} = kT \frac{d}{dz} \left[\ln(n) \right] = -\frac{d}{dz} (PE)$$
 (1.10)

where we have assumed that the force derived from some potential, that is, the difference in potential energy (PE). Then

$$d(\ln n) = -d(PE)/kT$$

that can be integrated to give

$$n = \cos t \, \mathrm{e}^{-PE/kT} \tag{1.11}$$

This equation gives the number density (pressure) as a function of the potential energy of a molecule, that is, mgz/kT, but it also constitutes the Boltzmann law, that is, the probability of finding a molecule in a determined energy state is proportional to the exponential of that energy divided by kT.

The pressure change with altitude is also an application of Equation (1.2). We start from the equation of hydrostatic equilibrium. The pressure change across a layer of thickness dz must be equal to the weight for unit surface of the atmospheric column of the same thickness:

$$dp = -\rho g dz \tag{1.12}$$

The z coordinate is oriented upward from the ground. Equation (2.3) can be easily integrated to give

$$p(z) = \int_{z}^{\infty} g\rho dz' \tag{1.13}$$

From this equation we see that the pressure as a function of altitude must depend on the density and then on the temperature.

1.1.1 The Scale Height

A very important connection can be made between thermodynamic quantities and a typical meteorological variable like the *geopotential*. This actually coincides with the gravitational potential when we take as reference the sea level. We can write

$$d\Phi = gdz = -\alpha dp \tag{1.14}$$

where Φ is the geopotential whose units are m⁻²s⁻². A quantity which is often used in meteorology is the *geopotential height Z* defined as

$$Z = \frac{\Phi}{g_0} = \frac{1}{g_0} \int_z^\infty g dz' \tag{1.15}$$

The geopotential height differs from the geometric altitude insofar as the acceleration of gravity decreases with altitude. However, in order to find appreciable differences, we need to consider altitudes of several tens of kilometers. In any case the introduction of the geopotential gives some useful relations. Starting from the gas equation and using the hydrostatic equilibrium we get

$$\frac{dp}{dz} = -\frac{pg}{RT} \tag{1.16}$$

from which we get

$$d\Phi = -RT\frac{dp}{p} \tag{1.17}$$

1.1 Simple Laws 5

and this can be integrated between pressure levels p_1, p_2

$$\Phi_1 - \Phi_2 = R \int_{p_1}^{p_2} T \frac{dp}{p} \tag{1.18}$$

If we divide both sides by g_0 , we obtain

$$Z_2 - Z_1 = \frac{R}{g_0} \int_{p_1}^{p_2} T \frac{dp}{p}$$
 (1.19)

The integral can be easily solved for an isothermal atmosphere to give

$$Z_2 - Z_1 = H \ln \left(\frac{p_1}{p_2} \right) \tag{1.20}$$

which is the same as

$$p_2 = p_1 \exp \left[-(Z_2 - Z_1) / H \right]$$

where $H = RT/g_0$ is the scale height. H gives an indication how fast atmospheric density and pressure decrease with altitude and has been defined only for an isothermal atmosphere. For example, for a temperature of 290 K, H is about 8 km. If we consider a layer defined by the pressures p_1 and p_2 to which correspond the geopotential heights Z_1 and Z_2 , then we can define an average scale height \overline{H} through Eq. (2.11)

$$Z_2 - Z_1 = -\overline{H} \ln \left(\frac{p_2}{p_1} \right) \tag{1.21}$$

where \overline{H} is the scale height calculated for \overline{T} .

1.1.2 The Potential Temperature

Application of the first law of thermodynamics gives the relation between the change in internal energy du, heat provided to the system δq , and work done dw

$$\delta q - dw = du \tag{1.22}$$

Equation (2.13) has some interesting consequences. Using the specific volume, it can be rewritten as $\delta q = pd\alpha + C_v dT$, where C_p is the specific heat per unit mass at constant pressure. We notice that $pd\alpha = -\alpha dp$ and we get an expression equivalent to the first law:

$$\delta q = C_{\nu}dT + RdT - \alpha dp = C_{p}dT - \alpha dp \tag{1.23}$$

It is convenient at this point to define the *enthalpy h*

$$h = u + p\alpha \tag{1.24}$$

and we find immediately $dh = C_p dT$. From the definition of geopotential, it follows another form of the first law of thermodynamics:

$$dq = d(h + \Phi) = d(C_p T + \Phi)$$
(1.25)

This very simple equation implies that if the motion of an air parcel is adiabatic (dq = 0), then the sum of the enthalpy and geopotential is conserved. Actually, we can get a very useful relation just putting dq = 0 in Eq. (1.25):

$$-\frac{dT}{dz} = \frac{g}{C_p} = \Gamma_d \tag{1.26}$$

This represents the *dry adiabatic lapse rate*, that is, the rate of change of temperature with altitude when the motion of the air parcel can be considered adiabatic. If we substitute $g = 9.81 \text{ ms}^{-2}$ and $C_p = 1004 \text{ j kg}^{-1} \text{ K}^{-1}$, we get $\Gamma_d = 9.8 \text{ K km}^{-1}$. Actually, the average gradient observed in the atmosphere is lower than this value, being about 6.5 K km⁻¹ and will be denoted by $\Gamma = -dT/dz$. Once we have introduced the adiabatic gradient, it is very simple to define the potential temperature θ . This would be the temperature assumed by an air parcel initially at pressure p brought adiabatically at pressure p_0 assumed as reference at 1000 hPa. Using Eq. (2.14) for an adiabatic and eliminating the specific volume through the gas equation, we have

$$\frac{C_p}{R}\frac{dT}{T} - \frac{dp}{p} = 0$$

which can be integrated from the reference pressure p_0 where the temperature is the potential temperature θ to the level p where the temperature is T. We get

$$\frac{C_p}{R} \ln \frac{T}{\theta} = \ln \frac{p}{P_0}$$

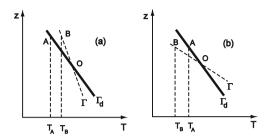
and then for the potential temperature

$$\theta = T \left(\frac{p_0}{p}\right)^{R/C_p} \tag{1.27}$$

The exponent R/C_p is about 0.286. The importance of the potential temperature is that it is a conservative quantity as we will see in the course of the book.

1.1 Simple Laws 7

Fig. 1.2 The static stability: on the *left* negative stability; on the *right* positive stability



1.1.3 Static Stability

Consider an air parcel at some altitude where the measured temperature gradient is Γ , with the dry adiabatic gradient being Γ_d . The air parcel does not contain water vapor and it is lifted from point O to A (Fig. 1.2a). Its temperature will change according to the dry adiabat and will assume the value T_A . This value is lower than the surrounding atmospheric temperature T_B , and at this point the density of the air parcel is greater than that of the surrounding air and will tend to return to the initial position. On the other hand, if Γ_d is less than Γ , the opposite will happen, and, as shown in Fig. 1.2b, the air parcel will move away from the initial point. The first case corresponds to a situation of positive static stability, while the second case is an example of negative static stability. It is interesting to relate the stability to the rate of change of the potential temperature with altitude. Suppose the air parcel of the previous examples has a volume dV, density ρ' , and temperature T'. The surrounding atmosphere, in hydrostatic equilibrium, satisfies the equation $dp/dz = -\rho g$. The air parcel will be subjected to a force per unit volume given by $-(\rho'-\rho) g$ and to acceleration $-(\rho'-\rho) g/\rho'$. As a function of temperature, we will get

$$\frac{d^2\delta z}{dt^2} = -g\frac{\Gamma_d - \Gamma}{T}\delta z \tag{1.28}$$

In writing Eq. (1.28), we have considered that the pressure inside and outside the air parcel is the same. Also $T \cong T'$ and $T - T' = (\Gamma_d - \Gamma) \delta z$ and δz is the vertical displacement of the air parcel.

The right-hand side of Eq. (2.18) has the dimension of frequency squared and it can be related to the potential temperature. Using the logarithmic derivative of the potential temperature

$$\frac{1}{\theta} \frac{\partial \theta}{\partial z} = \frac{1}{T} \frac{dT}{dz} + \frac{R}{C_p} \frac{\rho g}{p} = \frac{\Gamma_d - \Gamma}{T}$$
 (1.29)

and comparing Eqs. (2.18) and (2.19), we get

$$N^2 = \frac{g}{\theta} \frac{d\theta}{dz} \tag{1.30}$$

N is called the $Brunt-V\ddot{a}is\ddot{a}l\ddot{a}$ frequency and is a measure of the static stability. From Eq. (2.18), we have $\delta z = A \exp(iNt)$, so that for $N^2 > 0$ the air parcel will oscillate around its equilibrium position (positive stability) and if $N^2 < 0$ the air parcel will move away from the equilibrium position (negative stability). These two conditions correspond to

$$N^2 > 0 \rightarrow \frac{d\theta}{dz} > 0$$
 $N^2 < 0 \rightarrow \frac{d\theta}{dz} < 0$

In the case of positive stability, the potential temperature will increase with altitude and the opposite will happen for negative stability.

1.2 The Thermodynamics of Water Vapor

A topic in thermodynamics that is often neglected in general physics courses is that of the properties of condensable gases. In studying atmospheric physics, the obvious condensable gas is water vapor, at least for the atmosphere of the Earth. The pressure and atmospheric conditions on our planet are such as to allow the existence of water in its three main phases (gas, liquid, and solid). This peculiarity has a fundamental influence on the weather and the climate of our planet.

We will again start with a brief review of elementary thermodynamics. A good starting point is the introduction of the equation of Clausius—Clapeyron, which gives the saturation pressure of water vapor as a function of temperature.

1.2.1 The Equation of Clausius-Clapeyron

Water (just one chemical component) can appear on three different phases, that is, liquid, gas, and ice (solid). Once we have specified the number of component χ and the number of phases, ϕ , the Gibbs law gives the number of independent variable f needed to specify the system:

$$f = \chi - \phi + 2 \tag{1.31}$$

In the particular case of water, $\chi = 1$, so that $f = 3 - \phi$. This means that if the phase is just one (like gas), we need two variables to specify the state like pressure and temperature. If the phases are two like liquid and vapor, then we need to specify only one variable like temperature T. If all three phases are in equilibrium with f = 0, that means there is only one value for the variables. This is illustrated in Fig. 1.3 where we have traced the boundaries between the phase in a p, T plane.

We now consider a liquid in equilibrium with its vapor phase and we consider two isotherms as shown in Fig. 1.4a. When there is only the gas phase, the pressure

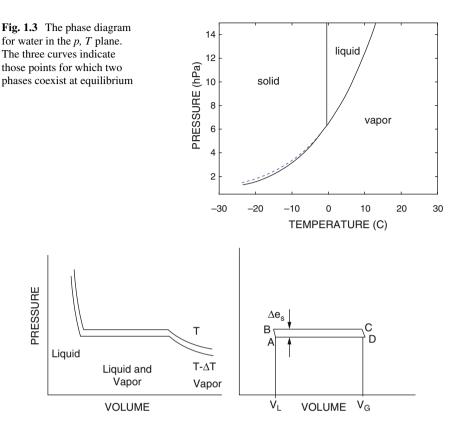


Fig. 1.4 Isothermal lines for water vapor (a, on the left) and the elementary Carnot cycle obtained by connecting the two isothermal at constant pressure. The liquid volume (V_L) and gas volume (V_G) are also indicated

increases as the volume decreases. When the vapor starts to condense and the liquid and gas phase coexist, then the pressure remains constant up to the point where all the gas has condensed. After that the pressure increases rapidly. We consider two isotherms, one at T and the other at $T - \Delta T$, and connect them with two adiabats to constitute an elementary Carnot cycle (see Fig. 1.4b). Starting from point A at pressure $e_s - de_s$ (with e_s being the saturation pressure) and temperature $T - \Delta T$, we execute a slight compression to pressure e_s to which corresponds a volume V_L (point B). In order to increase the saturation pressure, the temperature will increase to T. At constant temperature the gas expands to volume V_G (point C); from this point a new expansion is carried out to pressure $e_s - de_s$ and temperature $T - \Delta T$ (point D). Finally through adiabatic compression, the gas goes back to the initial condition. The isothermal expansion from B to C requires that some heat be provided: this is the *latent heat of condensation* L (2.5 10^6 J kg⁻¹). In the same expansion, the work done is e_s ($V_G - V_L$). On the other isotherm (D-A), heat must be subtracted and the work done will be $(e_s - de_s)$ ($V_G - V_L$). If