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Modern Portfolio Theory

Foundation, Analysis, and New Developments

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JACK CLARK FRANCIS
DONGCHEOL KIM

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To Harry Markowitz

Preface

Harry Markowitz introduced portfolio theory in a 1952 *Journal of Finance* article. That article has been widely referenced, frequently reprinted, and it was cited when Markowitz was awarded the Nobel prize. A few years later, professors James Tobin (Yale) and William Sharpe (Stanford) made important extensions to Markowitz's original model that both won Nobel prizes. Today, portfolio theory has grown to impact the finance and economics classrooms of universities, portfolio managers, financial service organizations, and many individual investors.

The extraordinary intellectual developments of Markowitz, Tobin, and Sharpe were furthered by increasing college enrollments, an explosive growth of information and computing technology, the global expansion of investment activity, and by decades of contributions by many different insightful authors. This book traces the valuable contributions by many different authors. Contributions involving utility analysis, single and multiple index models, non-normal probability distributions, higher-order statistical moments, investment decision criteria that go beyond the mean and variance framework, value at risk (VaR) models, Monte Carlo simulation models, the zero-beta portfolio, continuous time models, market timing, mutual fund portfolios, several portfolio performance evaluation models, arbitrage pricing theory (APT), and alternative trading systems (ATS) are reviewed and evaluated. The interactions between these diverse schools of thought are pulled together to form *Modern Portfolio Theory*.

Three editions of a book titled *Portfolio Analysis*, coauthored by professor Francis in 1971, 1979, and 1986, laid the foundation for *Modern Portfolio Theory*. The last edition of *Portfolio Analysis* included so many diverse topics we decided to name this latest book *Modern Portfolio Theory* (MPT) to reflect the continually growing number of additions to the book and the differing nature of some of the extensions to Markowitz's original portfolio theory.

MPT reports all important offshoots and recent developments to Markowitz portfolio theory. The book furnishes a concise review of portfolio theory and the derivative literature that can provide busy finance professionals a fast and efficient way to stay current on the theoretical developments in their field. In no particular order, we expect to sell this book to:

- Mutual fund executives working at the approximately 8,000 mutual funds in the United States, plus other mutual funds throughout the rest of the world.
- Security analysts working at the mutual funds in the United States, plus others throughout the rest of the world.
- Financial engineers working at the approximately 8,000 hedge funds in the United States, plus those at other hedge funds throughout the rest of the world.
- Investment researchers working at the few thousand pension funds in the United States.
- Portfolio managers working at mutual funds, pensions, hedge funds, trust funds in the trust departments of commercial banks, and other commingled portfolios.
- Sales-oriented financial analysts working at brokerage houses like Goldman-Sachs, Merrill Lynch, and hundreds of banks throughout the rest of the world.
- Professional organizations that run educational programs in finance; for instance, in the investments arena are the Chartered Financial Analysts (CFA) and Individual Investors programs. In the risk management field are Public Risk Management Association (PRIMA) and Global Association of Risk Professionals (GARP).

- Financial executives at the largest foundations throughout the rest of the world; for example, the Ford Foundation and the Rockefeller Foundation.
- Financial executives managing the investment portfolios at endowment funds at colleges, museums, and libraries in the United States (such as the TIAA-CREF and the CommonFund), plus other endowments throughout the rest of the world.
- Financial executives and analysts at multibillion-dollar sovereign wealth funds (SWFs) around the world.

Business schools can also use the book for a one- or two-semester investments course taught at the undergraduate, MS, MBA, or PhD level; or to supplement another book in an investments course. The book can also be used in a course about the economics of choice or uncertainty, taught in economics departments.

We worked to reduce the level of the math in most of the book's chapters without reducing the level of the content in the entire book. This was accomplished by putting the advanced math in a few designated highly mathematical chapters (like 7, 9, and 15), end-of-chapter mathematical appendixes, and footnotes, instead of scattering it throughout every chapter of the book. We did this to make the book more readily available to those who wish to avoid math. We inserted Chapter 4, titled Graphical Portfolio Analysis, so the book could be used by newcomers to portfolio theory. And, as mentioned above, the advanced mathematics can be avoided by skipping Chapters 7, 9, and 15 and the end-of-chapter appendixes. Skipping these more formal segments will not harm the flow of the book's logic.

The coauthors created several Excel spreadsheets that compute Markowitz efficient frontiers under various assumptions and circumstances. This user-friendly software is available at www.wiley.com/go/francis; it may be freely downloaded to the user's computer, used on the user's computer, and retained by the user. In addition, resources for professors can be found on Wiley's Higher Education website.

Chapter 1

Introduction

The number of alternative investments is overwhelming. Thousands of stocks, thousands of bonds, and many other alternatives are worthy of consideration. The purpose of this book is to simplify the investor's choices by treating the countably infinite number of stocks, bonds, and other individual assets as components of portfolios. Portfolios are the objects of choice. The individual assets that go into a portfolio are inputs, but they are not the objects of choice on which an investor should focus. The investor should focus on the best possible portfolio that can be created.

Portfolio theory is not as revolutionary as it might seem. A portfolio is simply a list of assets. But managing a portfolio requires skills.

1.1 The Portfolio Management Process

The portfolio management process is executed in steps.

Step 1. Security analysis focuses on the probability distributions of returns from the various investment candidates (such as individual stocks and bonds).

Step 2. Portfolio analysis is the phase of portfolio management that delineates the optimum portfolio possibilities that can be constructed from the available investment opportunities.

Step 3. Portfolio selection deals with selecting the single best portfolio from the menu of desirable portfolios.

These three phases are discussed briefly below.

1.2 The Security Analyst's Job

Part of the security analyst's job is to forecast. The security analyst need not forecast a security's returns for many periods into the future. The forecaster only needs to forecast security returns that are plausible for one period into the future. The length of this one-period forecasting horizon can vary within wide limits. It should not be a short-run period (such as an hour or a day), because portfolio analysis is not designed to analyze speculative trading. The forecasting horizon cannot be very long either, because it is not realistic to assume the security analyst is prescient. Between one month and several years, the portfolio manager can select any planning horizon that fits comfortably within the portfolio owner's holding period (investment horizon).

The security analyst's forecast should be in terms of the holding period rate of return, denoted r_1. For instance, for a share of common or preferred stock, r_1 is computed as follows.

[1.1]

$$\begin{pmatrix}\text{One} - \text{period}\\ \text{rate of return}\end{pmatrix} = \frac{\begin{pmatrix}\text{Price change during}\\ \text{the holding period}\end{pmatrix} + \begin{pmatrix}\text{Cash dividends paid during}\\ \text{the holding period, if any}\end{pmatrix}}{\begin{pmatrix}\text{Purchase price at the beginning}\\ \text{of the holding period}\end{pmatrix}}$$

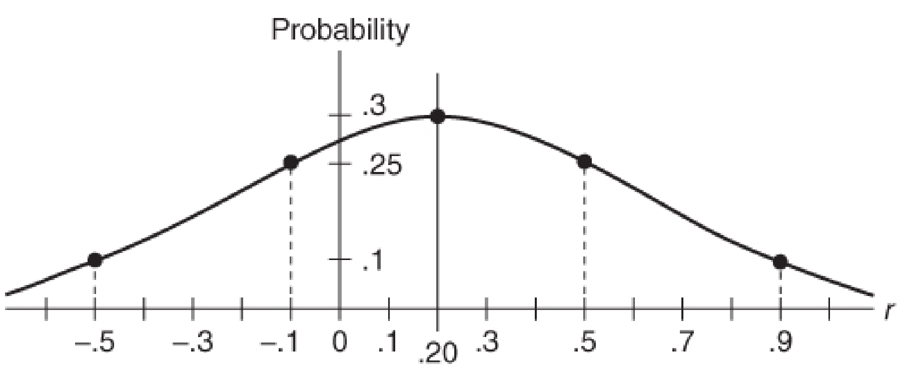
$$\text{or} \quad r_1 = \frac{(P_1 - P_0) + d_1}{P_0}$$

where P_0 denotes the price of a share of stock at the beginning of the holding period, P_1 represents the price at the end of the holding period, and d_1 stands for any cash dividend that might have been paid during the holding period (typically one month or one year).[1]

The security analyst should construct a probability distribution of returns for each individual security that is an investment candidate. The needed rates of return may be compiled from historical data if the candidate security already exists (that is, is not an initial public offering). The historically derived probability distribution of returns may then need to be adjusted subjectively to reflect anticipated factors that were not present historically. Figure 1.1 provides an example of a probability distribution of the rates of return for Coca-Cola's common stock that was constructed by a security analyst named Tom. This probability distribution is a finite probability distribution because the outcomes (rates of return) are assumed to be discrete occurrences.

Figure 1.1 Tom's Subjective Probability Distribution of Returns

A. Graph



B. Table

t	p_t	r_t
1	.1	.9 = 90%
2	.25	.5 = 50%
3	.3	.2 = 20%
4	.25	–.1 = –10%
5	.1	–.5 = –50%
	1.0	

The security analyst must also estimate correlation coefficients (or covariances) between all securities under consideration. Security analysis is discussed more extensively in Chapters 2 and 8. The expected return, variance, and covariance statistics are the input statistics used to create optimal portfolios.

1.3 Portfolio Analysis

Portfolio analysis is a mathematical algorithm created by the Nobel laureate Harry Markowitz during the 1950s.[2] Markowitz portfolio analysis requires the following statistical inputs.

- The expected rate of return, $E(r)$, for each investment candidate (that is, every stock, every bond, etc.).
- The standard deviation of returns, σ, for each investment candidate.
- The correlation coefficients, ρ, between all pairs of investment candidates.

Markowitz portfolio analysis takes the statistical inputs listed above and analyzes them simultaneously to determine a series of plausible investment portfolios. The solutions explain which investment candidates are selected and rejected in creating a list of optimal portfolios that can achieve some expected rate of return. Each Markowitz portfolio analysis solution also gives exact portfolio weightings for the investment candidates in that solution.

1.3.1 Basic Assumptions

Portfolio theory is based on four behavioral assumptions.

1. All investors visualize each investment opportunity (for instance, each stock or bond) as being represented by a probability distribution of returns that is measured over the same planning horizon (holding period).

2. Investors' risk estimates are proportional to the variability of the returns (as measured by the standard deviation, or equivalently, the variance of returns).

3. Investors are willing to base their decisions on only the expected return and risk statistics. That is, investors' utility of returns function, $U(r)$, is solely a function of variability of return (σ) and expected return [E(*r*)]. Symbolically, $U(r) = \mathrm{f}[\sigma, E(r)]$. Stated differently, whatever happiness an investor gets from an investment can be completely explained by σ and $E(r)$.

4. For any given level of risk, investors prefer higher returns to lower returns. Symbolically, $\partial U(r) / \partial E(r) > 0$. Conversely, for any given level of rate of return, investors prefer less risk over more risk. Symbolically, $\partial U(r) / \partial \sigma < 0$. In other words, all investors are risk-averse rate of return maximizers.

1.3.2 Reconsidering the Assumptions

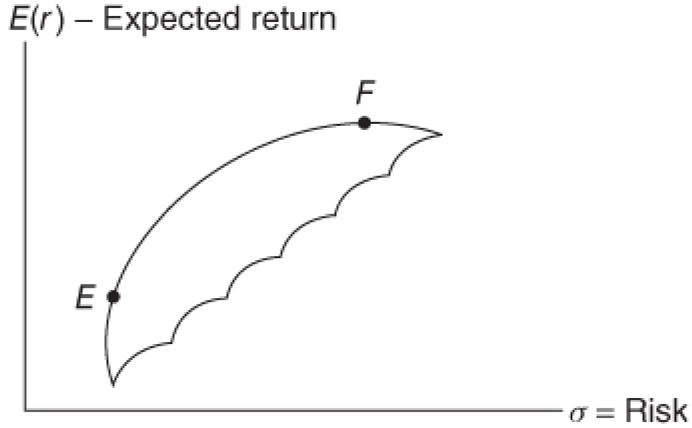
The four behavioral assumptions just listed are logical and realistic and are maintained throughout portfolio theory. Considering the four assumptions implies the most desirable investments have:

- The minimum expected risk at any given expected rate of return. Or, conversely,
- The maximum expected rate of return at any given level of expected risk.

Investors described by the preceding assumptions will prefer Markowitz efficient assets. Such assets are almost always portfolios rather than individual assets. The Markowitz efficient assets are called *efficient portfolios*, whether they contain one or many assets.

If all investors behave as described by the four assumptions, portfolio analysis can logically (mathematically) delineate the set of efficient portfolios. The set of efficient portfolios is called the *efficient frontier* and is illustrated in Figure 1.2. The efficient portfolios along the curve between points E and F have the maximum rate of return at each level of risk. The efficient frontier is the menu from which the investor should make his or her selection.

Figure 1.2 The Efficient Frontier



Before proceeding to the third step of the portfolio management process, portfolio selection, let us pause to reconsider the four assumptions listed previously. Portfolio theory is admittedly based on some simplifying assumptions that are not entirely realistic. This may raise questions in some people's minds. Therefore, we will examine the validity of the four assumptions underlying portfolio theory.

The first assumption about probability distributions of either terminal wealth or rates of return may be violated in several respects. First, many investors simply do not forecast assets' prices or the rate of return from an investment. Second, investors are frequently heard discussing the “growth potential of a stock,” “a glamor stock,” or the “quality of management” while ignoring the investment's terminal wealth or rates of return. Third, investors often base their decisions on estimates of the most likely outcome rather than considering a probability distribution that includes both the best and worst outcomes.

These seeming disparities with assumption 1 are not serious. If investors are interested in a security's glamor or growth, it is probably because they (consciously or subconsciously) believe that these factors affect the asset's rate of return and market value. And even if investors cannot define rate of return, they may still try to maximize it merely by trying to maximize their net worth: Maximizing these two objectives can be shown to be mathematically equivalent. Furthermore, forecasting future probability distributions need not be highly explicit. “Most likely” estimates are prepared either explicitly or implicitly from a subjective probability distribution that includes both good and bad outcomes.

The risk definition given in assumption 2 does not conform to the risk measures compiled by some popular financial services. The quality ratings published by Standard & Poor's are standardized symbols like AAA, AA, A, BBB, BB, B, CCC, CC, and C. Studies suggest that these symbols address the probability of default. Firms' probability of default is positively correlated with their variability of return. Therefore, assumption 2 is valid.[3]

As pointed out, investors sometimes discuss concepts such as the growth potential and/or glamor of a security. This may seem to indicate that the third assumption is an oversimplification. However, if these factors affect the expected value and/or variability of a security's rate of return, the third assumption is not violated either.

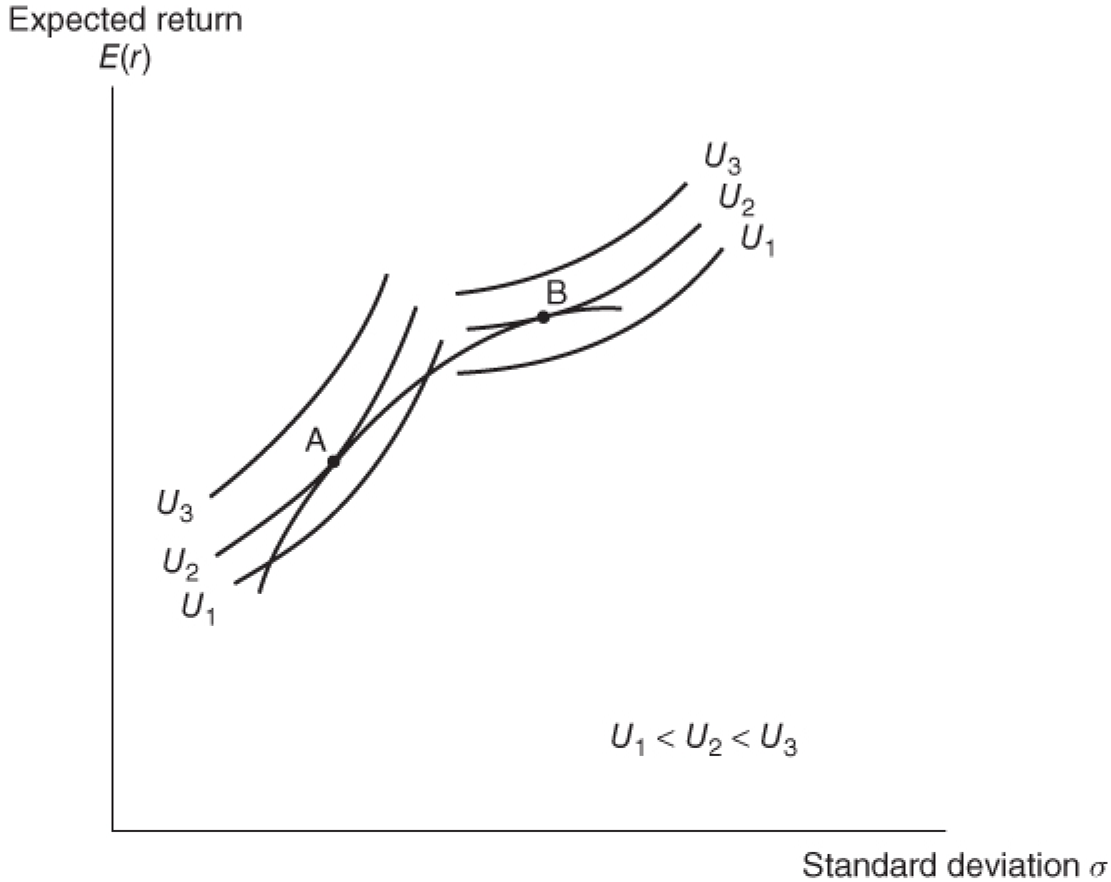
The fourth assumption may also seem inadequate. Psychologists and other behavioralists have pointed out to economists that business people infrequently maximize profits or minimize costs. The psychologists explain that people usually strive only to do a satisfactory or sufficient job. Rarely do they work to attain the maximum or minimum, whichever may be appropriate. However, if some highly competitive business managers attain near optimization of their objective and other business managers compete with these leaders, then this assumption also turns out to be realistic.

All the assumptions underlying portfolio analysis have been shown to be simplistic, and in some cases overly simplistic. Although it would be nice if none of the assumptions underlying the analysis were ever violated, this is not necessary to establish the value of the theory. If the analysis rationalizes complex behavior (such as diversification), or if the analysis yields worthwhile predictions (such as risk aversion), then it can be valuable in spite of its simplified assumptions. Furthermore, if the assumptions are only slight simplifications, as are the four mentioned previously, they are no cause for alarm. People need only behave *as if* they were described by the assumptions for a theory to be valid.[4]

1.4 Portfolio Selection

The final phase of the portfolio management process is to select the one best portfolio from the efficient frontier illustrated in Figure 1.2. The utility of returns function, which aligns with the four basic assumptions previously listed, is very helpful in selecting an optimal portfolio. Utility of return functions can be formulated into indifference curves in [σ, $E(r)$] space. Two different families of indifference curves that were created from similar but different utility of return functions are illustrated in Figure 1.3 to represent the preferences of two different investors. Figure 1.3 shows investor B achieves his maximum attainable happiness from investing in a riskier efficient portfolio than investor A's. In other words, investor A is more risk-averse than investor B.

Figure 1.3 Different Optimal Portfolios for Different Investors



Portfolio selection is made more difficult because security prices change as more recent information continually becomes available. And as cash dividends are paid, the expected return and risk of a selected portfolio can migrate. When this happens, the portfolio must be revised to maintain its superiority over alternative investments. Thus, portfolio selection leads, in turn, to additional security analysis and portfolio analysis work. Portfolio management is a never-ending process.

1.5 The Mathematics Is Segregated

What follows can be mathematical. However, the reader who is uninitiated in mathematics can master the material.