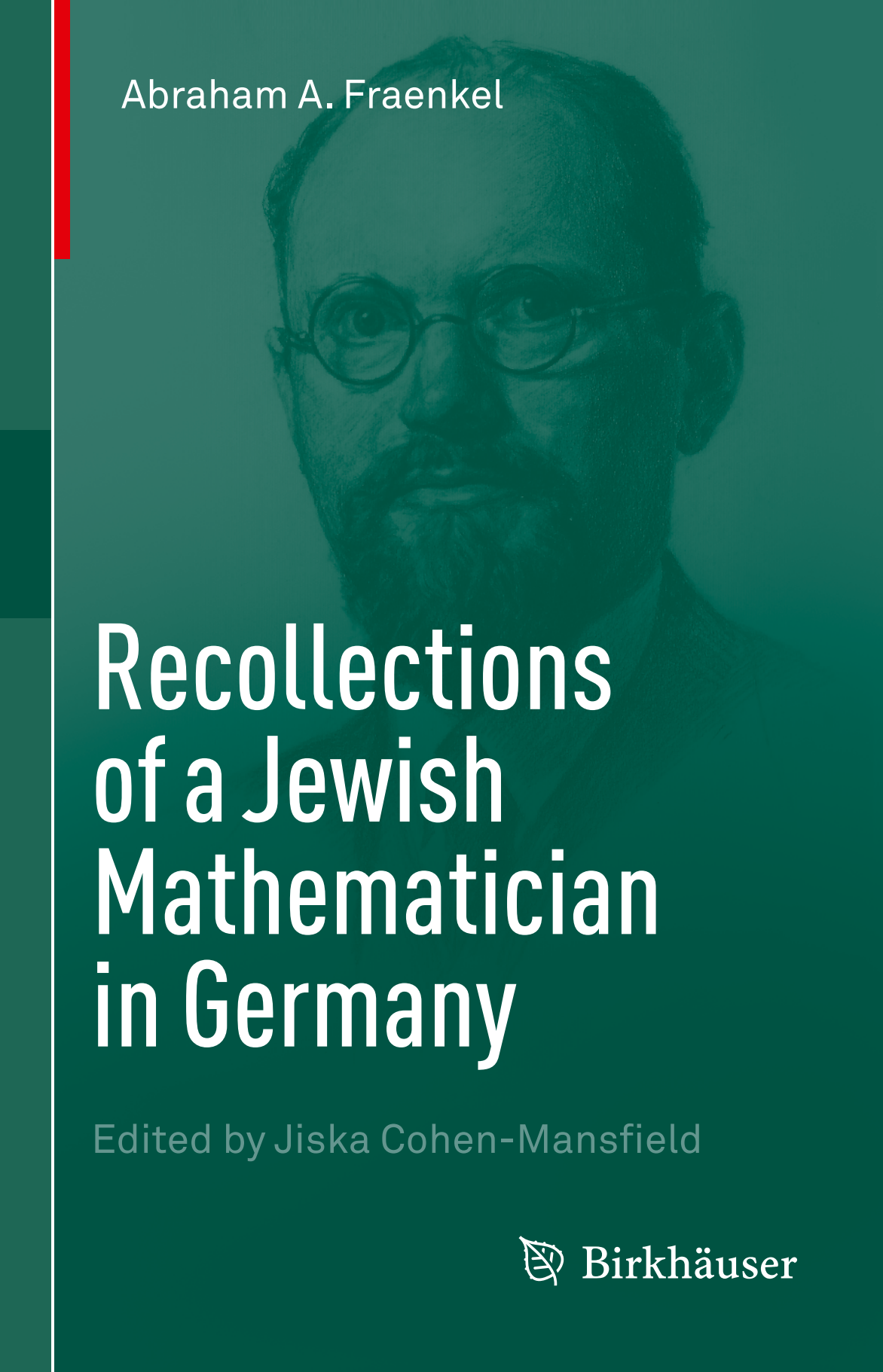

Abraham A. Fraenkel

Recollections of a Jewish Mathematician in Germany

Edited by Jiska Cohen-Mansfield

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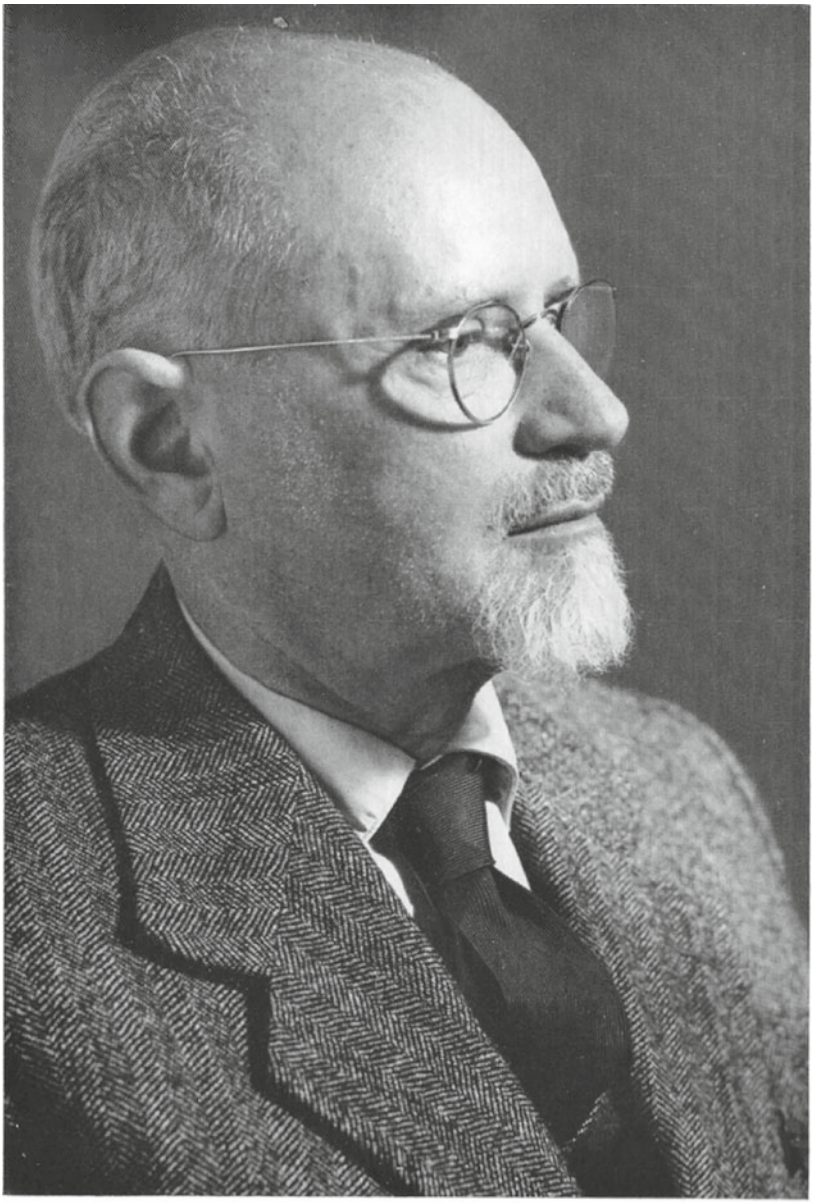

Abraham A. Fraenkel

This portrait was photographed by Alfred Bernheim, Jerusalem, Israel.

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Author
Abraham A. Fraenkel (1891–1965)
Jerusalem, Israel

Editor
Jiska Cohen-Mansfield
Jerusalem, Israel

Translated by Allison Brown

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well as family trees, have been added to this volume. This translation also includes new footnotes in which the translator and/or I clarified points in the text. These new footnotes are indicated with two asterisks in order to distinguish them from the original footnotes.

Jerusalem 2015 Jiska Cohen-Mansfield

Contents

Foreword to the 2016 English Edition

Professor Abraham HaLevi Fraenkel was my mathematical grandfather, that is to say the teacher of my teacher Professor Azriel Levy. The interaction I had with him was rather minimal: When I started my undergraduate studies at the Hebrew University of Jerusalem in 1962, Fraenkel was already emeritus and he passed away before I got my bachelor's degree. I did attend a course on the Jewish calendar that he taught as professor emeritus, but I was too shy to have any meaningful interaction with him. This being said, Fraenkel had a very profound impact on my career choices. In fact, he is indirectly responsible for my becoming a mathematician and especially for my interest in Set Theory.

I was 13 years old when, browsing through the books in a used bookstore in Netanya, the district town of the area in which I grew up, I ran into a series of four thin paperback volumes with the Hebrew title *Mavo Le-Mathematica* (Introduction to Mathematics). At that point, I hardly had any idea what "mathematics" was. My elementary school education in mathematics was limited to rather technical routine and boring arithmetical procedures. I started leafing through these books and randomly reading passages. Within a few minutes, it was clear to me, in spite of the fact that I did not fully understand what I was reading, that I was facing a building, very abstract but of sublime beauty. I fell in love with it and, right there on the spot, decided that the study of the architecture of that building would be the main theme of my life. I purchased the books and they still constitute the cornerstone of my mathematical library.

Fraenkel wrote that series of books over a period of several years from 1938 till 1945, but because of technical difficulties arising from the Second World War and Israel's War of Independence, their publication by the Hebrew University Press was delayed and spanned 15 years, from 1942 until 1957. Even from the perspective of more than half a century, I still consider this book to be by far the best of its kind. The volumes cover most of the important basic concepts of modern mathematics. Naturally, since it is Fraenkel, there is an excellent volume on the basics of Set Theory. (My fascination with the exposition in this volume is responsible for the fact that most of my mathematical work is in Set Theory). Besides its wide

coverage, it is unique in the connections it makes between developments in mathematics and the general culture, especially philosophy. Fraenkel was very careful in choosing the book's subjects, which he describes in full technical detail, but most importantly, there is a unique conceptual clarity to the basic notions and the motivations for their introduction.

In a deep sense, the conceptual clarity, the ability to see the essential features of the issues, and the succinct formulations that emerge from them are characteristic of Fraenkel's mathematical contributions. His two most famous contributions—the addition of the axiom of replacement to the standard axiom system of Set Theory, and the method of showing the independence of the axiom of choice from the Set Theory containing atoms—are not characterized by very elaborate technical developments, but are a major breakthrough in conceptual clarification, of finding the right formal explication of a notion that previously existed only intuitively and vaguely. His Ph.D. thesis on the p-adic numbers, essentially one of the first works introducing the important concept of "ring", has the same character of giving rigorous definition to concepts that were formulated only vaguely by his thesis supervisor, Kurt Hensel. The same clarity and the same ability to see the essential issues in a murky social or political situation are evident in Fraenkel's autobiography, even when he deals with domains that are very far from mathematical.

There are three major themes in this autobiography which have relevance to present-day contexts and can be examined from a contemporary perspective. The story of the book is mainly an account of an individual Jewish-German family, but the broader context is the last generation of Jewish-German society before its demise in the Second World War. More specifically, it is a description of the challenges facing an orthodox conservative minority that only a few generations prior to the described events had been enclosed within almost impenetrable physical and social boundaries, but which, when opportunities for integrating and operating in society at large opened up, at least formally, found itself faced with the tension between the desire to retain a traditional lifestyle and the pressure to integrate, or even assimilate, into general society. A variation on these challenges still exists today for traditional minorities in many developed countries. Similar dilemmas are currently faced by the Haredi community (strict orthodox Jews) in Israel, for instance, whether a core curriculum of general subjects ("*Limudei-liba*") should be included in the Haredi educational system.

The social and the cultural milieu of the several generations of Fraenkels described here is a very finely nuanced balance between strictly observant orthodoxy and a very active and eager participation in the general academic, political, and cultural environment. A very telling detail is the friendship that the strictly orthodox Fraenkel had with Christian theologians. Of course, this participation came to an abrupt end with the rise of the Nazi regime. An interesting twist is the critical attitude of major parts of the orthodox community to Fraenkel's Zionist involvement, including criticism of his acceptance of a position at the Hebrew University. The possibility of such a balance is definitely a lesson to be learned, in particular in present-day Israel.

The second theme, which requires more extensive commentary from a contemporary point of view, is the role played by Fraenkel's mathematical contributions in present-day Set Theory. Set Theory was created (or discovered—the right term is a matter of philosophical conviction) by George Cantor in the last decades of the nineteenth century. Cantor's version of Set Theory was to a large extent naive and intuitive. Especially naive was Cantor's unrestricted use of the principle of comprehension, according to which for every condition $\Phi(x)$ there exists the set that contains exactly those objects x which fulfil this condition. Towards the end of the nineteenth century, Cantor and others (e.g. Burali-Fori) became aware that such unrestricted use of the principle of comprehension leads to inconsistencies in Set Theory. The ultimate antinomy was discovered by Bertrand Russell (the famous "Russell's paradox") in 1901 when one applies the principle of comprehension to the property "x is a set that is not a member of itself". These antinomies created a crisis that threatened the very foundation of the theory and raised serious issues about the foundation of mathematics. A way out was suggested in 1908 by Zermelo who restricted the principle of comprehension by applying it only to the collection of elements that are already included in a given set. (The modified principle is called "the axiom of separation".) This requires also formulating a list of natural principles stipulating the existence of certain sets to which the restricted principle of comprehension can be applied.

Zermelo's system of principles (or "axioms") seemed to provide a sound basis for Set Theory, hopefully without including a contradiction. However, there was still a vagueness in Zermelo's formulation of the principle of separation (translated from the German):

> If the statement $\mathrm{U}(x)$ is definite for all members of the set M, then the set M has always a subset M_{U} which contains those members of M for which $\mathrm{U}(x)$ is true and only those members.[1]

The problem with this formulation is the vagueness of notions like "statement" or "definite".

Fraenkel in 1922 gave an explicit formulation of these concepts by specifying a class of functions defined by combinations of functions introduced by the other Zermelo axioms, and interpreting "definite statement" as statements of the form $f(x) \in g(x)$ or $f(x) \notin g(x)$ where the functions f, g are in the class. An equivalent formulation was given independently a year later by Skolem. Furthermore, Fraenkel realized that there was a natural axiom that was missing from Zermelo's axioms, which is implicitly used in many natural constructions. He formulated this axiom which he called "the axiom of replacement". The Zermelo axiom system as modified and augmented by Fraenkel became known as the Zermelo–Fraenkel axiom system or ZF. (When it includes the axiom of choice, it is denoted by ZFC.). ZFC very quickly became the canonical axiom system in which Set Theory is formalized. One reason for its almost universal acceptance was the fact that the

[1] Quoted in *Foundation of Set Theory* by A. Fraenkel, Y. Bar-Hillel, and A. Levy, 2nd edition, North Holland 1973, page 36.

axioms seemed to be very natural and such that they fit very well the intuitive notion of the concept of "set". Apparently, ZFC is a framework that is capable of including all of mathematics, and it seems to be free of the contradictions that afflicted the naive Cantorian Set Theory.

Set Theory, in the Zermelo–Fraenkel formulation, turned out to be a very fruitful mathematical theory, but many fundamental open problems persisted for a long time. The best-known open problem was the Continuum Hypothesis, which goes back to Cantor. In fact, it was the first problem on the list of central problems of mathematics presented by Hilbert at the Second International Congress of Mathematics in Paris in 1900. In 1938, Kurt Gödel proved that the Continuum Hypothesis could not be refuted in Set Theory. It still did not settle whether the Continuum Hypothesis was derivable in ZFC.

The next major breakthrough of Set Theory occurred in 1963 (two years before Fraenkel passed away) when Paul J. Cohen invented a technique (termed "the method of forcing") for constructing models of ZFC with varying properties. In particular, he constructed a model in which the Continuum Hypothesis failed. Hence, this central problem of Set Theory could not be decided on the basis of ZFC. The Continuum problem was not unique. Using the forcing method, many open problems and other mathematical fields, like Analysis, Algebra, and Topology, were shown to be undecided on the basis of ZFC.

In some sense, the phenomenon of independence was not unexpected. The famous theorem of Gödel (1931), known as the incompleteness theorem, claims that any mathematical theory rich enough to express some basic arithmetic facts (ZFC is definitely rich enough in this sense) is incomplete. Namely, it contains a statement which cannot be decided on the basis of the given theory. The surprise lay in the fact that the independent problems were not artificially constructed problems, but problems central to the field. This raised a deep philosophical problem: What is the meaning of independence? How do we settle the undecided problem? Is there a definite answer to the problem or does independence mean that the mathematical objects do not have an objective absolute existence? If, as many mathematicians believe, the mathematical objects represent an objective absolute reality of some kind, the way to get additional information about this reality and settle the independent problems is by studying extensions of ZFC.

Finding natural extensions of ZFC that would settle many of the undecided problems became a central research programme in contemporary Set Theory. These attempts assumed several directions like strong axioms of infinity (assuming that there are larger and larger sets), forcing axioms (intuitively meaning that a set whose existence can be imagined does exist) or canonical inner models. While there were several success stories where large classes of independent problems were settled by such extensions, none of the extensions of the Zermelo–Fraenkel Set Theory was able to gain the almost universal acceptance of the canonical natural assumptions about the universe of sets that ZFC did.

Fraenkel's second major contribution to Set Theory was also a source of many later developments. It concerns one of the axioms introduced by Zermelo: the axiom of choice (AC) in order to justify several natural constructions. Its

introduction as an axiom was initially controversial because of its non-constructive character. But now it is almost universally accepted. An interesting problem was whether the introduction of AC into the axiom system was not redundant, namely did it follow from the other axioms. Fraenkel in 1922 devised a method for showing the independence of the axiom. It did not apply to the accepted version of ZFC but a somewhat different version of Set Theory in which the universe of sets is constructed on the basis of an initial set of "atoms". Fraenkel's method started from a universe of sets with infinite sets of atoms and defined a subuniverse of sets that were invariant under some permutations of the atoms. The method was extended by the Polish mathematician Mostowski. The method is thus known as the Fraenkel–Mostowski method and was used to show the independence of many statements that follow from the axiom of choice. All these applications were for the version of ZFC with atoms.

Since the accepted version of ZFC is without atoms, this work left open the status of AC with respect to the atomless version of ZFC. The forcing method of Cohen once again came to the rescue. Part of Cohen's seminal work was to show the independence of AC with respect to the atomless ZFC. An interesting feature of Cohen's proof is that it has a clear affinity to the Fraenkel–Mostowski method. In fact, Cohen himself in his book about his method[2] points to this affinity. Cohen's work on the independence of AC was followed by a series of results which directly converted results obtained by the Fraenkel–Mostowski method, using forcing to get independent results also for the atomless version of ZFC.

The third theme that is worth commenting on is the very fundamental role of Fraenkel in the formation of the Hebrew University of Jerusalem. This aspect of Fraenkel's activity is represented only to a very limited extent in this volume because the time span described here concludes with Fraenkel's joining the Hebrew University and settling in Jerusalem. The next period, in which he made his most significant contributions to the university, was supposed to be covered in the subsequent volume of this autobiography. Sadly, Fraenkel passed away soon after the conclusion of this volume, so we do not have his version of his intensive activity in a leadership role at the young university.

Plans for establishing the Hebrew University of Jerusalem as part of the Zionist venture of recreating the Jewish commonwealth in Palestine had been taking shape since the beginning of the twentieth century. They become much more concrete after the First World War and the establishment of the British mandate for Palestine. The cornerstone for the campus was laid in 1918 and the opening ceremony took place in 1925. From the early stages, the character of the budding institution was a subject of great controversy. Many of the leaders of the university emphasized their ambition to create a research university of world caliber. (One needs to appreciate the boldness, or better the impertinence, of such a vision in view of the poor conditions and scant physical and academic resources available in Jerusalem in the 1920s.) Others, however, sought to establish a teaching institution whose

[2] *Set Theory and the Continuum Hypothesis,* P.J. Cohen, Benjamin 1966.

main mission would be to serve the needs of the small Jewish community in the country at the time, or offer the option of a college education for the many Jewish students from Eastern Europe, for whom admittance to their local universities was limited due to discriminatory policies. A similar dividing line existed between people who wanted the university to compete on the world scene and therefore to concentrate its research activity on subjects of universal interest as opposed to those who wanted the university to primarily serve the immediate needs of the country.

Prominent among the supporters of the first view was Albert Einstein, who had been involved since 1919 in the attempts to create the university. The other side can probably best be identified with Zionist leader Ze'ev Jabotinsky. The Zionist leader Chaim Weizmann can probably be described as the man in the middle. This controversy was bound up with many personal issues, severe criticism of the way Magnes was administering the university, and even external political preferences. Confrontations became acute in the late 1920s and the early 1930s. It reached the point where Einstein was so frustrated with Magnes that he withdrew his engagement with the university. This was a very serious blow to the project, since the involvement of a world academic leader like Einstein was one of the main assets of the young university.

This is precisely the period when Fraenkel joined the university. In retrospect, this move had a very deep impact on the future course of the institution. The greatest challenge that faced the new university was the recruitment of new faculty. This was an especially daunting task given the great chasm between the declared ambition of many of the founders of the university of creating a world-class research institution and the poor conditions in Jerusalem of the 1920s. The chances of attracting established academics to Jerusalem seemed rather slim. In fact, many members of the new faculty of the university in its initial years were rather young and junior. (It should be said, however, that many of them developed to become world-class scholars.) Fraenkel was one of very few who joined the young university, out of deep commitment to the Zionist idea, who had an established academic status as a senior scholar. In fact, one can claim that the external academic status of Fraenkel was much higher than that of any of the faculty of the university in the early 1930s. It is true that Fraenkel replaced another very distinguished mathematician, Edmond Landau of Götingen, who very seriously considered joining the university. He spent the winter term of 1928 in Jerusalem, but personal conflicts with the then chancellor of the university, Magnes, caused him to drop his plans to settle in Jerusalem. So when Fraenkel moved to Jerusalem in 1929, he was the best-known scholar among the small group of faculty members in Jerusalem. No other professor at the university besides Fraenkel had the same caliber of contacts with world-class academics like Einstein or Hilbert. If the ambition of the young university was to become a world-class institution, then recruiting academics of the class of Fraenkel was a necessary condition.

It is no coincidence that Fraenkel was Einstein's main contact in the university and his main source of information about it. When Einstein threatened to cut his involvement in the university, Fraenkel played a central role in convincing him to stay involved. Fraenkel's leadership position was enhanced substantially when,

following the pressure of Einstein and Weizmann, the governance structure of the university was reformed. The role of the chancellor was limited to external representation of the university (the title of the position was changed to "president"), and in parallel a position of academic head of the university, called "rector", was created. Fraenkel served as the second rector of the university and as such played a major role in shaping the academic future of the institution.

His position on the controversial issue of academic policy was clearly to put a great emphasis on the research excellence of the university and its international status, even if it meant a certain preference for issues and subjects which were of universal interest over issues of local interest. The fact that the Hebrew University in particular and Israeli science in general have an excellent world reputation is due to a large extent to this stance. This is especially evident in the field of pure mathematics, where Fraenkel's influence is directly felt.

This does not mean that Fraenkel was a typical ivory tower professor. He was committed to the role of the university in society at large. He played a very active role in the educational system. From 1933 until early 1950, Fraenkel chaired the university committee on high schools, which influenced the curricula and the pedagogic methodologies of the Hebrew high schools. He had a special interest in adult education. Besides his role as the chair of the committee on popular education, he spent enormous time and effort in delivering popular lectures on advanced topics in mathematics all over the country. As he mentions in this volume, he reached the remotest corners of the country, sometimes on horseback or donkey. His popular book *Mavo Le-Mathematica*, which, as mentioned above, had such a deep impact on my career, was written with the same goal of bridging the gap between pure research, which is the main role of the university, and general society. Fraenkel was a fine example of a balance between academic commitment to pure scholarship in the pursuit of knowledge for its own sake and social commitment.

Professor Abraham HaLevi Fraenkel was positioned at critical junctures of vastly different domains: The last decades of the German-Jewish community before its demise, the establishment of a firm foundation for Set Theory and Mathematics, and the formation of the new Jewish commonwealth in Israel, especially its academic and scientific infrastructure. In all these domains, he combined a conceptual clarity, deep knowledge of the relevant issues, and ideological commitment. This autobiography is a fascinating and illuminating testimony of a unique individual who was both an important player in and a keen observer of these different junctures.

The Hebrew University
Jerusalem, Israel
February 2015

Menachem Magidor

Foreword to the 1967 German Edition

Professor Abraham (Adolf) Fraenkel did not live to see the publication of this book. Early in the morning of October 15, 1965, the spry 74-year-old went for his daily swim in Jerusalem, where he had been living for almost four decades. A few hours later his heart stopped.

He was ready. He often spoke about death during his last few months. He believed he had completed his lifework and that it was well done. His autobiography, the first part of which is this book, was meant to be the "final chapter" of his complete works. He continued his scientific work until the end: six months before his death, he gave me his revised section for the forthcoming second edition of our joint book *Foundations of Set Theory*.[3] However, he was well aware that, in terms of his creativity, he was long past his prime. As he often mentioned in all seriousness, but with good spirit, mathematicians generally accomplish their greatest work before they turn 30. Indeed, he made his most outstanding contribution to mathematics very early on, at the age of 28, with his fundamental book, *Einleitung in die Mengenlehre* (Introduction to Set Theory), published in 1919.

While mathematical work and research were important to him, the scope of his life and work was much broader. Raised in an Orthodox household, deeply steeped in Jewish tradition, and a Zionist since adolescence, he also contributed towards creating a healthy, viable basis for a Jewish homeland, first in Palestine[4] and, later, in the State of Israel. He considered a good education to be the essential prerequisite for this, not only for youth but also for adults, in schools and at universities, not only in the cities but also in the countryside, and in the most remote kibbutz.

[3]The second (revised) edition, by A. A. Fraenkel, Y. Bar-Hillel, and A. Levy, *Foundations of Set Theory* (*Studies in Logic and the Foundations of Mathematics*, vol. 67) (Amsterdam: North-Holland/Elsevier, 1966). There was also a 1973 edition with the collaboration of Dirk van Dalen.**

[4]During the Ottoman Empire and the British Mandate, pre-state Israel was known as Palestine or Palestine/Land of Israel. For the sake of clarity, this book will use the formulation "Palestine/Land of Israel" to denote the region, and "*yishuv*" (Hebrew "settlement") to refer to the Jewish community there from 1860 to 1948, prior to the founding of the State of Israel.**

With unlimited willpower and iron self-discipline, he followed his schedule down to the last detail. This allowed him not only to carry out his teaching and research activities, but also to travel all over Israel, even to the most distant parts, from Dan to Eilat, to inspect middle and upper secondary schools and draft curricula, as well as to give hundreds of lectures. In addition, he also went swimming, hiking, and mountain climbing.

Until he became professor emeritus, Professor Fraenkel, together with Professor Michael Fekete, directed the Institute of Mathematics at the Hebrew University of Jerusalem. The international renown of the Hebrew University in the areas of mathematical logic, abstract set theory, and the foundations of mathematics can undoubtedly be largely attributed to his efforts. His students, Haim Gaifmann, Azriel Levy, Michael Rabin, Eliyahu Shamir, and Abraham Robinson, are among the best in their fields, and, with the exception of Robinson, all teach at the Hebrew University.

In 1938, Professor Fraenkel became the second rector of the Hebrew University, an office he held until the end of 1940. Afterwards, he spent many years as a member of the Administrative Council, chaired numerous commissions, and made his analytical skills and extensive expertise available in many aspects of academic life and research. His concerns were by no means limited to purely academic matters. For almost two decades, he chaired the Hebrew University athletic commission, organizing and, until an advanced age, also often participating in hikes, athletic events, and races.

As part of his great, sustained commitment to education, Professor Fraenkel founded, with others, the Center for Adult Education of the Hebrew University, where he served as director for many years. His intense commitment was also manifest in his unremitting readiness to give popular lectures to various groups on the foundations of mathematics, modern physics, and the Jewish calendar. He spared no effort to reach remote locations, on occasion even riding a donkey on the last leg of an arduous journey.

These activities, by no means independent of one another, were part of his lifelong effort to put into practice Orthodox German Jewry's *Torah im Derekh Eretz* (combining Torah [Jewish religious principles] with proper behavior in civic life). He did this himself and hoped thus to serve as a model to others.

While he was a profoundly religious man, Professor Fraenkel also showed true tolerance. Many of his students, including myself, were not religious, but this never affected his attitude towards us. He firmly believed that religion and science were two separate domains that should not be intertwined. The physical worldview was to be based on purely scientific findings. He felt that it was not rational to try to convert non-believers into believers. Attempts to use political power for religious coercion, as by Israel's National Religious Party, were repugnant to him. This is one of the reasons why he never joined that party.

His dream was to see the entire Jewish people united and unified in Israel. For this reason, he often prayed in Yemenite and other Oriental[5] synagogues, where he

[5]Oriental Jews are Jews of North African and Middle Eastern origin.**

was welcomed and honored. Professor Fraenkel greatly enjoyed taking guests on a personal Friday evening synagogue tour whenever he could. With skill and knowledge, he would explain the particularities of various customs and traditions, down to the smallest detail. Those tours gave many non-Jews, as well as quite a number of Jews, their first objective and vivid picture of these "exotic" congregations. Afterwards, they would be invited to a Sabbath dinner at his home, featuring an Israeli version of Bavarian Jewish custom.

Professor Fraenkel's most lasting impression was undoubtedly left on his own students. In describing what he meant to them, the word "teacher" is entirely inadequate. As an inspector of middle schools and academic secondary schools, he was always on the lookout for mathematically gifted children. He discovered several child prodigies, whom he supported as best he could. To spur interest in mathematics, he wrote a book, *Introduction to Mathematics*, in Hebrew, in which his clear and exciting presentation gave many pupils their first insights into the field. He also contributed significantly to developing an appropriate Hebrew terminology for higher mathematics.

As tolerant as Professor Fraenkel was in general, there were two traits that he could not and would not abide among his students—and not only them: a lack of punctuality and incompetence. Few things aggravated him more than students attempting to cloak incompetence and insufficient understanding in rhetoric, platitudes, or vague formulations. He would take such students to task in a way they would never forget.

Gifted students were assured of his personal favour. The distinguished accomplishments of so many of them were clearly not only due to their scientific aptitude but equally to their teacher's interest and consistent support.

To illustrate this, let me share a personal experience. In the spring of 1937, at one of Professor Fraenkel's seminars on the foundations of mathematics, I gave a talk on logical and semantic antinomies. He liked it and, although he was the veritable master of basic research in mathematics, immediately suggested that I, a 22-year-old novice at the time, expand my talk into a joint project with him, which was indeed published in a French journal two years later.[6] The translator, a French-Jewish mathematician, was unfortunately killed a short time later by the Nazis. That was my first international publication.

What attracted me to Professor Fraenkel was not only his personal interest in my growth and progress and his harsh rejection of all non-scientific metaphysics but also his own farsightedness. He was never just a mathematician. While he paid rigorous attention to the philosophical and logical foundations of mathematics, he did not confine himself to them. He tended towards Platonism as a philosophy of mathematics, namely that mathematical entities fully exist as abstract objects, even at times when this view was not very popular. However, he also gave the best and clearest interpretation of intuitionist views, which he personally did not support.

[6]Fraenkel, A. A., and J. Bar-Hillel (1939). "Le Problème des antinomies et ses développements récents." *Revue de Métaphysique et de Morale* 46:225–242.**

Although I did not share Professor Fraenkel's Platonism, favoring instead an ontology-free philosophy of mathematics, this did not mar our student–teacher relationship, or our later collaboration, in the least. He was fully aware that it was impossible to prove Platonism to be the only tenable mathematical philosophy. This view appealed to him personally, and he managed to weather the various foundational crises rather well.

Professor Fraenkel's knowledge of philosophy went far beyond the philosophy of mathematics. During the 1920s, he kept in personal contact with many neo-Kantians and phenomenologists in Germany. However, he never really warmed up to their approaches, probably because their statements about mathematics seemed too unclear and irresponsible.

Thus, Professor Abraham Fraenkel will be remembered as a great mathematician, for whom nothing human was foreign; a strict teacher who did not tolerate superficial knowledge, but warmly supported genuine talent; a *Talmid Chacham* (Jewish scholar), a true student of sages, always strict with himself in religious matters, but tolerant towards others; a Zionist, who held the education of the youth in Israel and the unity of the Jewish people close to his heart, every day of his life; and, while superficially austere and pedantic, this did not conceal his warm personality. How this personality was shaped, and then changed when as a German Jew he returned to Israel, making his way from Munich to Jerusalem, is depicted in the autobiography he left us.

The Hebrew University
Jerusalem, Israel
April 1967

Y. Bar-Hillel

Chapter 1
My Ancestors

Most people have eight great-grandparents, namely, the parents of their four grandparents. While that is the case for my parents, wife, children, and grandchildren, my siblings and I have only six great-grandparents. My father's father, Wilhelm Fraenkel,[1] and my mother's mother, Rosa Neuburger, née Fraenkel, were siblings, so their parents, Abraham and Nanette Fraenkel, count twice. Indeed, the oil-painting portraits I have of them [see below] show a remarkable resemblance to two of my own children.

So, in addition to Abraham Fraenkel, I had only two other great-grandfathers: Benjamin Hirsch Auerbach, father of Rahel Fraenkel, my paternal grandmother;

[1] See the Fraenkel family tree p. 204.**

A.A. Fraenkel, *Recollections of a Jewish Mathematician in Germany*,
DOI 10.1007/978-3-319-30847-0_1

and Joël Neuburger, father of Isidor Neuburger, my maternal grandfather. I have nothing notable to report about Joël Neuburger and his family, who lived in Fürth, Bavaria. Benjamin Hirsch (Zvi Benjamin) Auerbach, born on June 21, 1808, in Neuwied, however, was all the more important. The eldest of 16 children of Abraham Auerbach (1763–1845) and his wife Ester Rebecca, née Oppenheim (1785–1864), he received his Ph.D. in philosophy and Semitic languages at the University of Marburg an der Lahn. In 1837, he married Lea Fraenkel (b. Witzenhausen 1814, d. Halberstadt 1884), daughter of Daubchen and Isaak Eisenmann Bodenheim-Fraenkel. Benjamin Hirsch Auerbach (d. 1872) and his wife Lea had seven children.

In the nineteenth century, Benjamin Hirsch Auerbach, the rabbi in Halberstadt, was one of Germany's leading rabbis. Together with Samson Raphael Hirsch and Azriel Hildesheimer, he was among the founders of "neo-Orthodoxy." He became renowned for several writings displaying his profound Talmudic scholarship, especially *Nachal Eshkol*, a commentary on Rabbi Abraham ben Isaac of Narbonne's *Sefer ha-Eshkol*. There is no need to write about him and his numerous descendants, since their monumental family tree is available in *The Auerbach Family*,[2] published by Siegfried M. Auerbach.

I have three authentic sources of information on my great-grandparents who were most significant from a genetic point of view, Abraham Fraenkel (1792–1858)[3] and his wife Perl Nanette, née Neubauer (1808–1881). The first source is Abraham Fraenkel's Hebrew entries in the first volume (*Bereshit,* Genesis) of the splendidly printed *Chumash Derekh Selulah* (which includes a Judeo-German—i.e., Western Yiddish—translation and commentaries), which was published between 1801 and 1803 in Fürth. These entries, covering the period from 1827 to 1843, refer to the birth of his seven children. The second source is his last will and testament, with the heading "Memorandum of Abraham Fraenkel (sic!), addressed during his lifetime to his four beloved children,[4] Sigmund, Wolf, Jacob, and Rosa, written in Munich on February 3, 1857." It includes 40 folio pages in German, followed by the remark, "I would like to write a conclusion to my memorandum, which is intended solely as a recapitulation of all the aforementioned, in the Jewish national script, which I praised above, and is very precious and dear to me." This conclusion consists of nine pages in Judeo-German. Both parts contain many Biblical verses as well as quotations from the *Aggadah*, all magnificently written in square Hebrew script with *nikkud* (vowels). The entire will is in excellent condition and includes an admonition, to all his children as a group and to

[2]London: Perry Press, 1957.

[3]There are no clear-cut conventions for spelling the name: Fränkel, Fraenkel, [and even Fraenckel, in a few cases]. By and large, though not consistently, the first one is considered the earlier version, roughly up to my parents' generation. For the sake of clarity, Fraenkel is the form used in this book.

[4]Three of his seven children died young.**

each one individually, to preserve the Jewish tradition and adhere to religious law. The last page states the following[5]:

תקריבו אלי כלכם כאיש אחד, לא במעשה אלא ברוחי ובמחשבתי, ליתן לכם את
ברכתי, ברכת אברהם אביכם, לכם ולזרעיכם אחריכם, כולן על הברכה תעמדון,
והמקום יהיה עד ישקיף וירא ה' מן השמים ויענה ויאמר אמן, אמן. אמן על ההווה
ואמן על העתיד. . . ואסיים במה שהתחלתי, ליתן שבח והודיה לאל, יוצרי, עושי
וקוני. על כל הטובות והחסדים אשר עשית עמדי מהיום היותי על האדמה עד היום
הזה. בידך אפקיד רוחי פדיתה אותי ה' אל אמת, בטוב אלין אקיץ ברחמים טובים. אמן
יום ג' טית שבט תרי"ז פה עיר מינכען הבירה. נאם הקטן אברהם פרענקעל הלוי.

Attached to the will is a draft text for his gravestone, which reads as follows:

פ"נ
העפר דאברהם פרענקעל הלוי מקורו מק"ק פיורדא
תנצ"בה[6]
Abraham Fraenkel

born in Fürth on November 27, 1792, and died on [. . .], 18. . ., was a teacher, *shochet* [ritual slaughterer], and cantor in Hofheim, near Hattersheim, in the duchy of Nassau, and then cantor, secretary, and, later, member of the administration, executive committee, and management of three charitable organizations in the Jewish community, and also a wholesaler in the royal capital of

MUNICH
God putteth down the high, and lifteth up the low.
Peace to his ashes
(by his own decree, in his lifetime)

The third source is my mother, who at 97 is mentally still fully alert, here in Jerusalem in June 1965,[7] who told me some things about Nanette Fraenkel, her grandmother. For the first 13 years of my mother's life, starting in 1868, they lived together in the home of her mother, Rosa Neuburger (Nanette's daughter).[8]

[5]"Come all of you, approach me together as one person, not in deed, but in my spirit and in my thought. To give you my blessing, the blessing of Abraham your father, for you and your offspring after you. All of you shall stand by the blessing. God will be a witness and shall look down from heaven and respond and say Amen, Amen. Amen for the present and Amen for the future . . . and I shall end as I have started, giving praise and gratitude unto God, my creator and maker, for all the good and mercy Thou hast done unto me from when I came upon the earth until this day. Into Thine hand I commit my spirit, Thou hast redeemed me, O Lord, God of truth. May I sleep with goodness and awaken with mercy. Amen. Completed on Tuesday, the ninth day of the Hebrew month of Shevat, 5,617 (1857) in the capital city of Munich. The word of the insignificant, Abraham HaLevi Fraenkel."**

[6]Here is buried the dust of Abraham HaLevi Fraenkel, of the holy community of Fürth, May his soul be bound in the bonds of life.**

[7]Charlotte (Chaya Sara) Fraenkel, née Neuburger, died on October 24, 1965.**

[8]In addition to these authentic sources, there are also some less reliable data from people who did genealogical research on our family.

Rabbi Sigbert Neufeld asserted in an essay "*Vom Ries gauüber Wien nach Elbing*," ("From Riesgau to Vienna to Elbing"), in *Das Neue Israel* 14, (Zurich) 1962, that our Fraenkel family came from the Jewish community in Riesgau, on the border between Franconia and Swabia, i.e., from Oettingen, Wallerstein, Spielberg, Harburg, and other towns. According to the same source,

Abraham Fraenkel came from Fürth, a long-established community near Nuremberg. Indeed, Munich's Jewish community, not founded until the early nineteenth century, recruited their most esteemed early members from Fürth. Suffice it to mention the family of Seligmann Feuchtwanger (1786–1852), who was born and died in Fürth. His four sons, Jacob Loew, Elkan, Moritz, and David, settled in Munich, and, together with Abraham Fraenkel and his descendants, formed the core of the Orthodox Jewish community.[9] For the services in their congregation, the printed edition of the Fürth *minhag* (custom) book was authoritative. Jacob Loew (1821–1890) was cofounder of the still-existing J. L. Feuchtwanger Bank.[10]

Characteristic of the legal situation of Jews in Munich in the first third of the nineteenth century is the fact that Abraham and Nanette Fraenkel's wedding in 1826 took place outside Munich, specifically in Kriegshaber, near Augsburg. This was because of the Bavarian edict of 1813, which decreed that "the number of Jewish families in communities where they presently reside should not be increased, but . . . gradually decreased." This edict was not permanently abolished until 1861.

Abraham Fraenkel's childhood in Fürth, Hofheim, and, then, in Munich, must have been very modest and subject to religious and other challenges. At the beginning of his last will and testament, he thanked God, "who lifted me from the lowest position to highly honored positions; from servant of the congregation . . . to a congregational leader . . ., from a recipient of donations to a wealthy distributor of alms, and, thus, from a tolerated stranger to a citizen and wholesaler in this city." His position as *chazan* (Jewish prayer leader) is verified by his mahzor (prayer book for the holidays), which I possess. It is an excellent, first edition (from 1800) of Wolf Heidenheim of Rödelheim's mahzor, which later circulated in many editions.

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the family assumed the name Fraenkel, derived from their place of origin Franconia (Franken, in German), while other families took the names Riess or Riesser, Oettingen, and Wallerstein, for similar reasons. David Fraenkel, who served as a rabbi in Dessau and Berlin and Moses Mendelssohn's teacher, and Councilor of Commerce Jonas Fraenckel, the benefactor of Breslau's Jewish Theological Seminary, are assumed to have been among them.

In addition, Dr. Siegfried Asher of Haifa and Dr. Yomtov Bato of Tel Aviv were also interested in the background of Abraham Fraenkel of Fürth-Munich, partly because of his Levitical descent. Traces led to Vienna, specifically to Rabbi Israel ben Koppel Halevi Fraenkel, born there around 1640. After the expulsion of Jews from Vienna, in 1670, he was active in several places, with his last post as the district rabbi in Würzburg, which he held until his death in 1700. The name Koppel also came up in our family, as Abraham Fraenkel's brother and a son had this name. The essay by Dr. Yomtov Bato is particularly insightful: "Koppel Fraenkel und seine Nachfahren. Die Schicksale einer deutsch-jüdischen Familie im Wandel von mehr als drei Jahrhunderten" ("Koppel Fraenkel and his descendants: The fate of a German-Jewish family over the course of more than three centuries"), *Israelitisches Wochenblatt* (Zurich), July 3, 1964.

[9]See Nathan Drori, Susan Edel et al., eds., *The Feuchtwanger Family: The Descendants of Seligmann and Fanny Feuchtwanger* (Tel Aviv: Feuchtwanger Family committee, 2009), printed by DoroTree Technologies (Jerusalem).**

[10]The J. L. Feuchtwanger bank, established in 1857, was liquidated with the rise of the Nazis, in 1937. In 1936, it was reestablished in Israel as the I. L. Feuchtwanger bank, which closed in 1967. Additional details can be found in Drori and Edel, ibid., pp. xxxii–xliii.**

In the *Yom Kippur* (Day of Atonement) volume, Abraham Fraenkel not only wrote his own name in German and Hebrew, but also the titles of the *Selichot*[11] selected for the *Shacharit*,[12] *Musaf*,[13] and *Mincha*[14] services, as well as the text of ויאתיו (*v'ye'etayu*), a *piyyut* (a liturgical song or poem). According to the mahzor, this was recited only in the *Musaf* for *Rosh Hashanah* (the Jewish New Year), but Cantor Abraham Fraenkel evidently also included it on *Yom Kippur*, as was the practice in Eastern Europe, and as far west as Berlin.

When Abraham Fraenkel was about 35, he married the young Nanette Neubauer. He must have already accomplished much in life, since his bride came from a respected and wealthy family and was well versed not only in Jewish knowledge, as was common for girls, but also in French and English. The marriage produced seven children, three of whom died at a very young age. The other four were still alive at the turn of the twentieth century: the firstborn Yitzhak Seckel Sigmund, born on August 16, 1827; Zechariah Benjamin Wolf (later Wilhelm), born on December 20, 1830, who was my grandfather; Koppel Jacob, born on October 9, 1833; and Rosl Rosalie Rosa, born on February 26, 1843, who was my grandmother and named after Abraham Fraenkel's mother. After the entry (in his *Chumash Derekh Selulah* mentioned above) for his firstborn, Abraham Fraenkel noted, וימל אברהם את יצחק בנו.[15]

My memories of my father's parents are all the more vivid, since, until my grandfather's death, which was most of my time in Munich, we lived together in the same building at 30 Klenzestrasse, close to Gärtnerplatz and the Gärtner Theater. The very old-fashioned house that belonged to my grandfather included two four-room apartments on each of the first two upper floors facing the street. The separating walls had been removed to convert them into two larger, but extremely uncomfortable, apartments for my grandparents and my parents. For two decades, with five children and attending servants, my mother suffered the ordeal of living in that apartment, with much hardship, but without complaint. The central building housed the offices of the "A. Fraenkel, Wholesale Wool Business," founded by Abraham Fraenkel. The warehouse for storing the bales of wool was across the courtyard. For us children, the most important space was the courtyard between the central building and the warehouse. It was spacious and made beautiful by chestnut trees. My siblings and I, and often our cousins too, played there in the summer, and built snow mountains and tunnels there in the winter.

[11] Repentance prayers and poems for *Yom Kippur* and the preceding weeks, as well as for other fast days.**

[12] *Shacharit* is the morning prayer service.**

[13] *Musaf* is the additional service, recited on the Sabbath, Rosh Chodesh (the first day of a new month), and on holidays.**

[14] *Mincha*: afternoon prayer service.**

[15] "And Abraham circumcised Yitzhak," Genesis 21:4, which refers to the entries in the first source mentioned on p. 2.**

When he married my grandmother Rahel Auerbach (1839–1915), my grandfather Wolf changed his name to Wilhelm Fraenkel (1830–1907). According to the Frankfurt authorities' marriage register, this took place on December 10, 1858. The marriage produced seven children, all born in Munich. Like many children at that time, their oldest son Adolf Abraham (1859–1868) died young of diphtheria. The other children were: Sigmund Aviezri[16] (1860–1925), who was my father; Heinrich (1862–1940); Toni (1865–1922), who married Abraham Auerbach of Cologne; Emil (1867–1942), the only one who was always in good spirits and was sensitive to all things poetic; Emma (1868–1928), who married Leo Mainz of Frankfurt am Main; and Berta (1875–1961), who married the physician Dr. August Feuchtwanger of Munich. The three siblings who still lived in Munich in 1933 emigrated to Haifa and Jerusalem between 1935 and 1939. Additional information about them, their children, and their grandchildren can be found in the aforementioned Auerbach family tree.

Rahel (third from left, seated) and Wilhelm (center, seated) Fraenkel with their children and children-in-law

Wilhelm and Rahel came from rather different backgrounds. Although his religious stance was Orthodox, it was far from the strict, militant stance Rahel brought from her parental home, which was absolutely unknown in Munich at that time, although Jacob Loew Feuchtwanger was not averse to it. They also differed

[16]The name Aviezri appears again among his descendants. It is the name of the grandfather (who died in 1767), and not the father, of his grandfather B. H. Auerbach. This can be explained by the fact that B. H. Auerbach's father was called Abraham (1763–1845), the name chosen by the paternal side for Sigmund's older brother, who died early [see the Fraenkel family tree in the appendix].