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# Leveraged Exchange-Traded **Funds** Price Dynamics and Options Valuation



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Tim Leung • Marco Santoli

## Leveraged Exchange-Traded Funds

Price Dynamics and Options Valuation



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To Kelly and Vivian

### Preface

Leveraged exchange-traded funds (LETFs) are relatively new financial products liquidly traded on major exchanges. They have gained popularity with a rapidly growing aggregate assets under management (AUM) in recent years. Furthermore, there are now derivatives written based on LETFs. This book aims to provide an overview of the major characteristics of LETFs, examine their price dynamics, and analyze the mathematical problems that arise from trading LETFs and pricing options written on these funds.

When writing this book, we aim to make it useful not only for graduate and advanced undergraduate students but also for researchers interested in financial engineering, as well as practitioners who specialize in trading leveraged or non-leveraged ETFs and related derivatives.

In the first part of the book, we assume very little background in probability and statistics in our discussion of the price dynamics of LETFs. Nevertheless, new insights and trading strategies are discussed with mathematical justification and illustrated with a host of examples using empirical data. Our emphasis is on the risk analyses of LETFs and associated trading strategies. The second part focuses on the risk measurement for LETFs, and we provide a number of formulas for instant implementation. In the final part, we present the analytical and empirical studies on the pricing and returns of options written on LETFs. Our main objective is to examine a consistent pricing approach applied to all LETFs. This allows us to identify any price discrepancies across the LETF options markets. As the market of ETFs continues to grow in terms of market capitalization and product diversity, there are plenty of new problems for future research. In the final chapter, we point out a number of new directions. We would like to express our gratitude to several people who have helped make this book project possible. Parts of the book are based on the thesis of my Ph.D. student and coauthor, Marco Santoli, who has been funded by the Department of Industrial Engineering and Operations Research at Columbia University throughout his doctoral study. Various chapters have been used in several courses at Columbia University and have benefited from students' feedback and questions. Several Columbia Ph.D. and master's students, who participated in exploratory projects on ETFs, have also helped shape the materials.

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New York City, NY, USA Thanksgiving Day, 2015 Tim Leung

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