S*pringer* **M***onographs in* **M***athematics*

Igor Chudinovich Christian Constanda

Variational and Potential Methods for a Class of Linear Hyperbolic Evolutionary Processes

Springer

Igor Chudinovich, MS, PhD, DSc
Professor of Mathematics, Faculty of Mechanical, Electrical, and Electronic Engineering
The University of Guanajuato, Salamanca, GTO, Mexico

Christian Constanda, MS, PhD, DSc
Charles W. Oliphant Professor of Mathematical Sciences, The University of Tulsa
600 South College Avenue, Tulsa, Oklahoma 74104, USA

British Library Cataloguing in Publication Data
Chudinovich, Igor
Variational and potential methods for a class of linear hyperbolic evolutionary processes. – (Springer monographs in mathematics)
1. Plates (Engineering) – Mathematical models 2. Boundary element methods 3. Differential equations, Hyperbolic
4. Differential equations, Linear
I. Title II. Constanda, C. (Christian)
515.3′535
ISBN 1852338881

Library of Congress Cataloging-in-Publication Data
CIP data available.

Mathematics Subject Classification (2000): 35C15; 35D05; 35E05; 35L15; 35L20; 35Q72; 45F15; 74H20; 74H25; 74K20

Springer Monographs in Mathematics ISSN 1439-7382
ISBN 1-85233-888-1
Springer is a part of Springer Science+Business Media
springeronline.com

Typesetting: Camera-ready by the authors
12/3830-543210 Printer on acid-free paper SPIN 10991749

For Olga and Lia
and the younger generation
Genia and Dan

Preface

Variational and boundary integral equation techniques are two of the most useful methods for solving time-dependent problems described by systems of equations of the form

$$\frac{\partial^2 u}{\partial t^2} = Au,$$

where $u = u(x,t)$ is a vector-valued function, x is a point in a domain in $\mathbb{R}^2$ or $\mathbb{R}^3$, and A is a linear elliptic differential operator. To facilitate a better understanding of these two types of methods, below we propose to illustrate their mechanisms in action on a specific mathematical model rather than in a more impersonal abstract setting. For this purpose, we have chosen the hyperbolic system of partial differential equations governing the nonstationary bending of elastic plates with transverse shear deformation. The reason for our choice is twofold. On the one hand, in a certain sense this is a "hybrid" system, consisting of three equations for three unknown functions in only two independent variables, which makes it more unusual—and thereby more interesting to the analyst—than other systems arising in solid mechanics. On the other hand, this particular plate model has received very little attention compared to the so-called classical one, based on Kirchhoff's simplifying hypotheses, although, as acknowledged by practitioners, it represents a substantial refinement of the latter and therefore needs a rigorous discussion of the existence, uniqueness, and continuous dependence of its solution on the data before any construction of numerical approximation algorithms can be contemplated.

The first part of our analysis is conducted by means of a procedure that is close in both nature and detail to a variational method, and which, for this reason, we also call variational. Once the results have been established in the general setting of Sobolev spaces, we carry out the second part of the study by seeking useful, closed-form integral representations of the solutions in terms of dynamic (retarded) plate potentials.

The problems discussed in this book include those with Dirichlet and Neumann boundary conditions (corresponding, in particular, to the clamped-edge and free-edge plate), with elastic (Robin), mixed, and combined displacement-traction (simply supported edge) boundary data, transmission (contact) problems, problems for plates with homogeneous inclusions, plates with cracks, and plates on a generalized elastic foundation. For each of them, the variational version is formulated and its solvability is examined in spaces of distributions; subsequently, the solutions are found in the form of time-dependent single-layer and double-layer potentials with distributional densities that satisfy nonstationary integral equations. The analysis technique consists in using the Laplace transformation to reduce the original problems to boundary value problems depending on the transformation parameter, and on establishing estimates for the solutions of the latter that allow conclusions to be drawn about the existence and properties of the solutions to the given initial-boundary value problems. The transformed problems are solved by means of specially constructed algebras of singular integral operators defined by the boundary values of the transformed potentials.

The distributional setting has the advantage over the classical one in that it enables the method to be applied in less smooth domains—for example, in regions with corners and cuts. Furthermore, Sobolev-type norms are particularly useful in the global error analysis of numerical schemes, but such analysis falls outside the scope of this book and we do not pursue it.

To the authors' knowledge, this is the first time that so many typical initial-boundary value problems have been considered in the same book for a model in conjunction with both variational and boundary integral equation methods. The text provides full details of the proofs and is aimed at researchers interested in the use of applied analysis as a tool for investigating mathematical models in mechanics. The presentation assumes no specialized knowledge beyond a basic understanding of functional analysis and Sobolev spaces.

We want to emphasize that the book does not intend to explain the mechanical background of plate theory. Details of that nature and a fuller discussion of the limitations of the model that we have chosen as our object of study can be found in the article

> J.R. Cho and J.T. Oden, A priori modeling error estimate of hierarchical models for elasticity problems for plate and shell-like structures, *Math. Comput. Modelling* **23** (1996), 117–133.

Ours is a purely mathematical that aims to acquaint the interested reader with two of the most powerful and general techniques of solution for this type of linear problem. We reiterate that the theory of bending of plates with transverse shear deformation has been selected merely as an application vehicle because of its unusual features and lack of previous strict mathematical treatment. The book is a natural complement to our earlier monograph [7], where we investigated the corresponding equilibrium problems.

Some of the results discussed below have already been announced in concise form in the literature (see [4]–[6] and [8]).

The authors would like to acknowledge help and support received from various quarters during the preparation of this book. I.C. wishes to thank his former colleagues in the Mathematical Physics and Computational Mathematics section of the Department of Mathematics and Mechanics at Kharkov National University, and his current colleagues in the Department of Mechanical, Electrical, and Electronic Engineering of the University of Guanajuato in Salamanca—in particular, Drs. Igor Chueshov, Arturo Lara Lopez, and René Jaime Rivas, for playing an instrumental role in arranging his move to Mexico. C.C. wishes to thank Dr. Bill Coberly and his other colleagues at the University of Tulsa for a departmental atmosphere that has proved highly conducive to the writing of mathematical books.

Last but by no means least, we would like to place on record the debt of gratitude that we owe our wives, *sine quibus non,* who have guided us wisely, patiently, and selflessly, by word and by deed, to exciting and challenging new shores.

Igor Chudinovich

Professor of Mathematics
University of Guanajuato
Salamanca, Mexico

Christian Constanda

C.W. Oliphant Professor
of Mathematical Sciences
University of Tulsa, USA

April 2004

Contents

1

Formulation of the Problems and Their Nonstationary Boundary Integral Equations

1.1 The Initial-Boundary Value Problems

All problem statements in this chapter are formal; rigorous versions will be presented after the introduction of the necessary function spaces.

Below we consider initial-boundary value problems for the time-dependent homogeneous equations of the model with homogeneous initial data. In Chapter 9, we indicate how the general case can be reduced to the homogeneous one.

By an *elastic plate* we understand an elastic body that occupies a region $\bar{S} \times [-h_0/2, h_0/2]$ in $\mathbb{R}^3$, where S is a domain in $\mathbb{R}^2$ bounded by a simple closed curve ∂S and $0 < h_0 = \text{const} \ll \operatorname{diam} S$ is called the *thickness.*

Throughout the book we use the following notation and conventions.

Unless otherwise specified, Greek and Latin subscripts and superscripts in all formulas take the values $1, 2$ and $1, 2, 3$, respectively, and summation over repeated indices is adopted.

The standard inner product in $\mathbb{R}^3$ is $(a, b) = a_i b_i$.

A generic point in $\mathbb{R}^2$ referred to a Cartesian system of coordinates in the middle plane $x_3 = 0$ of the plate is written as $x = (x_1, x_2)$.

$X = (x, t)$, where t is the time variable.

Partial derivatives are denoted by $\partial_\alpha = \partial/\partial x_\alpha$ and $\partial_t = \partial/\partial_t$.

A superscript T denotes matrix transposition. A superscript $*$ denotes conjugation and transposition of a complex matrix.

The columns of a matrix M are denoted by $M^{(i)}$.

Both matrix-valued functions and scalar functions are simply referred to as functions. If $\mathcal{Y}$ is a space of scalar functions and g is a matrix-valued function, then $g \in \mathcal{Y}$ means that each entry of g belongs to $\mathcal{Y}$.

A three-component vector $q = (q_1, q_2, q_3)^{\mathrm{T}}$ may be written alternatively as $q = (\bar{q}^{\mathrm{T}}, q_3)^{\mathrm{T}}$, where $\bar{q} = (q_1, q_2)^{\mathrm{T}}$.

S^+ is the finite domain enclosed by ∂S, and $S^- = \mathbb{R}^2 \setminus (S^+ \cup \partial S)$.

The boundary ∂S is a C^2-curve with a uniquely defined outward (with respect to S^+) normal $n = (n_1, n_2)^{\mathrm{T}}$.

We write

$$G = S \times (0, \infty), \quad G^{\pm} = S^{\pm} \times (0, \infty), \quad \Gamma = \partial S \times (0, \infty).$$

If φ is a smooth function defined in S^+ (S^-), then φ^+ (φ^-) denotes the limiting value (if it exists) of φ as its argument tends to ∂S from within S^+ (S^-). If φ is not smooth but has a trace on ∂S, then the latter is denoted by $\gamma^+\varphi$ ($\gamma^-\varphi$). Since there is no danger of ambiguity, the notation remains the same for functions defined in G^+ (G^-) and their limiting values (traces) on the boundary Γ.

The operators of restriction from $\mathbb{R}^2$ (or $S^+ \cup S^-$) to $S^{\pm}$, or from $\mathbb{R}^2 \times (0, \infty)$ (or $G^+ \cup G^-$) to $G^{\pm}$, are denoted by $\pi^{\pm}$.

Operators of extension from ∂S to $S^{\pm}$, or from Γ to $G^{\pm}$, are denoted by $l^{\pm}$, respectively.

Δ is the Laplacian and δ_{ij} is the Kronecker delta.

$\mathcal{L}$ and $\mathcal{L}^{-1}$ are, respectively, the Laplace transformation with respect to t, and its inverse. The Laplace transform of a function $u(x, t)$ is denoted by $\hat{u}(x, p)$, where p is the transformation parameter.

Other notation will be introduced as the need arises.

Suppose that the material is homogeneous and isotropic, of density ρ and Lamé constants λ and μ, which satisfy the inequalities [9]

$$\lambda + \mu > 0, \quad \mu > 0, \quad \rho > 0. \tag{1.1}$$

If we denote by t_{ij}, ε_{ij}, v_i, and f_i, respectively, the components of the stress tensor, deformation tensor, displacement vector, and body force vector, then the behavior of the plate as a three-dimensional elastic body under prescribed initial and boundary conditions is governed by three main groups of equations, namely (see [14] and [17]),

the kinematic formulas

$$\varepsilon_{ij} = \tfrac{1}{2}(\partial_i v_j + \partial_j v_i); \tag{1.2}$$

the stress-strain relations (generalized Hooke's law)

$$t_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}; \tag{1.3}$$

the equations of motion

$$\partial_j t_{ij} + f_i = \rho \partial_t^2 v_i. \tag{1.4}$$

In addition,

$$t_i = t_{ij} n_j$$

are the components of the stress vector on ∂S.

The model of bending of plates with transverse shear deformation that we intend to study here postulates a displacement field of the form

$$\begin{aligned} v_\alpha(x, x_3, t) &= x_3 u_\alpha(X), \\ v_3(x, x_3, t) &= u_3(X). \end{aligned} \tag{1.5}$$

This assumption is valid only for plates whose ratio of thickness to diameter falls within a certain range (see the Preface).

Expressions (1.5) and the geometry of the plate suggest a way of simplifying equations (1.2)–(1.4). This is done by means of a well-known procedure that involves the use of the averaging operators $\mathcal{I}_\alpha$ and $\mathcal{J}_\alpha$, $\alpha = 0, 1$, defined by

$$(\mathcal{I}_\alpha g)(X) = h_0^{-1}\big[x_3^\alpha g(x, x_3, t)\big]_{x_3=-h_0/2}^{x_3=h_0/2},$$

$$(\mathcal{J}_\alpha g)(X) = h_0^{-1} \int\limits_{-h_0/2}^{h_0/2} x_3^\alpha g(x, x_3, t)\, dx_3.$$

Specifically, setting

$$\begin{aligned} N_{\alpha\beta} &= \mathcal{J}_1 t_{\alpha\beta}, \\ N_{3\alpha} &= \mathcal{J}_0 t_{3\alpha}, \\ q_\alpha &= \mathcal{J}_1 f_\alpha + \mathcal{I}_1 t_{\alpha 3}, \\ q_3 &= \mathcal{J}_0 f_3 + \mathcal{I}_0 t_{33}, \\ h^2 &= h_0^2/12, \end{aligned}$$

system (1.4) yields the plate equations of motion

$$\begin{aligned} \partial_\beta N_{\alpha\beta} - N_{3\alpha} + q_\alpha &= \rho h^2 \partial_t^2 u_\alpha, \\ \partial_\alpha N_{3\alpha} + q_3 &= \rho \partial_t^2 u_3. \end{aligned} \tag{1.6}$$

Also, from (1.2), (1.3), and (1.5), we obtain the plate constitutive relations

$$\begin{aligned} N_{\alpha\beta} &= h^2\big[\lambda(\partial_\gamma u_\gamma)\delta_{\alpha\beta} + \mu(\partial_\alpha u_\beta + \partial_\beta u_\alpha)\big], \\ N_{3\alpha} &= \mu(\partial_\alpha u_3 + u_\alpha). \end{aligned} \tag{1.7}$$

Finally, substituting (1.7) into (1.6) leads to the alternative equations of motion

$$B(\partial_t^2 u)(X) + (Au)(X) = q(X), \quad X \in G^+ \text{ or } X \in G^-, \tag{1.8}$$

where

$$B = \operatorname{diag}\{\rho h^2, \rho h^2, \rho\},$$

A is the matrix differential operator with entries [9]

$$A_{\alpha\alpha} = -h^2\mu\Delta - h^2(\lambda+\mu)\partial_\alpha^2 - \mu \quad (\alpha \text{ not summed}),$$
$$A_{33} = -\mu\Delta,$$
$$A_{12} = A_{21} = -h^2(\lambda+\mu)\partial_1\partial_2,$$
$$A_{\alpha 3} = -A_{3\alpha} = \mu\partial_\alpha,$$

and

$$u = (\bar{u}^{\mathrm{T}}, u_3)^{\mathrm{T}},$$
$$q = (\bar{q}^{\mathrm{T}}, q_3)^{\mathrm{T}}.$$

It is easily verified that, under conditions (1.1), A is a strongly elliptic operator and satisfies Gårding's inequality [18].

The quantities $N_{\alpha\beta}$ and $N_{\alpha 3}$ are the averages across the thickness of the plate of the bending and twisting moments with respect to the middle plane $x_3 = 0$, and of the transverse shear forces [9]; q_α and q_3 are combinations of the body moments and forces and of the moments and forces acting on the faces $x_3 = \pm h_0/2$.

Similarly, setting

$$N_\alpha = \mathcal{J}_1 t_\alpha,$$
$$N_3 = \mathcal{J}_0 t_3,$$

we obtain

$$N_1 = h^2\big[(\lambda\partial_\alpha u_\alpha + 2\mu\partial_1 u_1)n_1 + \mu(\partial_1 u_2 + \partial_2 u_1)n_2\big],$$
$$N_2 = h^2\big[\mu(\partial_1 u_2 + \partial_2 u_1)n_1 + (\lambda\partial_\alpha u_\alpha + 2\mu\partial_2 u_2)n_2\big],$$
$$N_3 = \mu(\partial_\alpha u_3 + u_\alpha)n_\alpha,$$

which can be written as

$$N_i = (Tu)_i,$$

where T is the matrix boundary operator with entries

$$T_{11} = h^2\big[(\lambda+2\mu)n_1\partial_1 + \mu n_2\partial_2\big],$$
$$T_{22} = h^2\big[(\lambda+2\mu)n_2\partial_2 + \mu n_1\partial_1\big],$$
$$T_{33} = \mu n_\alpha\partial_\alpha,$$
$$T_{12} = h^2(\lambda n_1\partial_2 + \mu n_2\partial_1),$$
$$T_{21} = h^2(\mu n_1\partial_2 + \lambda n_2\partial_1),$$
$$T_{3\alpha} = \mu n_\alpha,$$
$$T_{\alpha 3} = 0.$$

From what has been said above, it is obvious that Tu is the vector of the averaged moments and shear force acting on the lateral part $\partial S \times (-h_0/2, h_0/2)$

of the boundary. The vector u is referred to as the displacement vector since it characterizes the latter uniquely in terms of the assumption (1.5).

In Chapters 2–8, we deal almost exclusively with the homogeneous equation (1.8), that is,

$$B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G^+ \text{ or } X \in G^-. \tag{1.9}$$

To (1.9) we adjoin appropriate boundary conditions and homogeneous initial conditions. The functions occurring on the right-hand side in all the boundary conditions below are prescribed.

The symbolic name of each problem that we consider starts with a "D" to indicate that it is a dynamic problem. The remaining letters are fairly obvious initials related to the problem type and/or boundary condition type.

Thus, the classical interior and exterior problems ($\mathrm{DD}^\pm$) with Dirichlet boundary conditions consist, respectively, in finding functions $u \in C^2(G^\pm) \cap C^1(\bar{G}^\pm)$ such that

$$\begin{gathered} B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G^\pm, \\ u(x,0+) = (\partial_t u)(x,0+) = 0, \quad x \in S^\pm, \\ u^\pm(X) = f(X), \quad X \in \Gamma. \end{gathered}$$

In the interior and exterior initial boundary-value problems ($\mathrm{DN}^\pm$) with Neumann boundary conditions, we seek solutions $u \in C^2(G^\pm) \cap C^1(\bar{G}^\pm)$ of

$$\begin{gathered} B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G^\pm, \\ u(x,0+) = (\partial_t u)(x,0+) = 0, \quad x \in S^\pm, \\ (Tu)^\pm(X) = g(X), \quad X \in \Gamma. \end{gathered}$$

Consider two open arcs ∂S_1 and ∂S_2 of ∂S such that

$$\begin{gathered} \mathrm{mes}(\partial S_\alpha) > 0, \\ \overline{\partial S_1} \cup \overline{\partial S_2} = \partial S, \\ \partial S_1 \cap \partial S_2 = \emptyset. \end{gathered}$$

The interior and exterior initial-value problems ($\mathrm{DM}^\pm$) with mixed boundary conditions consist in finding $u \in C^2(G^\pm) \cap C^1(\bar{G}^\pm)$ satisfying

$$\begin{gathered} B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G^\pm, \\ u(x,0+) = (\partial_t u)(x,0+) = 0, \quad x \in S^\pm, \\ u^\pm(X) = f(X), \quad X \in \partial S_1 \times (0,\infty), \\ (Tu)^\pm(X) = g(X), \quad X \in \partial S_2 \times (0,\infty). \end{gathered}$$

In the interior and exterior initial-boundary value problems ($\mathrm{DC}_1^{\pm}$) with combined boundary conditions of the first kind, we look for $u \in C^2(G^{\pm}) \cap C^1(\bar{G}^{\pm})$ such that

$$\begin{gathered} B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G^{\pm}, \\ u(x, 0+) = (\partial_t u)(x, 0+) = 0, \quad x \in S^{\pm}, \\ u_3^{\pm}(X) = f_3(X), \quad X \in \Gamma, \\ (Tu)_\alpha^{\pm}(X) = g_\alpha(X), \quad X \in \Gamma. \end{gathered}$$

If the boundary conditions are of the second kind, then the solution $u \in C^2(G^{\pm}) \cap C^1(\bar{G}^{\pm})$ satisfies

$$\begin{gathered} B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G^{\pm}, \\ u(x, 0+) = (\partial_t u)(x, 0+) = 0, \quad x \in S^{\pm}, \\ u_\alpha^{\pm}(X) = f_\alpha(X), \quad X \in \Gamma, \\ (Tu)_3^{\pm}(X) = g_3(X), \quad X \in \Gamma. \end{gathered}$$

If the regions $S^{\pm} \times [-h_0/2, h_0/2]$ are occupied by two different elastic materials with Lamé constants $\lambda_{\pm}$, $\mu_{\pm}$ and densities $\rho_{\pm}$, respectively, then the initial-boundary value problem (DT) with transmission (contact) boundary conditions consists in finding a pair of functions $u_{\pm} \in C^2(G^{\pm}) \cap C^1(\bar{G}^{\pm})$ such that

$$\begin{gathered} B_{\pm}(\partial_t^2 u_{\pm})(X) + (A_{\pm} u_{\pm})(X) = 0, \quad X \in G^{\pm}, \\ u_{\pm}(x, 0+) = (\partial_t u_{\pm})(x, 0+) = 0, \quad x \in S^{\pm}, \\ u_+^+(X) - u_-^-(X) = f(X), \quad X \in \Gamma, \\ (T_+ u_+)^+(X) - (T_- u_-)^-(X) = g(X), \quad X \in \Gamma, \end{gathered}$$

where $A_{\pm}$, $B_{\pm}$, and $T_{\pm}$ have the obvious meaning.

Consider an open arc ∂S_0 of ∂S that models a crack, and let

$$\begin{gathered} \Omega = \mathbb{R}^2 \setminus \overline{\partial S}_0, \\ \partial S_1 = \partial S \setminus \overline{\partial S}_0, \\ G = \Omega \times (0, \infty), \\ \Gamma_i = \partial S_i \times (0, \infty), \quad i = 0, 1. \end{gathered}$$

We write $u \in C^k(\bar{G})$, $k = 0, 1, 2, \ldots$, if the restrictions $u_{\pm}$ of u to $G^{\pm}$ are, respectively, of class $C^k(\bar{G}^{\pm})$ and the limiting values on Γ_1 of u_+ and all its derivatives up to the order k coincide with those of u_-. (These values may differ on Γ_0.) In the initial-boundary value problem (DKD) with Dirichlet boundary conditions, we seek $u \in C^2(G) \cap C^1(\bar{G})$ satisfying

$$B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G,$$
$$u(x, 0+) = (\partial_t u)(x, 0+) = 0, \quad x \in \Omega,$$
$$u_+^+(X) = f^+(X), \quad X \in \Gamma_0,$$
$$u_-^-(X) = f^-(X), \quad X \in \Gamma_0.$$

The problem (DKN) with Neumann boundary conditions consists in finding $u \in C^2(G) \cap C^1(\bar{G})$ such that

$$B(\partial_t^2 u)(X) + (Au)(X) = 0, \quad X \in G,$$
$$u(x, 0+) = (\partial_t u)(x, 0+) = 0, \quad x \in \Omega,$$
$$(Tu_+)^+(X) = g^+(X), \quad X \in \Gamma_0,$$
$$(Tu_-)^-(X) = g^-(X), \quad X \in \Gamma_0.$$

Let $\mathcal{K}$ be a (3×3)-matrix of the form

$$\mathcal{K} = \begin{pmatrix} \bar{\mathcal{K}} & 0 \\ 0 & k_{33} \end{pmatrix},$$

where $k_{33} > 0$ and the (2×2)-matrix $\bar{\mathcal{K}} = h^2(k_{\alpha\beta})$ is positive definite. In the interior and exterior initial-boundary value problems ($\mathrm{DD}_{\mathcal{K}}^{\pm}$) for a plate on an elastic foundation with Dirichlet boundary conditions, we look for $u \in C^2(G^\pm) \cap C^1(\bar{G}^\pm)$ such that

$$B(\partial_t^2 u)(X) + (Au)(X) + \mathcal{K}u(X) = 0, \quad X \in G^\pm,$$
$$u(x, 0+) = (\partial_t u)(x, 0+) = 0, \quad x \in S^\pm,$$
$$u^\pm(X) = f(X), \quad X \in \Gamma.$$

The corresponding problems ($\mathrm{DN}_{\mathcal{K}}^{\pm}$) with Neumann boundary conditions consist in finding functions $u \in C^2(G^\pm) \cap C^1(\bar{G}^\pm)$ satisfying

$$B(\partial_t^2 u)(X) + (Au)(X) + \mathcal{K}u(X) = 0, \quad X \in G^\pm,$$
$$u(x, 0+) = (\partial_t u)(x, 0+) = 0, \quad x \in S^\pm,$$
$$(Tu)^\pm(X) = g(X), \quad X \in \Gamma.$$

Throughout what follows, we work frequently with the Laplace transforms of vector-valued functions $u(X) = u(x, t)$, $t \in \mathbb{R}$, which vanish for $t < 0$; that is,

$$\hat{u}(x, p) = \mathcal{L}u(x, t) = \int_0^\infty e^{-pt} u(x, t)\, dt.$$