Water Waves and Ship Hydrodynamics

2nd Edition

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An Introduction

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Preface to the Second Edition

This book is a revision and extension of the book published by R. Timman, A.J. Hermans and G.C. Hsiao based on the lecture notes of courses presented by Timman at the University of Delaware in 1971 and by Hermans at the Technical University of Delft. The main topic of the original text is based on linearised free surface water wave theory. For many years the first edition of the book is used by Aad Hermans as material for a course in ship hydrodynamics presented to Master students in applied mathematics and naval architecture at the Technical University of Delft. Influenced by the progress in the research in water waves and especially in ship hydrodynamics the contents of the course has changed gradually. For instance in offshore engineering the topic like the low-frequency motion of objects moored to a buoy has become an important issue during this period. Therefore an introduction in this field has been added. For didactic reasons the very simple rather abstract problem of the motion of a vertical wall is added. The reason to do so is that most effects that play a role can be treated analytically, while for a general three dimensional object some terms can only be obtained numerically. The use of numerical programs is normal practice in this field, therefor an introduction in the theory of integral equations is presented and some specific problems which may arises, such how to avoid non-physical resonance at the so called irregular frequencies may be avoided. In the first edition a derivation of the structure of the equations of motion in all six degrees of freedom is presented. Because the functions derived there are not easily computed in a practical case, we restrict ourselves to the derivation of the equation of motion in one degree of freedom.

Delft, The Netherlands A.J. Hermans

Preface to the First Edition

In the spring of 1971, Reinier Timman visited the University of Delaware during which time he gave a series of lectures on water waves from which these notes grew. Those of us privileged to be present during that time will never forget the experience. Rein Timman is not easily forgotten.

His seemingly inexhaustible energy completely overwhelmed us. Who could forget the numbing effect of a succession of long wine-filled evenings of lively conversation on literature, politics, education, you name it, followed early next day by the appearance of the apparently totally refreshed red-haired giant eager to discuss mathematical problems with keen insight and remarkable understanding, ready to lecture on fluid dynamics and optimal control theory or a host of other subjects and ready to work into the evening until the cycle repeated. He thought faster, knew more, drank more and slept less than any of the mortals; he literally wore us out. What a rare privilege indeed to have participated in this intellectual orgy. Timman's lively interest in almost everything coupled with his buoyant enthusiasm and infectious optimism epitomised his approach to life, No delicate nibbling at the fringes, he wanted every morsel of every course.

In these times of narrow specialisation, truly renaissance figures are, if not extinct, at least a highly endangered species. But Timman was one of that rare breed. His knowledge in virtually all areas of classical applied mathematics was prodigious. I still marvel that while I was his doctoral student in Delft in the late fifties working on a problem in electromagnetic scattering he had at the same time students working in water waves, cavitation, elasticity, aerodynamics and numerical analysis. He was a boundless source of inspiration to his students in all of these varied fields.

His inattention to detail is legendary but this did not hamper his ability to focus on what was really important in a problem. With a wave of his large hand he would dismiss unimportant errors while concentrating on central ideas, leaving to us the task of setting things right mathematically. This nonchalant attitude toward minus signs and numerical factors was probably deliberate. He wanted people to see the forest, not the trees; to focus on the heart of the problem, not inconsequential superficialities. He had little use for the all too prevalent penchant for examining someone's work looking for errors. He would read a paper looking for the gold, not the dross; looking for what was right, not what was wrong.

Of course this did not make life easy for those around him but it did make it interesting. This will be attested to by George Hsiao and Richard Weinacht whose revised version of the notes from Timman's water wave lectures appeared as a University of Delaware report. Timman and Hsiao then planned to further revise and expand these notes and publish them in book form, but the project came to an abrupt halt with Reinier Timman's untimely death in 1975. It might have remained unfinished had not Aad Hermans' visit of Delaware in 1980 breathed new life into it. Together George Hsiao and Aad Hermans have completed the task of revising the notes, reorganising the presentation, restoring the factors of 2 which Timman had cavalierly omitted, and adding some new material. The first four chapters are based substantially on the original notes, while the fifth chapter and appendices have been added.

It is gratifying to see the completion of these notes. It is not unreasonable to hope that they will provide a useful introduction to water waves for a new generation of mathematicians and engineers. This area was perhaps first among equals in the broad spectrum of Timman's interests. If these notes succeed in stimulating a new generation to concentrate on the challenging problems remaining in this field, they will serve a fitting memorial to a remarkable man whose like will not be soon seen again.

Newark, Delaware R.E. Kleinman
March, 1985

Acknowledgements to the First Edition

The authors wish to thank Professor Willard E. Baxter for his active interest in the publication of these lecture notes. We are grateful to Professor Richard J. Weinacht who spent a great deal of time helping one of us (GCH) in preparing the original draft of the water wave notes on which the present first four chapters were based. We owe special debt to Professor Ralph E. Kleinman without whom this project probably would never have started.

We would like to express our appreciation to Mr. G. Broere for making most of the drawings, and to Mrs. Angelina de Wit for typing the final manuscript.

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March, 1985

A.J. Hermans
G.C. Hsiao

Contents

Chapter 1
Theory of Water Waves

This chapter contains the formulation of boundary and initial boundary value problems in water waves. The basic equations here are the Euler equations and the equation of continuity for a non-viscous incompressible fluid moving under gravity. Throughout the book, in most considerations the motion is assumed to be irrotational and hence the existence of a velocity potential function is ensured in simply connected regions. In this case the equation of continuity for the velocity of the fluid is then reduced to the familiar Laplace equation for the velocity potential function.

Water waves are created normally by the presence of a free surface along which the pressure is constant. For the irrotational motion, on the free surface one than obtains the non-linear Bernoulli equation for the velocity potential function from the Euler equation. Based on small amplitude waves, linearised problems for the velocity potential function and for the free surface elevation are formulated.

At first we follow the derivation as can be found in [17, 19] to obtain equations for the wave potential in still water and as a superposition on a constant parallel flow potential. The coefficients in the free surface equations are constant. Then we derive linear equation for the superposition of small amplitude waves on a flow disturbed by some three dimensional object. If we consider the magnitude of the steady velocity vector to be small, we obtain for the time-dependent wave potential function a linear equation with non-constant coefficients.

1.1 Basic Linear Equations

The theory of water waves, to be presented here, is based on a model of non-viscous incompressible fluid moving under gravity. The equations of motion will be expressed in a right-handed system of rectangular coordinates x, y, z. In the Euler representation they read

$$u_t + uu_x + vu_y + wu_z = -\frac{1}{\rho} p_x,$$

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$$v_t + uv_x + vv_y + wv_z = -\frac{1}{\rho}p_y - g, \tag{1.1}$$

$$w_t + uw_x + vw_y + ww_z = -\frac{1}{\rho}p_z.$$

Here $u = u(x, y, z, t)$, $v = v(x, y, z, t)$, $w = w(x, y, z, t)$ are velocity components in the corresponding x, y, z direction; $p = p(x, y, z, t)$ is the pressure; ρ is the density of the fluid, a constant, and g is the gravitational acceleration. The continuity equation is

$$u_x + v_y + w_z = 0. \tag{1.2}$$

In most of the considerations the fluid motion is considered to be *irrotational*. This gives the additional set of equations

$$\begin{aligned} u_y - v_x &= 0, \\ v_z - w_y &= 0, \\ w_x - u_z &= 0, \end{aligned} \tag{1.3}$$

which guarantees in a simply connected region the existence of a velocity potential φ with

$$\begin{aligned} u &= \varphi_x, \\ v &= \varphi_y, \\ w &= \varphi_z. \end{aligned} \tag{1.4}$$

From (1.2) we see that φ satisfies Laplace's equation,

$$\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0. \tag{1.5}$$

This greatly facilitates the theory.

In general, however, solutions of Laplace's equation will not show wave character, since the equation is elliptic. Waves are created by the presence of a free surface and are intimately related to the free surface condition.

1.2 Boundary Conditions

At the moving boundary the condition for a non-viscous fluid is very simple. The fluid velocity normal to the surface has to be equal to the normal component of the velocity of the surface itself. If the equation of the surface is given by

$$y = F(x, z, t), \tag{1.6}$$

we denote the velocity of a point on the surface by (U, V, W). A normal to the surface has the direction cosines

$$\left(\frac{F_x}{\sqrt{F_x^2+F_z^2+1}}, \frac{-1}{\sqrt{F_x^2+F_z^2+1}}, \frac{F_z}{\sqrt{F_x^2+F_z^2+1}}\right) \tag{1.7}$$

and the surface (or boundary) condition reads

$$\frac{uF_x - v + wF_z}{\sqrt{F_x^2+F_z^2+1}} = \frac{UF_x - V + WF_z}{\sqrt{F_x^2+F_z^2+1}} = \frac{-F_t}{\sqrt{F_x^2+F_z^2+1}}, \tag{1.8}$$

because

$$F_x U - V + F_z W + F_t = 0$$

for a point on the moving surface. Hence from (1.8) we have

$$v = F_t + uF_x + wF_z = \frac{\mathrm{d}F(x,z,t)}{\mathrm{d}t}, \tag{1.9}$$

which expresses the fact that, once a fluid particle is on the surface, it remains on the surface.

We will usually denote the bottom surface by $y = H(x, z, t)$, so that (1.9) reads

$$v = H_t + uH_x + wH_z. \tag{1.10}$$

Mostly in our considerations the bottom is fixed, that is H is independent of t, so that the term H_t in (1.10) vanishes.

The waves are created at the free surface, which is characterised by the condition that along this surface the pressure is a constant. Hence in addition to the kinematic equation

$$v = \eta_t + u\eta_x + w\eta_z, \tag{1.11}$$

for the free surface $y = \eta(x, z, t)$, we have the condition

$$p = \text{constant}, \tag{1.12}$$

along $y = \eta(x, z, t)$. There are two ways of formulating these conditions:

a. From the equations of motion (1.2), we find by inspection, in the case of irrotational motion, the Bernoulli equation

$$\varphi_t + \frac{1}{2}(u^2 + v^2 + w^2) + gy + \frac{p}{\rho} = f(t) \tag{1.13}$$

in which, because of the constant pressure, one can normalise φ to result in the dynamical free surface condition

$$\varphi_t + \frac{1}{2}(\varphi_x^2 + \varphi_y^2 + \varphi_z^2) + g\eta = \text{constant}. \tag{1.14}$$

b. The second way expresses that

$$\frac{\partial p}{\partial s_x} = 0, \qquad \frac{\partial p}{\partial s_z} = 0, \tag{1.15}$$

where s_x and s_z are coordinates on the free surface, which have their projections in the x and z directions, respectively. This gives[1]

$$\begin{aligned} \frac{\partial p}{\partial s_x} &= \frac{\partial p}{\partial x}\cos(x, s_x) + \frac{\partial p}{\partial y}\cos(y, s_x) = 0, \\ \frac{\partial p}{\partial s_z} &= \frac{\partial p}{\partial z}\cos(z, s_z) + \frac{\partial p}{\partial y}\cos(y, s_z) = 0 \end{aligned} \tag{1.16}$$

or

$$\begin{aligned} p_x + p_y\eta_x &= 0, \\ p_z + p_y\eta_z &= 0. \end{aligned} \tag{1.17}$$

Substituting (1.17) into (1.2), we have the relation

$$\begin{aligned} u_t + uu_x + vu_y + wu_z + \eta_x(v_t + uv_x + vv_y + wv_z) &= 0, \\ w_t + uw_x + vw_y + ww_z + \eta_z(v_t + uv_x + vv_y + wv_z) &= 0, \end{aligned} \tag{1.18}$$

which are also valid for rotational flow.

In this way the basic equations are derived. The further development of the theory is based on small parameter expansions of these equations. To do so an appropriate small dimensionless parameter has to be specified. Depending on the case considered, different formulations arise. In the next section we consider the case of a fixed bottom and where the water region is horizontally extended to infinity while no floating objects are present. This simplifies the theory considerably. Later we take other effects into account as well.

1.3 Linearised Theory

In this section we discuss two different cases, where we may obtain linearised equations for different situations. In the first one we assume that the waves are superimposed on a steady constant parallel flow field (current), while the second one deals with a wave field superimposed on a steady flow field, which obeys a simplified free surface condition. This steady flow may be generated by a slowly moving vessel. For fast moving objects one may need a more general non-linear theory for steady and unsteady boundary conditions. We will deal with some of these problems in future chapters.

[1]Note that $\cos(x, s_z) = \cos(z, s_x) = 0$.

1.3.1 Small Amplitude Waves in a Steady Current

The simplest approximation is the case where the deviation η of the free surface above a certain standard level, which is taken as $y = 0$, is small. We assume that

$$\eta(x, z, t) = \varepsilon\bar{\eta}(x, z, t), \tag{1.19}$$

where ε is a small dimensionless parameter. In addition we assume the bottom slope to be small of the same order of magnitude in ε and put

$$y = -h + \varepsilon h_1, \tag{1.20}$$

which will lead to the boundary condition

$$v = \varepsilon(h_{1t} + uh_{1x} + wh_{1z}) \tag{1.21}$$

from (1.10). For the free surface we obtain from (1.11) and (1.14)

$$v = \varepsilon(\bar{\eta}_t + u\bar{\eta}_x + w\bar{\eta}_z) \tag{1.22}$$

together with

$$\varphi_t + \frac{1}{2}(\varphi_x^2 + \varphi_y^2 + \varphi_z^2) + \varepsilon g\bar{\eta} = \text{constant}. \tag{1.23}$$

Now, for the solution of (1.5), we assume an expansion

$$\varphi(x, y, z, t) = \varphi_0 + \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + \cdots, \tag{1.24}$$

and substitute it in (1.5) and boundary condition (1.21). Equating to zero the coefficients of like powers of ε, we get first that all φ_k's are harmonic functions. Moreover, we have from (1.20) and (1.21)

$$\left.\begin{aligned} v_0 &= \varphi_{0y} = 0, \\ v_1 &= h_{1t} + u_0 h_{1x} + w_0 h_{1z}, \end{aligned}\right\} \quad \text{at } y = -h. \tag{1.25}$$

Similarly, we expand $\bar{\eta}$ in (1.19) in the form

$$\bar{\eta} = \eta_1 + \varepsilon\eta_2 + \varepsilon^2\eta_2 + \cdots, \tag{1.26}$$

and find from (1.22) the free surface condition at $y = \varepsilon\bar{\eta}$,

$$\begin{aligned} v_0 &= 0 \quad \text{and} \\ v_1 &= \eta_{1t} + u_0\eta_{1x} + w_0\eta_{1z} \end{aligned} \tag{1.27}$$

together with

$$\begin{aligned} &\varphi_{0t} + \frac{1}{2}(\varphi_{0x}^2 + \varphi_{0y}^2 + \varphi_{0z}^2) = \text{constant}, \\ &\varphi_{1t} + u_0 u_1 + v_0 v_1 + w_0 w_1 + g\eta_1 = 0 \end{aligned} \tag{1.28}$$

from (1.23).

The first approximation $\varphi_0, u_0 = \varphi_{0x}, v_0 = \varphi_{0y}, w_0 = \varphi_{0z}$, corresponds to a permanent flow. If we take the special case

$$\begin{aligned} u_0 &= \text{constant}, \\ v_0 &= 0, \\ w_0 &= \text{constant}, \end{aligned}$$

we can transform to a coordinate system with the x-axis in the direction of this constant flow and denote the velocity by U. In this case we have $\varphi_0 = Ux$ and the constant in (1.23) is equal to $\frac{1}{2}U^2$. Then we have the boundary condition from (1.25),

$$v_1 = Uh_{1x} \quad \text{at } y = -h, \tag{1.29}$$

and at the free surface $y = \varepsilon\bar{\eta}$, the coefficient of ε for (1.27) and (1.28) gives

$$\begin{aligned} \varphi_{1y} &= \eta_{1t} + U\eta_{1x}, \\ \varphi_{1t} &+ U\varphi_{1x} + g\eta_1 = 0. \end{aligned} \tag{1.30}$$

Instead of putting this condition (1.30) at $y = \varepsilon\bar{\eta}$, we put it at $y = 0$. Assuming that φ_1 admits an expansion in powers of $\varepsilon\bar{\eta}$, we then have

$$\begin{aligned} \varphi_{1x}(x, \varepsilon\bar{\eta}, z) &= \varphi_{1x}(x, 0, z) + \varepsilon\bar{\eta}\varphi_{1xy}(x, 0, z) + \cdots \\ &= \varphi_{1x}(x, 0, z) + \varepsilon\eta_1\varphi_{1xy}(x, 0, z) + \mathrm{O}(\varepsilon^2), \end{aligned}$$

which leads to a modification of the terms of second order or higher. Hence the first approximation gives the following set of linear equations for φ_1 and η_1:

$$\begin{aligned} &\varphi_{1xx} + \varphi_{1yy} + \varphi_{1zz} = 0, \\ &\varphi_{1y} = h_{1t} + Uh_{1x} \qquad \text{at } y = -h, \\ &\left.\begin{aligned} &\varphi_{1y} = \eta_{1t} + U\eta_{1x} \\ &\varphi_{1t} + U\varphi_{1x} + g\eta_1 = 0 \end{aligned}\right\} \quad \text{at } y = 0. \end{aligned} \tag{1.31}$$

For a fixed flat bottom, h_1 is constant so that $h_{1x} = h_{1t} = 0$. For smooth functions, one can easily eliminate η_1 in the surface condition and obtain the formulation for the first-order approximation (dropping subscript 1):

$$\begin{aligned} &\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0, \\ &\varphi_y = 0 \qquad \text{at } y = -h, \\ &U^2\varphi_{xx} + 2U\varphi_{xt} + \varphi_{tt} + g\varphi_y = 0 \quad \text{at } y = 0. \end{aligned} \tag{1.32}$$

Here the surface elevation η can be computed by

$$\eta = \frac{-1}{g}\left(\varphi_t + U\varphi_x\right). \tag{1.33}$$

Now given initial conditions, problems defined by (1.32) can be solved by means of the Laplace or Fourier transform method. As for illustration, we shall consider a few simple examples in Chap. 2.

1.3.2 Small Amplitude Waves in a Small Velocity Flow Field

Here we derive a free surface for unsteady waves superimposed on the steady free surface generated by a steady velocity field. This steady field may be generated by an object positioned in a constant parallel flow field. In general this leads to a very complicated condition, however if the magnitude of the velocity is small it can be simplified significantly. If no waves are present the magnitude of the velocity is characterised by a small non-dimensional Froude number $F = \frac{U}{\sqrt{gL}}$, where L is some length scale that plays a role in the problem, for instance the length of the disturbing object. It is assumed that this Froude number is small. In Sect. 5.2 we consider the diffraction of short waves if the steady flow field is generated by a parallel flow and is disturbed by a blunt object such as a sphere or a circular cylinder. In these cases we take for L the radius of the sphere or cylinder. Here we derive the free surface condition for such a case.

The easiest way is to follow the derivation, presented in Sect. 1.3.1, to determine a useful formulation for the steady potential. In this case of constant water depth the only small parameter is the Froude number $F = \frac{U}{\sqrt{gL}}$. Again we assume that the deviation of the free surface around $y = 0$ will be small. However we can not say that the free surface elevation is of $O(F)$. The order of magnitude of the elevation follows from the derivation and will turn out to be $O(F^2)$. For the steady case the kinematic free surface condition (1.11) becomes

$$v = u\eta_x + w\eta_z. \tag{1.34}$$

We assume that u, v and w are of the same order of magnitude $O(F)$. Hence for small values of η the kinematic condition reduces to

$$v = 0 \quad \text{at } y = 0. \tag{1.35}$$

The dynamic free surface condition now determines the order of magnitude of the corresponding free surface elevation. If we assume that in the far field the potential equals the unperturbed parallel flow Ux we obtain

$$\eta = \frac{-1}{2g}(u^2 + v^2 + w^2 - U^2). \tag{1.36}$$

Because of the specific form of the free surface condition (1.35) the steady potential described here is called the *double body* potential. For this potential we use the notation φ_r, the velocity components are written as $(u_r, v_r, w_r) = \nabla\varphi_r$ and the free surface elevation as η_r. If one is interested in the total steady potential one must derive an appropriate free surface condition also describing the *wavy pattern*. This will be done in Sect. 2.4. Our goal here is to derive a linearised free surface condition for the unsteady wave potential.

We assume that the potential φ can be decomposed as follows:

$$\varphi(x, y, z, t) = \varphi_r(x, y, z) + \varphi_0(x, y, z) + \varphi_w(x, y, z, t). \tag{1.37}$$

The potential φ_0 describes the steady wave pattern if waves are not present. Later we will show that this potential $\varphi_0 = \mathrm{o}(\varphi_r)$, while as we have seen $\varphi_r = \mathrm{O}(F)$. For this reason we neglect this term in the low Froude number small wave expansion and write

$$\varphi(x, y, z, t) = \varphi_r(x, y, z) + \varphi_w(x, y, z, t). \tag{1.38}$$

The free surface elevation $\eta(x, z, t)$ is assumed to be of the form

$$\eta(x, z, t) = \eta_r(x, z) + \eta_0(x, z) + \eta_w(x, z, t). \tag{1.39}$$

The function $\eta_0 = \mathrm{o}(\eta_r)$, while $\eta_r = \mathrm{O}(F^2)$, so we neglect η_0 and write

$$\eta(x, z, t) = \eta_r(x, z) + \eta_w(x, z, t). \tag{1.40}$$

We assume that the elevation of the free surface above the level $y = \eta_r(x, z)$ is small $\mathrm{O}(\varepsilon)$. The condition for the wave potential at the bottom remains the same as before, however the free surface condition changes significantly. In principle the two small parameters are independent of each other. If the small Froude number is large compared with ε, we may introduce a new coordinate system $(x', y' - \eta_r(x', z'), z')$. The additional terms in the Laplace equation are small and may be neglected. The additional terms in the free surface condition may be neglected as well. If the two parameters are of the same order of magnitude we may linearise with respect to $y = 0$ directly, else it is defined at $y = \eta_r$. The kinematic condition as in (1.27), becomes

$$v_w = \eta_{wt} + u_r \eta_{wx} + w_r \eta_{wz}, \tag{1.41}$$

and if we use the surface condition the dynamic condition becomes

$$\varphi_{wt} + u_r \varphi_{wx} + w_r \varphi_{wz} + g\eta_w = 0. \tag{1.42}$$

We eliminate η_w by means of differentiation of (1.42) with respect to t, x and z respectively. The additional terms due to differentiation along the double body free surface η_r are $\mathrm{O}(F^3)$ and may be neglected. For the wave potential we obtain the following formulation (we omit the primes):

$$\begin{aligned}
&\varphi_{wxx} + \varphi_{wyy} + \varphi_{wzz} = 0, && \\
&\varphi_{wy} = 0 && \text{at } y = -h, \\
&\left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial x} + w_r \frac{\partial}{\partial z}\right)^2 \varphi_w + g\frac{\partial \varphi_w}{\partial y} = 0 && \text{at } y = 0.
\end{aligned} \tag{1.43}$$

The coefficients in the free surface condition depend on the local velocity. Although the formulation for the wave potential is linear, no simple solutions for a wave pattern can be given. In the case of the diffraction of short wave by a smooth object we will use an asymptotic wave theory. This method is developed in acoustic and electromagnetic theory, it is generally called the *ray method*. In Chap. 5 we present this asymptotic method.