Mathematical Biosciences Institute Lecture Series 1.5
Stochastics in Biological Systems

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Stochastic Neuron Models

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Mathematical Biosciences Institute Lecture Series

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Mathematical Biosciences Institute Lecture Series
Volume 1: Stochastics in Biological Systems

Stochasticity is fundamental to biological systems. While in many situations the system can be viewed as a large number of similar agents interacting in a homogeneously mixing environment so the dynamics are captured well by ordinary differential equations or other deterministic models. In many more situations, the system can be driven by a small number of agents or strongly influenced by an environment fluctuating in space or time. Stochastic fluctuations are critical in the initial stages of an epidemic; a small number of molecules may determine the direction of cellular processes; changing climate may alter the balance among competing populations. Spatial models may be required when agents are distributed in space and interactions between agents form a network. Systems evolve to become more robust or co-evolve in response to competitive or host-pathogen interactions. Consequently, models must allow agents to change and interact in complex ways. Stochasticity increases the complexity of models in some ways, but may smooth and simplify in others.

Volume 1 provides a series of lectures by well-known international researchers based on the year on Stochastics in Biological Systems which took place at the MBI in 2011–2012.

Michael Reed, Richard Durrett
Editors

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Preface

In this book we describe a large number of open problems in the theory of stochastic neural systems, with the aim of enticing probabilists to work on them. These include problems arising from stochastic models of individual neurons as well as those arising from stochastic models of the activities of small and large networks of interconnected neurons. We sketch the necessary neuroscience background to these problems so that probabilists can grasp the context in which they arise. We focus on the mathematics of the models and theories, rather than on descriptions of empirical systems. Although considerable insight into the behavior of these models has been developed through computer simulation, mathematical results are, at present, limited. We hope to inspire a probabilistic attack on these open problems that will advance theoretical neuroscience.

Several themes appear herein. A stochastic model of a dynamical system is often formulated first in terms of local Poisson rates, with a discrete state space of counts. A large N approximation, which recently has been termed the "linear noise approximation," and can be thought of as a stochastic process version of a linear approximation, appears in a number of contexts here.

Another theme is oscillations sustained by noise. If a deterministic model has oscillations that are damped to a fixed point, a stochastic version of the model will have sustained, stochastic, oscillations called "quasicycles." The phenomenon appears in the context of a single neuron where stochastically sustained oscillations provide a useful model of subthreshold oscillations. It appears at the population level where stochastically sustained oscillations provide an explanation of observed population gamma-band rhythms. The same phenomenon also appears spatially, where patterns that would be damped in a deterministic model are sustained by stochasticity and are called "quasipatterns."

Our mathematical focus is on stochastic dynamical systems defined by small sets of interacting stochastic differential equations. The text heavily emphasizes work done by the authors together with several collaborators. We thank them for their important contributions to that work. We are also grateful for support from the

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