

Sir,

I understand from Mr. Richard Askey, Wisconsin, U.S.A.,
you have contributed for the sculpture in memory
of my late husband Mr. Srinivasa Ramanujan. I am happy
about this event.

Ken Ono

Amir D. Aczel

Thank you very much for your good gesture and wish
success in all your endeavours.

My Search for Ramanujan

Yours faithfully,

S. Janaki Ammal

How I learned to count



45, Muthiah Mudali H
Krishnampet, Madras-6

INDIA

Date 17.3.1



Springer

My Search for Ramanujan
How I Learned to Count

Ken Ono • Amir D. Aczel

My Search for Ramanujan
How I Learned to Count



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To the memory of Basil Gordon and Paul Sally and to Andrew Granville

PREFACE

Although this book is a first-person narrative in my voice, it represents collaborative work between me and my friend and coauthor, Amir D. Aczel. This book was his idea, and without him it would never have been written. I am deeply saddened by the fact that Amir passed away unexpectedly before work on the book was complete. Amir was a passionate man who took great pride in spreading scientific ideas through his writing. I will miss him. He still had so much to give to the world.

Ever since I was a teenager growing up in a suburb of Baltimore, I have been enchanted by the story of Ramanujan. I first learned about him when my father, a prominent mathematician, received a letter from Ramanujan's widow, sixty years after her husband's death. Srinivasa Ramanujan Iyengar was an enigmatic Indian mathematician who, without formal education in advanced mathematics, was able to derive thousands of unproved, yet valid, mathematical formulas and identities. As a mathematician, I have spent my career extending and proving some of Ramanujan's results, largely in an attempt to understand the soul and spirit of this genius who died at the young age of thirty-two. Along the way, I found myself and came to terms with my past.

This book is the result of my strong need to tell the world about how the story of this man and his mathematics helped transform me. I was once an emotionally frail, dispirited sixteen-year-old high-school dropout on the run from myself. Today, I am content. I have a loving family, I am a successful mathematician, and I have a rich spiritual life. As the character Jerry puts it in Edward Albee's play *The Zoo Story*, "Sometimes a person has to go a very long distance out of his way to come back a short distance correctly."

Acknowledgments

Amir and I could not have written this book without the help and support of many people. First of all, we thank our families: our wives, Debra Aczel and Erika Ono, and my parents, Sachiko and Takashi Ono, for their tough love and unwavering support. I thank my colleagues Krishnaswami Alladi, George

PREFACE

Andrews, Dick Askey, and Bruce Berndt for their shared enthusiasm for this story. We express our deepest gratitude to SASTRA University, in India, for converting Ramanujan's childhood home in Kumbakonam into a museum and for establishing the SASTRA Ramanujan Prize. I am indebted to Krishnaswami Alladi for playing a central role in honoring the memory of Ramanujan and for hosting me on my many trips to India. I thank Emory University and the Asa Griggs Candler Fund for their financial support, and we thank Matthew Brown, director of the film *The Man Who Knew Infinity*, and Pressman Films for their cooperation. The film is based on the superb book *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, by Robert Kanigel. We are grateful to Robert for writing this exceptional biography twenty-five years ago.

We are indebted to Henna Cho, Carol Clark, Melissa Mouly Di Teresa, Danny Gulden, Robert Schneider, Marc Strauss, and Sarah Trebat-Leder, who provided many helpful comments on earlier drafts of this book. Without their help, the book would have fallen far short of its intended goals. We thank our editor, Marc Strauss, for encouraging us to tell this story. And we thank our copyeditor, David Kramer, for beautifying and polishing our manuscript. He improved our book in uncountably many ways. Finally, we thank our literary agent, Albert Zuckerman, for helping us make this book a reality.

I am one of the luckiest mathematicians in the world. I have been guided by three amazing men, without whose friendship and guidance I would certainly have had nothing to write about. We dedicate this book to them, my mentors: Paul Sally, Basil Gordon, and Andrew Granville. Sally rescued me when I was an unmotivated undergraduate at the University of Chicago. Gordon taught me how to do mathematics for its own sake. Granville taught me how to become a professional mathematician. These men reformed, transformed, inspired, and coached me, and they made me what I am today, an active, spiritually aware mathematician with a story to tell.

Atlanta, GA
Boston, MA
December 2015

Ken Ono
Amir D. Aczel

PROLOGUE: MY HAPPY PLACE

I cannot sleep. I have a lot on my mind. It is 5:30 a.m. on May 28, 2015, and I am sitting on the lanai of an oceanview room at the Makena Beach and Golf Resort on the island of Maui. The doves are cooing. The gentle ocean breeze and the soothing sounds of waves crashing on the white sandy beach below define this heavenly moment. I am waiting for the first hint of the sun's rays in what I expect will be an absolutely glorious sunrise, with the Haleakala volcano as a backdrop.

I have never been happier.

Last night, I enjoyed a lovely evening with my wife, Erika, complete with a delicious sunset dinner at a fancy Italian restaurant on the beach in Kihei. This week we are celebrating our twenty-fifth wedding anniversary, doing many of the things we love most: mountain biking in the Makawao Forest Preserve, scuba diving in search of manta rays and green sea turtles, hiking the lava fields that formed this gorgeous island, surfing the waves at Kalama Beach, among other activities that define our active lives. Our teenage children, Aspen, who is an undergraduate at Emory University, and Sage, a rising junior at Centennial High School, in Roswell, Georgia, are home alone, enjoying ten days of freedom from their parents. They are great kids; we couldn't be more proud of the young adults that they have become.

Although the last twenty-five years seem to have passed by in a blur, Erika and I are blessed in that life has not passed us by. We have spent much of this week reminiscing about the path we have taken—our college years in Chicago, our years in Los Angeles, where I earned my doctorate, my postdoctoral tour of America, the birth of our kids, and so on. We have been leading rich lives. We have traveled the world, and we have many good friends. We are at peace with who we are as a couple, as parents, and as individuals. I am a successful mathematician, a professor at Emory University, and I am considered a leader in my field. I am a well-known mentor of young mathematicians. To borrow an over-used phrase, "I'm living the dream."

How have I been lucky enough to get to where I am today?

PROLOGUE: MY HAPPY PLACE



With Erika on Maui (May 27, 2015)

That is the question that keeps me awake. It is a question of grace and gratitude, a question that I ask every day in wonder, almost out of fear that I will be awakened from this lovely dream to discover that none of my life has actually happened. It is a question that will haunt me for the rest of my life. Erika knows this about me. But even she will be surprised when she reads some of the details of the story I am about to tell, events that I haven't shared with anyone before.

It wasn't that long ago that my waking thoughts, the parental voices in my head, were about inadequacy and fear of failure. There was a time when I couldn't even imagine wanting to live long enough to witness my thirty-second birthday. In a moment of weakness and despair twenty-three years ago, when I was twenty-four, I came within seconds of taking my own life. In a torrential rainstorm, I veered over the double yellow line of a Montana highway, near a place called Ronan, with the intention of driving headlong into an oncoming logging truck. I swerved back into my lane at the sound of the trucker's frantic horn, and I pulled over and came to a stop on that lonely strip of asphalt in the middle of a vast wilderness. I sat there alone, with the engine running, protected from the driving rain, for an eternity, trying to figure out what I was going to do with the life I had just failed to destroy. The harrowing voices, the product of my confusing and frustrating childhood as one of three sons of tough-loving, hard-driving Japanese-American parents, had nearly dragged me to my death.

Most of my current acquaintances know nothing of my past. I have done my best to erase my childhood by pretending to myself that my life began when I was sixteen years old. I ran away from my former life, and I have successfully managed to live my adult life in self-imposed amnesia. I almost never talk about my life before college; the memories are too painful. Instead of coming to terms with the past, I dumped it into a black hole, a massive part of my history from which memories could not escape.

I was so tortured by those voices that I dropped out of high school in the naive hope that I could escape my feelings of inadequacy. I have spoken to not more than one or two of my former classmates and friends since that day over thirty years ago. I didn't return phone calls from former friends who were wondering what had become of me. I made a clean break. I simply abandoned my former self. Although I didn't understand it at the time, I now feel that I was committing a metaphorical ritual suicide, what the Japanese call *seppuku*.

I was raised by "tiger parents," a term applied to Asian-American parents who raise their children in an overly strict way (by Western standards) with the goal of fostering an academically competitive spirit. This form of upbringing is intended to direct a child toward success, and it has one goal—to raise extraordinarily successful children, conquerors of their fields. Tiger parents set high academic standards, and they severely restrict nonacademic activities with this long-term goal in mind. That is how my parents showed their love for me, by demanding nothing less than the best. But the cost was enormous. It almost killed me.

My professional success confirms, and perhaps even justifies, the merits of this philosophy. There is certainly much truth to the undeniable fact that my parents fostered qualities in me that have been essential for my success. I am ambitious and competitive. I am restless, anxious to take on the next challenge. I thank my parents, whom I love deeply, for instilling these qualities in me.

This style of parenting, however, especially when carried to the degree that I experienced, has the potential of crushing a child's spirit. Children can grow up emotionally unfulfilled and starving for recognition. Those voices in my head that nearly drove me mad, my despair and feelings of worthlessness, all attest to that fact.

What vanquished those voices and gave me back my soul? The answer involves a mysterious genius from India, a man whose story inspired me at my lowest points, and whose ideas have powered my career.

Let me tell it to you as I remember it.

Makena Beach, Maui, HI
May 2015

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Part I
My Life Before Ramanujan

Chapter 1

~

TIGER BOY

Lutherville, Maryland (1976–1984)

I'm sitting on the couch watching *Gilligan's Island*, every second-grader's favorite sitcom. It's the episode where the headhunters from a neighboring island attack the motley crew of castaways. As usual, the klutzy skinny first mate Gilligan accidentally saves the day, in this episode by scaring off the headhunters.

I really should be doing the geometry problems that my parents assigned me, but they aren't home, and I love *Gilligan's Island*. If my parents find out, I'll be in super big trouble. But I'm prepared. I have a washcloth and a small pink plastic basin filled with ice water. The TV is on low enough that I will be able to hear my parents pull into the driveway, which will give me just enough time to turn off the TV and cool the back of the set with the ice-cold washcloth.

My second-grade portrait



Does this seem over the top?

In her book *Battle Hymn of the Tiger Mother*, Amy Chua, a Yale law professor, writes about what she calls tiger parenting, a traditional strict form of child-rearing popular in Asia and among Asian-American families. This “tiger mom” ideology accurately describes the approach of my Japanese immigrant parents, Sachiko and Takashi Ono. Like other tiger parents, they believed that their children could be “the best” students and that academic achievement is a reflection of successful parenting. Indeed, if their children are not at the top of their class, then the parents aren’t doing their job. My parents went a step further. If I wasn’t the best student, then I would bring shame on my family. It was understood that it was my duty to be “the best.”

My parents in 1999 (photo by Olan Mills)



I emerged from my early childhood with the voices of my parents in my head that continually rebuked me for my inadequacy and my inability to live up to their unrealistic expectations:

Ken-chan, your parents are disappointed in you. You are embarrassment. Look at that professor’s children. Unlike you, they study all of time, and they what you should be. You sloppy. You spoiled. Your mother sacrificed her life for you, so you do your part. What wrong with you? You want play all of time?

Those voices told me that my parents would love me only if I was both a star student and a brilliant musician. Those voices told me that it was wrong to relax and have fun and hang out with friends. When I did those things, those voices made certain that I suffered tremendous pangs of guilt.

I now understand that many children today hear similar voices. Tragically, some of these children will succumb to those voices and take their own lives. Moreover, those suicides often occur in clusters, a phenomenon that has recently become a source of concern in communities like Palo Alto, where elevated academic expectations are rampant and such parenting is common.

Those voices are symptoms of an anxiety disorder that has been the focus of considerable recent study by clinical psychologists. Their research suggests that children of tiger parents are often burdened with anxieties that last a lifetime. The research also offers a possible biological explanation for this phenomenon.

When we realize that we have made a mistake, a predictable electroneurological process called *error-related negativity* (ERN) is triggered in the medial prefrontal cortex of our brains. It acts as a reset button for the brain. It is now believed that the strength of ERN is negatively impacted by prolonged exposure to harsh criticism.

I didn't need this research to understand the validity of its conclusions. I have firsthand knowledge. I became desperate for the love and approval of my parents, and when I failed and failed again to obtain it, my life began to unravel.

For you to understand how all this came to be, I will have to explain my family. My parents raised my two brothers and me under the assumption that we were somehow genetically predestined, with each son to follow a well-defined path that my parents determined in response to the talents and strengths we exhibited in our early years. Our job was simple—stay on track and succeed in the lives that our parents had prescribed for us. I felt that I never had a choice.

My oldest brother, Momoro, was gifted in music. He was a child prodigy. You know the kind—the cute Asian-American third-grader with a bowl haircut, dressed in a tuxedo, dazzling television audiences with a precocious rendition of a Tchaikovsky piano concerto. He was going to be a concert pianist who performed at Carnegie Hall.

Santa, the middle son, had a different path. He was often described as the black sheep of the family, which is ironic, because he is the one who will go on to be the most successful son. My parents felt that he was unlikely to amount to much of anything, so he was expected to be an ordinary company man, whatever that meant. As a second-grader, I understood only that it referred to something that my parents viewed as significantly below concert pianist and university professor.

I was being groomed to be a mathematician in the image of my father. But I was also expected to be an outstanding musician. I was only in second grade,

but I already had the next twenty-five years mapped out for me. I was to attend one of the best Ivy League universities, earn a PhD in mathematics, and then secure a professorship at a top university.

My parents showed their love for us, which I didn't understand at the time, by defining our long-term professional goals and offering opportunities that powered us toward them. Their entire focus was on those goals, in the belief that we would reach our happy places by achieving them.

The rules that they made were simple to follow, reducing each of our lives to an individual formula. To achieve the goals that our parents had set for us was going to be easy—as easy as basic algebra: just as plugging $x = 2$ into the formula $y = 3x + 1$ gives the value $y = 7$, I was given a simple formula for becoming a mathematician—I had to be a straight-A student who earned top scores on all my tests. And for good measure, I was expected to become an accomplished violinist.

Everything outside of the formula was considered extraneous, and if we ever strayed in the slightest from our formulas, we were subjected to a litany of rebukes and threats that discouraged and humiliated us.

Here is a vivid example of what I am talking about. I was in third grade. Each year in elementary school, we took the Iowa Tests of Basic Skills, standardized tests that were administered as a tool for improving instruction. These tests evaluated our skills in grammar, reading comprehension, and mathematics. When my parents received my scores, they were shocked at my poor performance: ninety-eighth percentile in math and ninety-seventh percentile in reading comprehension. They summoned me to the kitchen and sat me down at the head of the table. Pacing behind me, they rebuked me for my embarrassingly inadequate performance:

Ken-chan, one in fifty people did better in math. Even more better in English. That's fifty thousand kids in country. You aren't even among best fifty thousand. Harvard only accepts few thousand. If you don't fix, you might end up at University of Maryland, or, God forbid, Towson State. You must get ninety-ninth percentiles. That professor's children all got these scores, and look how successful they are. We sacrificing everything for you three boys, and this how you thank us? If you won't do better, then get out of house!

Then they left the room, while I sat alone at the table crying, with only the ticking of the clock and the rumble of the fridge to keep me company. Half an hour later I'd hear the shuffling of slippers, which signaled their return, and they

repeated the diatribe again. I wanted to sink through the floor, or at least to escape to the privacy of my room.

Between me and my two brothers, incidents like this were common in my early childhood. One of us would be scolded while the other two would cower in their bedrooms, thankful that it wasn't their turn. Truth be told, I was on the receiving end of these rants much less often than my two older brothers.

My parents forbade just about everything that was not directly tied to our formulaic lives. There was no room for *Gilligan's Island*. Santa was forbidden from attending his high-school prom. When he sneaked out to attend despite their prohibition, my parents tracked him down and brought him home. Imagine the humiliation of being pulled from the prom in front of your classmates. My parents' view was that this time-honored rite of passage was not for serious students, not part of the formula. Perhaps they also wanted to protect him from the bad influences they imagined he would be exposed to.

Here is another example. Let me explain why I can't watch the *Star Wars* movies. When the first *Star Wars* film came out in 1977, I begged my parents to let me go with my friends to the movie theater to see it. Many of my friends saw the film several times, and so I had plenty of opportunities. After incessant pleading, my parents finally relented, but under one condition—that I write a three-page essay explaining the importance of the film and the deeper meaning of the story. I was a nine-year-old who simply wanted to delight in the droids and bizarre space creatures that everyone was talking about. Instead, the film became an academic exercise complete with edits and revisions. To this day, I can't watch any of the sequels. The mere thought of *Star Wars* stirs up painful memories.

But when it came to academics and music, my parents offered me the best. They bought me computers: I was one of the first kids at school to get an Apple II. They enrolled me for violin lessons at the Peabody Institute. Peabody is an internationally renowned conservatory that has trained famous musicians like singer Tori Amos and pianist André Watts. My parents engaged Yong Ku Ahn, a distinguished Peabody Institute professor, to teach me to play the violin. And the violin they bought for me was the product of a nineteenth-century Italian craftsman. I didn't understand until much later that beginning violinists don't usually take their first lessons from a well-known virtuoso, and they don't typically play an instrument whose value is roughly that of a brand-new car.

As part of my mathematical training, my father had me write computer programs to collect data for his mathematics graduate students. I actually enjoyed doing this, and it was the source of some of the very few memories I have of the parental attention and approval that I so badly craved. Even though I didn't know

anything about class numbers of quadratic forms, I was able to write computer programs that computed tables of them. I was delighted that I, a kid in elementary school, could help produce results that were published in a PhD thesis.

But these opportunities always came with a price, and when it came to music, the price was more than I was able or willing to pay. I took lessons from Professor Ahn for almost ten years. Each week, both my parents drove me to his home, and then they sat in on my lesson, taking note of my progress. The pressure was unendurable. Instead of concentrating on my playing, I constantly had my eyes on my parents, looking for evidence of their approval or chagrin. They never told me that I had done well; I believe that the thought of giving praise never crossed their minds. And on the long drive home, they would rehash the lesson, reminding me of the mistakes I had made and emphasizing the improvements that were expected of me for the next lesson.

Despite the fact that I became an accomplished violinist, making second chair in the first violin section of the Peabody Preparatory Orchestra, there is very little that I can say that might put a positive spin on my musical career. I hated the violin.

One day, when I was in tenth grade, I simply quit. I have picked up a violin only once since then, and that was five years later, when Erika, who would later become my wife, brought me to her home for the first time to meet her parents. In my desire to impress them, I unthinkingly broke my black-hole rule: I mentioned that I had been an accomplished violinist. To my surprise, they actually wanted to hear me play. And to my chagrin, there was a violin—Erika's sister's—in the house. They no doubt thought that I would be delighted to play for them, and they had every reason to expect to hear an "accomplished violinist" produce lovely music. I tried to play, but I couldn't. Instead, I sat alone in Erika's sister's bedroom, holding her violin while I tore myself apart inside. How could I possibly explain? How could I make them understand? They had no idea of what they were asking me to do.

Still in the bedroom, I finally worked up a bit of courage and began softly playing Bach's *Partita Number 3* from memory. The sight of my reflection in the mirrored closet doors aroused painful long-dormant memories, and it was more than I could bear. After a few minutes, I stopped playing, and I told Erika and her parents that I just couldn't perform for them. At least, I thought, they had overheard enough to know that I had indeed been a competent violinist. And I was grateful that they had apparently recognized that I was in the grip of an inner struggle, and I was relieved when they let the incident pass without pressing me for details. When this book is published and Erika's parents read this