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On Hermeneutics, Geometrical Models and Artificial Intelligence

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On Hermeneutics, Geometrical Models and Artificial Intelligence

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Preface

In several aspects this book is a sequel to our book "On Communication" (Klüver and Klüver 2007). Yet it is of course possible to read and understand this new book of ours without having to read our publication from 2007. When we refer to concepts, definitions, and research results from this book we shall always give the necessary explanations. In other words, we shall do the same as for example J. R. R. Tolkien or J. Rowling did in their sequels to the first novel(s), namely referring to that what had happened before to Frodo and Harry.

As our books before this one could also never have been written without the labor(s) of many students, in particular those who wrote their MA-theses supervised by us. We mentioned their names in the text where they contributed to this book by implementing certain computer programs, designed by us, or performed experiments with computers and/or human probands. We emphasize this fact because our experiences with these students again demonstrated to us that it is nowadays still possible to do ambitious research without much support from funding institutions. The old and venerable paradigm of the unity of research and teaching that characterized the modern universities since the great reformers Humboldt and Schleiermacher is still valid in the sense that there are no better research assistants than engaged graduate students.

Yet there are always exceptions, even from this rule. We wish to thank Wolfram Mach for his help to realize a cooperation with his firm, the *Deutsche Telekom*, in order to develop one of our systems, namely the so-called self-enforcing network (SEN) that is described in Chapter 4. Our thanks of course also go to his superiors who supported the cooperation.

It is a bit unusual that only one author of two writes the preface. My wife and co-author Christina insisted that I should do it because the theoretical and mathematical ideas and the text are principally my responsibility. I should according to her explicitly say this in the preface or else she would not accept a co-authorship. I reluctantly agreed to do this but I wish to emphasize that this book could never have been written without her and that her contributions to this book are much more than just giving me technical help or encouraging the students when they became desperate with respect to their MA-thesis. Christina is, after all, the professional computer scientist in our team and so her indispensable task was to supervise and consult the programmers and to test and validate the programming results.

For a long time Christina and I have been a successful research team and we intend to keep matters this way. Nothing is better for a happy marriage than satisfying common work.

Essen, Germany Jürgen Klüver

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Chapter 1
Introduction: Historical Methodical and Conceptual Frames

A book that simultaneously deals with hermeneutical problems on the one hand and with Artificial Intelligence and the construction of mathematical models on the other seems to achieve the impossible or, to speak in mathematical terms, to solve the quadrature of the circle. Since the rise of the modern physical sciences at the Renaissance and the beginning of Enlightenment mathematics is *the* language of the sciences; hermeneutics in contrast is the traditional methodical tool of the humanities and – to apply a famous quotation of Kipling – never the twain shall meet. In particular the quest for an Artificial Intelligence is a branch of computer science and hence a science based on the construction of mathematical models. Therefore, to combine these very different schools of rational thinking seems to be an enterprise that can only be done in metaphorical ways and hence rather useless ones.

We of course believe otherwise or we would not have written this book. But we are quite aware of the fact that the analysis of hermeneutical problems by mathematical models and the attempt to base some principles of a hermeneutical Artificial Intelligence on such models is nothing less than the attempt to bridge the gulf between the sciences and the humanities. In other words, simultaneously dealing with hermeneutics and mathematical constructs is the walking of a path whose end may be the goal of a unified science. To be sure, we do not intend to reach that old goal in this book. We just try to demonstrate that the conception of a hermeneutical Artificial Intelligence may be more than a metaphor and that it is now already possible to give some preliminary results of our research in this direction. Yet as we know that in the end the possibility of a combination of hermeneutics and Artificial Intelligence has much to do with the old quest for a unified science we start in the first subchapter with a brief historical sketch of classical attempts to reach that goal.

In the center of the wide and widely differentiated field of hermeneutics is the concept of *understanding*. Whole sub disciplines of the social sciences, in particular qualitative social research and cultural anthropology, deal with the question how it is methodically possible to *understand* humans, i.e. their behavior, their world views, beliefs and so on. The same is true for the humanities like history or literature analysis insofar as they deal with texts, written by humans. In all these fields the core problem is to understand, i.e. to interpret the products of human activity. Yet there is a common consent that all human activities must be understood as *socially* founded ones and that, therefore, understanding always means understanding in some social

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sense. That is why we define our subject as *social* understanding. We shall clarify these rather general and a bit vague remarks in the next chapters, but we wish to emphasize already in the beginning that we deal only with the problems of understanding humans as social actors and human social actions. The hermeneutical problem of the interpretation of certain texts is not our subject and not our problem in this book although we show some models how humans possibly understand specific texts.

Neither qualitative social research nor the traditional hermeneutical disciplines are mathematically orientated – on the contrary because usually mathematical methods seem to be very unsuited for the tasks of understanding. Yet we have already shown that some problems of qualitative social research, for example, can be successfully analyzed by several mathematical methods of computer simulations (Klüver and Klüver 2007; Klüver et al. 2006). The subject of this book is to continue and systematize these ideas, namely to demonstrate the possibility of a partial integration of mathematics and hermeneutics.

When we speak of a combination of hermeneutics and mathematical methods, though, we have to clarify our usage of "mathematical methods". We do not mean, of course, the classical mathematical methods of the natural sciences, namely differential equations, nor do we mean statistical methods. Both mathematical methods have been immensely fruitful not only in the natural sciences but also in the fields of economics and, in the case of statistical methods, also in psychology and quantitative social research. The mathematical methods we shall use to analyze hermeneutical problems are chiefly the so-called techniques of Soft Computing, namely especially artificial neural networks and cellular automata, and computer simulations based on such models. In particular we try to demonstrate that the analysis of the topology of artificial neural networks can be an important contribution to understand understanding. In other words, we do not only construct artificial neural networks that are able to perform some operations or simulations respectively of some basic hermeneutical processes but we also intend to demonstrate that a geometrical, i.e. topological analysis of such networks is a way to understand the empirical processes that these networks simulate. Hence, on the one hand we use these networks not only for the task to simulate processes of understanding. Parts of the cognitive sciences have developed this approach with encouraging success and with different modeling techniques, which is also called the field of cognitive modeling (cf. McLeod et al. 1998; Polk and Seifert 2002; Thagard 1996). We also look on the other hand for mathematical, i.e. topological characteristics of our networks and specific algorithmic operations that can explain the specific behavior of the networks and hence probably of the humans whom these networks simulate. The second approach is still rather new even in the cognitive sciences. Therefore, with the term "mathematical methods" we simultaneously mean the construction of simulation models and the mathematical analysis of these models. In the following chapters we show examples of both procedures.

To put our procedure into a nutshell: The first step is of course the identification and definition of the respective problems like, for example, certain processes of restricted information assimilation by human individuals (see below Chapter 5); after that we have to construct a suited mathematical model, usually in form of an artificial neural network and to implement it into an according computer program; by performing experiments we try to find out observable regularities and in addition try to explain these regularities by the mathematical characteristics of our model. If the models behavior is sufficiently similar to the human cognitive processes that should be modeled we not only have a possible explanation for these processes but also the programs as components of a future Artificial Intelligence: If we can construct suited mathematical models and according computer programs we also may be capable to construct artificial systems that are able to "understand", at least in some ways.[1]

But first we give some overviews to the historical and conceptual framework of our subject.

1.1 Unifications of the Sciences

We only understand what we can make (Source: *Giambattista Vico*)

In the seventeenth century the philosophers and mathematicians Descartes and Leibniz developed the ideas of a *mathesis universalis* (Descartes) and a *mathesis characteristica* respectively (Leibniz). These concepts expressed the idea of a universal mathematical science, i.e. a science with a universal mathematical language where all problems can be formulated in a mathematical manner and can be solved by arithmetical calculations. Accordingly Leibniz declared "*calculemus*" (let us calculate) if some differences of opinion should occur.[2]

It was not by chance that at the beginning of the modern natural sciences the dream of a unified mathematically formulated science simultaneously emerged. Both Leibniz and Descartes had made important contributions to mathematics and hence they believed that it must be possible to unify all forms of rational thinking by the new and powerful mathematical tools that they and others had developed. The rapid progress of mathematics and physics in the following decades and centuries seemed to be a verification of the dream of Descartes and Leibniz. Possibly Leibniz got the idea of a universal science because he invented the system of dual numbers, i.e. the possibility to represent all numbers as combinations of 1 and 0. As 1 and 0 represent the simplest alternative – yes or no – all problems should be expressed as a combination of such simple alternatives. Three centuries later Shannon in his mathematical theory of information formalized the same basic idea with his famous concept of "bit": One bit is the simplest form of selection, namely between 1 and 0 (Hartley 1928; Shannon and Weaver 1949).

[1] It is not important here if the artificial systems "really" understand or if they are "only" simulating understanding. We shall deal with the discussion about "strong" versus "weak" AI in Section 1.4.

[2] Actually the basic idea of such a formal calculus is much older. Already in the thirteenth and fourteenth century the scholastic philosopher Raimundus Lullus developed the program of an *ars combinatoria*, i.e. a formal method for combining different concepts in order to solve philosophical problems. Yet the idea of Lullus, naturally, found not much interest. Nowadays it reminds of certain modern computer programs that operate in a similar manner.

Yet already in the eighteenth century the Italian philosopher Giambattista Vico denied the possibility of even a natural science by declaring a famous argument: God understands the world but only because he created it. Therefore, humans cannot understand nature because they did not make it. They just can understand their own creations, namely human history, society, and of course creations of their mind like literature or the arts. In modern terms Vico argued for the possibility of the humanities and denied the possibility of the sciences, as far as they deal with nature.[3]

Vico apparently did not see that the natural scientists had found a methodical equivalent for the creation of nature with the experimental method. Quite literally natural scientists recreate parts of material reality during their experiments in the laboratory and they are even able to create material artifacts that did not exist before the according experiments; this is quite usual in the chemistry of synthetic materials. The intended experiments with the new *Large Hadron Collider* (LHC) at CERN (autumn 2009) have the goal to demonstrate the recreation of early states of the Universe, perhaps even states like the Big Bang. In this sense physics and other natural sciences have taken Vico's postulate quite literally: They understand nature because they can recreate and even create parts of it – they can make it. In this sense they realized the famous word of Vico "*verum et factum convertuntur*".[4] It is not without irony that the natural sciences successfully applied the principle of Vico and that the humanities were not able to do it in a strict and systematical manner. We certainly often (although not always) understand our own creations but, in comparison to the strict and successful methods of the natural sciences, only in an informal manner.

In any case, whether he had intended it or not, Vico laid the foundations for the growing gulf between the natural sciences and the humanities. The dream of a universal science with a common mathematical terminology had been a product of the rationality of the Enlightenment. During Romanticism and in particular inspired by the philosophers of German Idealism like Herder, Schelling and Hegel the methodical concept of the humanities evolved.

According to the German term *Geisteswissenschaften* (sciences of the mind) the subject of the humanities should be, just as Vico postulated, the creations and products of the human mind. The fact that the humanities could not apply the successful methods of the natural sciences was not understood as a deficit of the humanities but as evidence for the distinguishing marks of their subject. The human mind and its products cannot be understood via the mathematical and experimental methods of the natural sciences but must be analyzed in a special manner. Therefore, the humanities are rational ways of thinking too but because of the complexity of their subjects they have rational methods and standards in their own right. In consequence the humanities evolved in parallel to the natural sciences, became institutionalized in the modern universities, and created a rational universe of their own. In the middle

[3] In contrast to the possibility of the natural sciences Vico accepted the possibility of a science of mathematics because the objects of mathematics are also our own products (Vico 1947).

[4] That what is true and what has been made converge (our translation).

of the twentieth century the gulf between the world of natural sciences and that of the humanities seemed so absolute that C. P. Snow characterized this situation by his famous remark about the "Two Cultures" (Snow 1963).

The main reason for this still existing and very influential gulf between the two cultures of scientific enterprise may be characterized by the old and venerable term of *Hermeneutics*. The concept of Hermeneutics can be translated as the art of interpreting and hence understanding human actions and their according products like society, the arts, literature and so on. It is principally not possible, so the early founders and advocates of the humanities, to explain the world of the human mind by the methods of the natural sciences, although these have been and are very successful in dealing with material reality. Human beings cannot be explained this way but must be understood in a hermeneutical way. *In nuce*, the task of the natural sciences is to *explain* material reality in a mathematical and experimental way; the according task of the humanities is to *understand* human beings in a hermeneutical manner. As it is not possible to reduce one methodical way to the other or to combine both methodical procedures, according to the partisans of the hermeneutical approach the differences between the two cultures is a methodical necessity. It is not by chance, by the way, that the gulf between the different methodical ways even exists within the social sciences, namely the difference between the so called qualitative and the quantitative methods of social research. We shall systematically deal with the concepts of hermeneutics, explaining, and understanding in the next chapters.

Despite this differentiation between the two cultures since the nineteenth century there have always been attempts to realize the idea of a universal science. The well-known program of a "Unified Science" of the philosophical school of Logical Positivism (e.g. Carnap 1950) is just one of them. All sciences should, according to this program, be only based on empirical facts and formal, i.e. mathematical logic; scientific theories, regardless of which domain, are then nothing else than logical combinations of empirical facts. By the way, it is interesting and not by chance that despite the failure of this program one important branch of Artificial Intelligence research, namely the research in and construction of rule based systems or expert systems respectively follows the same idea: Expert systems mainly consist of (a) facts that are stored in a so called knowledge base and (b) logical rules for the combination of these facts, e.g. in order to give a diagnosis for certain failures of systems like diseases or technical failures, and to derive a "therapy" from the diagnosis, like medical therapies or technical repairs. Yet the program of Logical Positivism failed for several reasons and with respect to the problem of hermeneutics the program of the Unified Science was not a solution of the problem but simply a denying of its existence.[5]

[5] The program of a "real" Artificial Intelligence, realized by the construction of sophisticated expert systems, failed too and probably for the same reasons: This program also simply denied the fact that human thinking is indeed partly only understandable in terms of hermeneutics. It must be noted, however, that expert systems are nowadays widely applied to all kinds of problems, e.g. in medical, technical, and economical domains.

Despite or perhaps because of the failure of the program of a Unified Science of the logical positivists many other attempts were made with different concepts and methods. To name only a few: Under the influence of Wiener`s creation of cybernetics (Wiener 1948) many scholars tried to translate the "art of steering", as the Greek word cybernetics must be translated, into certain fields of the humanities, e.g. into the methodology of education (Cube 1965); concepts like feed back systems and feed back loops are still prominent in different scientific disciplines. Similar ideas were developed in the theory of autopoietic systems by the biologist Maturana (1975) and the theory of synergetic (Haken 1984), i.e. the theory of the effects of the combination of different factors. Yet in the humanities all these approaches were either neglected or only taken as sources of new concepts. The scholars in the humanities, as for example the theoretical works of the sociologist Luhmann, used these concepts only metaphorically and neglected the fact that a scientific usage of concepts from the natural sciences only makes sense if one uses the according scientific methods too (cf. Klüver 2000; Mayntz 1990).

The newest and perhaps most promising quest for a unified science is the research program on complex systems theory. The according research is often identified with the famous Santa Fé Institute for Research on Complex Systems (for a general description cf. Waldrup 1992). The basic concept of these lines of research is to treat empirical domains as complex dynamical systems. Accordingly research is done on general properties of complex dynamical systems via the use of certain computer programs like cellular automata on the one hand and the reformulation of scientific subjects in terms of complex systems theory and according empirical research on the other. The program of research in complex systems theory is formulated from its beginning as an interdisciplinary enterprise that shall include and unify all scientific disciplines as well as the humanities. Because our own work is also based on the general foundations of complex systems theory we shall deal with some concepts of this approach in a later subchapter.[6]

The important attempts to unify the sciences have always been, like the program of Logical Positivism and in accordance to the dreams of Leibniz and Descartes, attempts to introduce mathematical and experimental methods into the fields of the humanities. To be sure, there exist attempts to "understand" nature in a hermeneutical way, i.e. to reformulate the natural sciences according to the methodical program of the humanities. One of the best-known attempts in this direction is the *Farbenlehre* (theory of colors) by Goethe (cf. e.g. Böhme 1980). But the success of the natural sciences was and is so overwhelming that a unified science in a concrete meaning of this term can only be either a reduction of hermeneutical methods to those of the mathematical sciences, i.e. to develop the concept of a mathematical and experimental hermeneutics, or to combine both methodical ways. It is probably not surprising that we undertake in this study the first way, mainly because a simple

[6]We are certainly by far not the only scholars in the social and cognitive sciences, who use the concepts and methodical approaches of "complexity science". For a detailed and systematic overview of the influence of complexity science on the social sciences cf. Castellani and Hafferty (2009); an impression of the merging of complexity science and cognitive ones gives the collection of Polk and Seifert (2002).

combination of both methods would be strictly speaking just mirroring the same situation as it is now: There would be still fields of hermeneutical thinking on the one hand and those of the application of natural sciences methods on the other and the two domains would be as separated as ever. In other words, the goal of a unified science demands an integration of the hermeneutical methods into the field of mathematical thinking – a mathematical hermeneutics.

Even if one accepts the fact that the program of Logical Positivism was far too restricted to give a foundation even only for the natural sciences, other attempts to found the humanities on the mathematical and experimental methods of the natural sciences also failed or captured only fragments of the subjects of the humanities, as we already mentioned. The most important reasons for these failures are certainly the impossibility of repeatable experiments in most of the humanities on the one hand and the fact that the usage of the traditional mathematical methods of the calculus on the other hand yield only in very few cases important results in the humanities.[7] To be sure, the application of the tools of differential equations has been successful in economics and experimental methods have been applied with equal success in psychology. But these disciplines do not belong any more to the humanities proper and psychology has not succeeded in combining experimental *and* mathematical methods with the exception of elaborated statistical methods. Accordingly the core of the humanities is still and exclusively founded on hermeneutical methods and uses concepts of the natural sciences, if at all, only in a metaphorical way.

If practically all attempts to unify the different sciences, i.e. to unify the two cultures, have failed the question arises if such attempts are indeed fruitless because the task is impossible. Hence, why another attempt despite the discouraging experiences of the past?

Our belief that another attempt to lay down the foundations of a mathematical hermeneutics would be worthwhile, namely the proof that it is possible to analyze the classical hermeneutical problems in a mathematical way, is founded chiefly on three scientific achievements of the last century. The first one is the rise of modern structural mathematics and mathematical logic, which allow the mathematical treatment of hermeneutical problems in a fashion more suited to these problems than by the classical methods of the calculus. The second one is the mentioned program of research on complex dynamical systems. In our belief the respective conceptual and methodical foundations of this program offer a new chance for another attempt that is not restricted like its predecessors. The third and for us most important one is the invention of the computer and the possibility to introduce the method of computer simulation and hence computer experiments into the field of the humanities. We shall deal with these achievements in the next subchapters.

In particular, even only sketches of characteristics of an Artificial Intelligence, founded on these three achievements, would be a step according to the postulate of Vico: We can understand our mind because we can make an artificial one. Perhaps this is a bit more than just a hopeless dream or a utopia respectively.

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[7] Examples for such cases are given, for example, in Epstein (1997).

In the rest of the chapter we shall discuss some fundamental problems concerning the questions what meaning the term of Artificial Intelligence has in our conceptual frame and why in particular philosophers of Artificial Intelligence often understand hermeneutics as a proof of the impossibility of a "real" Artificial Intelligence. The assumption that an Artificial Intelligence is principally not able to understand in a hermeneutical manner can be found, as we shall demonstrate, even in contemporary Science Fiction.

1.2 The Importance of Structural Mathematics and Computer Models

Since the rise of modern science in the sixteenth and seventeenth century its success was always explained by the combination of experimental method and mathematical theory construction. Only the language of mathematics apparently was suited for the generation of exact and precise knowledge and only by such knowledge science was able to serve as the foundation of modern technology, i.e. science based technology, and become this way the "first of productive forces" (Habermas 1968) and the foundation of modern economics. The fact that theoretical physics was and probably still is the paradigm of science proper is in particular explainable by the mathematical formulation of its theories. Therefore, it is no wonder that the famous Einstein equation $E = mc^2$ is for many laymen and scientists alike *the* symbol of science as it should be. Accordingly, no field of knowledge may be called as "science" if the respective knowledge cannot be formulated in mathematical terms.[8]

Because those disciplines that deal with human beings, society, and human cognition were for a long time not able to construct mathematical models and theories these fields of knowledge were not "sciences" but "humanities". To be sure, for example in psychology already in the nineteenth century the German psychologist Wundt and his school tried to formulate mathematical laws of human perception and contemporary psychology has gained a lot of statistically interpreted knowledge. That is why many psychologists define themselves rather as natural scientists than as representatives of the humanities. In economics, to take another example, Keynes and his followers have developed at least in macroeconomics mathematical tools akin to those of theoretical physics and have applied these tools with remarkable success to macro-economical processes. Therefore, economists alone of the social scientists may win a Nobel Prize. Yet it is an undeniable fact, as we remarked, that at the core of the sciences of man – the humanities – there are none or only very simple mathematical models. Sciences in the sense that mathematical models are the foundation of theory construction and of the interpretation of empirical data are, with the exception of macroeconomics, still only the natural sciences.

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[8] Already at the beginning of modern (natural) science one of its founders, namely Galileo Galilei, postulated this thought when he declared, "the book of nature is written in mathematical letters". More than a century later Immanuel Kant, the probably greatest mind of the Enlightenment, remarked that each theory of nature is only so far science as it contains mathematics (Metaphysical Foundations of the Natural Sciences).

The reason for this fact is probably that the classical mathematical tools of, e.g. theoretical physics are not suited for the modeling of "real" complex systems like social or cognitive ones. When the renowned evolutionary biologist Richard Lewontin once remarked that in comparison to the complexity of social systems the problems of molecular biology seem to be "trivial" (Lewontin 2000) then this may be a reason for the difficulties of constructing mathematical models for the problems of the "soft" sciences. Indeed, social-cognitive systems are, e.g., characterized by their capability of changing their own structure, they frequently consist of different levels that permanently interact, and in particular they generate their own complexity by the continuous interdependence between social and cognitive processes: the cognition of social actors is dependent on their social milieu and the social milieu is constructed by the thoughts, world views, and the according actions of these actors. Problems of such complexity are not to be found in the complex natural systems – the natural sciences deal with systems that are in comparison "simple".[9] The classical mathematical methods that are very successful for the analysis of these systems were not constructed to deal with social-cognitive complexity.

Fortunately, at least for the development of mathematical foundations of the social-cognitive sciences, during the nineteenth century a new kind of mathematical thinking emerged that can be called "structural mathematics": Mathematics became "pure" in the sense that it was defined as the general theory of formal structures. These structures may be algebraic ones like those expressed in group theory, logical ones like those George Boole formulated in his algebra of logic, or topological ones as foundations for the problems of the continuum. Mathematics finally became a science in its own right and not primarily a tool for the natural sciences. This development is in particular clearly characterized by Cantor's theory of transfinite sets: Mathematics emancipated itself from empirical reality. On first sight it seems a bit paradoxically that just the *pure* mathematics also became the foundation of the computer and the according simulation software; on a second sight the fact that each computer basically consists of many coupled logical nets can quite easily explain this paradox. In the famous work of Bourbaki, which influenced the mathematical curricula for more than a generation, the whole building of mathematics then was reconstructed on the foundations of set theory and mathematical logic; Bourbaki, hence, can be understood as the peak of this development.[10]

In our opinion it is not by chance that nearly at the same time of Bourbaki, i.e. since the fifties of the last century, new mathematical modeling techniques emerged. On the one hand the new structural mathematics had been fully developed at this time and the first accordingly revised mathematical textbooks began to determine mathematical education. On the other hand based on the work of early pioneers

[9] "Simple" is, of curse, to be understood only in a relative manner. Many of the greatest minds of the human species have demonstrated how difficult it is even to understand these "simple" systems.

[10] Several social theorists rather early saw the possibilities the new mathematical ways could offer to the social-cognitive sciences. For example, Kurt Lewin, one of the founders of *gestalt* theory, spoke of a "topological psychology" and postulated the introduction of vector algebra into the social-cognitive sciences (Lewin 1969). Bourbaki, by the way, was the pseudonym of a group of French mathematicians.

like Konrad Zuse and John von Neumann the first computers were built and offered new ways for the construction of mathematical models. Indeed, the influence of the computers on the ways of mathematical thinking became so strong that one of the most famous representatives of early research in Artificial Intelligence (AI), namely Douglas Hofstadter, spoke of a renaissance of the experimental mathematics of the eighteenth century by the computer (Hofstadter 1985).

To be sure, the well established and proven mathematical methods like statistics and differential equations still are at the core of the usage of mathematics via the computer. Yet at that time new mathematical models were invented and analyzed that were oriented at natural processes and in particular developed in order to solve problems of life, mind and society. For example: In 1943 McCulloch and Pitts described the first mathematical model of a "neural network", i.e. a model of the operations of the brain. The operation of this artificial network was called by its inventors a "logical addition", which meant the characterization of brain processes in terms of logic and mathematics. This was the start of the development of artificial neural networks, the main tool we use for our purposes in this book. A whole branch of computer science, the so-called neuro-informatics, concentrates on the development and applications of artificial neural networks. Neural networks, as we shall call them in this book, are basically nothing else than "dynamical" graph structures, whose connections can be varied according to certain rules and which can be described by graph theoretical and topological characteristics. In this sense they are an offspring of structural mathematics. Yet they can be applied to practical problems and thoroughly studied with respect to their fundamental features only by their implementation in according computer programs. Therefore, neural networks are a fascinating type of mathematical models that are based on structural mathematics and on their usage as computer programs likewise.

A second example is the development of cellular automata in the Fifties of the last century, mainly connected with the name of John von Neumann.[11] Von Neumann, a mathematical universal genius, was interested in the construction of models of living systems, in particular with respect to their ability of self-reproduction. Cellular automata consist of a grid of cells and rules of interaction between the cells. By these rules the cells change their state in dependency of the state of other cells. Similar as in the case of neural networks cellular automata can be characterized by a certain "geometry" or topology respectively, namely the topological definition of neighborhood, and by certain rules of interaction. Yet cellular automata are also only useful by implementing them into respective computer programs. Even simple cellular automata unfold rather complex dynamics that are impossible to analyze without the use of computers. The probably most famous cellular automaton, the Game of Life by Conway, is an instructive example that even a cellular automaton with very simple rules can be thoroughly analyzed only via a computer program.[12]

[11] Actually von Neumann got the basic idea for cellular automata from Stanislav Ulam, the mathematical father of the hydrogen bomb.

[12] Conway did his first experiments with the Game of Life without using computers. He instead used little black and white plates like those known from the game of Go. Records tell us that soon his working room and other rooms were full of these plates and that it was literally impossible to

Evolutionary algorithms, developed nearly at the same time in the late Sixties by Holland and Rechenberg, are a third example. As the name suggests, evolutionary algorithms are constructed as formal models of biological evolution, i.e. of the processes of mutation and recombination of genes and of natural selection. These algorithms are mainly used as optimization methods and are also useful only as specific computer programs. Yet some of their main characteristics, in particular their convergence behavior, can be understood by applying some results of metrical topology, as for example Michalewicz (1994) has demonstrated. Again we find the combination of structural mathematics and the usage of computer programs.[13]

Last but not least the so-called Fuzzy-Methods are basically nothing else than an extension of classical mathematical logic and classical set theory, *the* foundations of modern mathematics. Although their invention by Zadeh in the Sixties was motivated by the attempt to capture some aspects of human thinking that are not exactly describable by classical logic Fuzzy-Methods have also demonstrated their fruitfulness by their implementation as specific computer programs, in particular as extensions of so-called production systems or expert systems respectively. In all these cases, to put it into a nutshell, structural mathematics and the computer together made it possible to develop and to apply new forms of mathematical models, which can be applied to problems outside of the range of traditional mathematics.[14]

These new modeling techniques, of which we shall chiefly use neural networks in this book, hence may be called mutual children of structural mathematics and the computer. They are the main *methodical* reason why we think that a new attempt to apply mathematical techniques to problems of hermeneutics is worthwhile. A more *conceptual* reason is, as we mentioned, the framework of complex systems theory, with which we shall briefly deal in the next subchapter.

A certain terminological remark is in this context in order. Frequently a distinction is made between *mathematical* models on the one hand and *computational* models on the other; the last term is used when formal models are based on computer programs and when the modeled system's behavior is not given by the solutions of according equations but by the algorithms of the respective programs and the simulation runs of the program. In our opinion this distinction is rather superfluous. Of course for example neural networks are nothing else than certain mathematical algorithms and the models of social and cognitive processes, built via the construction of adequate neural networks or other techniques, are consequently mathematical

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capture different developments of his cellular automaton in dependency of different initial states (cf. Levy 1993).

[13] Holland, Rechenberg, and other pioneers in the field of evolutionary algorithms based their models on the famous "modern synthesis" in evolutionary biology (Huxley 1942). In the meantime, though, recent developments in evolutionary biology demonstrate that the modern synthesis is too simple to capture the complexity of biological evolution. In particular it is apparently necessary not only to consider simple genes as the fundamentals of evolution but also to differentiate between "toolkit" genes – the sort of genes functionally known since Mendel – and so-called regulator genes (cf. Carroll 2006).

[14] Basically of course a computer itself is nothing else than a huge ensemble of logical circuits, i.e. a technical application of mathematical logic.

models. To be sure, as we already mentioned, these are not mathematical models in the tradition of the mathematical natural sciences, mainly based on the calculus. But because the complex systems the social and cognitive sciences have to deal with need another form of mathematical modeling it seems rather strange to us not to speak of mathematical models in this context. It may well be the case that in particular evolutionary theories in the social, cognitive and even biological sciences need such new forms of mathematical models because they are at their logical and structural core algorithmic ones (cf. Dennett 1995; Klüver 2003). Therefore, the difference between mathematical models and computational models should be renamed as the difference between classical mathematical approaches and those new ones that are better suited to model social and cognitive complexity.

By the way, even a superficial glance at formal systems like, e.g., cellular automata and Boolean networks shows the mathematical basic features of them. We already mentioned the fact that these systems can and must be described on the one hand by their respective topology (see below Section 1.3.1). On the other hand rules like the transition functions of cellular automata or the logical functions of Boolean networks must be understood as basic algebraic properties of these systems. For example, the set of transition rules of a reversible cellular automaton is in terms of algebra an algebraic group, although not necessarily an Abelian one. This example demonstrates that these formal systems can be understood as classical mathematical structures, namely by a combination of algebraic and topological structures; we shall come back to this aspect below. In future publications we shall give some results of according investigations.[15]

1.3 A Short Glossary on Complex Systems, Simulations, and Communication Theory

1.3.1 Complex Systems, Attractors, and Trajectories

If one wants to model some empirical domain the first step, of course, is to determine the conceptual characteristics of the model. In other words, from which perspective shall the model be constructed? To be sure, there are many different ways to solve that problem. Yet in science it has become rather common to describe the respective domain as a "system", which means in the most general sense that the domain is understood – and described – as a set of "elements" that are connected by specific "relations". If one simply wants a static description of that system it is enough to characterize the relations and define that way the "structure" of that system as the set of all relations between the elements. A simple example is, for instance, a social group. The elements of that system are the members of that group and the social relations are, e.g., defined by a hierarchical structure: some members are more important

[15] Reversible cellular automata are t-invariant systems, i.e. they allow computing their states in the future and the past. An Abelian group is one with commutative operations.

than others and may influence the members with a lower social status. In the behavioral sciences groups with an informal social hierarchy are called groups with a "pecking order". Another example for a system is our planetary system, consisting of the elements "Sun" and the formerly nine planets.[16] Their relations are described by Newton's law of gravitation, which means that the relations are defined by the mutual gravitational attraction of the planets.

Yet a purely structural description of systems is often not enough. Most interesting empirical domains are characterized by a certain dynamics, which means that these systems change over time. Because of this fact the mathematical tools of the calculus, i.e. differential equations, had become the most prominent mathematical methods to analyze the time dependent behavior of systems. Already one of the first mathematical laws of physics, namely the famous law of fall by Galileo Galilei, is written as a differential equation $ds/dt = g$, that is acceleration in dependency of the time t. Hence, a definition of a system must contain in addition rules of interaction between the elements that describe the behavior of the system during a certain time of observation.

The modeling of physical systems has an old tradition in contrast to that of social and cognitive ones and definitions developed for the natural sciences are not necessarily useful for the social and cognitive sciences. Therefore, it is necessary to give some general definitions of the dynamical behavior of complex systems.

At a certain time t a complex system is in a state S_t, which is usually defined by an aggregation of the states of the respective system's elements. For example, a social group may consist of nine members and these members are in a certain state of mind, i.e., they are angry, happy, or indifferent. Then the state of this group can e.g., be defined by the arithmetical mean of the emotional states of the members or by other mathematical forms of aggregation. The state of the whole group can accordingly be described as, for example, "more frustrated than happy with respect to all members". If a new member enters the group then the (emotional) state of the group will change, according to the state of the new member. In addition, if the group members interact according to the social rules of the group hierarchy then the emotional state of some members may change too – they become even more frustrated or happier. The new state(s) of our system hence is a result of the rule governed social interactions of the group members.

Accordingly the state of the solar system is defined by the number of the different planets and their geometrical positions at time t in the space the solar system occupies. As these geometrical positions change according to Kepler's and Newton's laws, which means according to the gravity caused attractions, the new state(s) of the solar system is again a result of the interactions between the system's elements.

In a more general and formal sense we obtain the following definition:

If f denotes the set of the rules of interaction of a certain system, if S_1 denotes the first state, i.e. an initial state, and $f^n(S)$ the applications of the rules of interaction on the state S for n times, then

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[16] Recently the former planet Pluto is not considered as a planet anymore.

$$f^n(S_1) = S_{n+1}. \tag{1}$$

The dynamics of a complex system is accordingly defined as the succession of the system's states generated by the iterative application of the rules of interaction. Note that the term "application" means in the empirical sense that the elements of the system interact according to the specific rules; in a model theoretical sense "application" of course means that the rules are applied on the model. For the sake of brevity we speak in both cases of the application of the rules.

By defining the state of a system as we did in the previous examples we may also define the set of all possible states of a system. "Possible" means all states that are generated under all conditions that may determine the system's fate. For example, a predator-prey system may reach a state where the predators have eaten all prey, which will cause the predators to starve. In this case the system will vanish but this is nevertheless a possible state. Another possible extreme state may be that all predators starve because they could not get the prey. Then the prey's population will permanently grow until it reaches the carrying capacity of their biological environment, i.e., the maximum number of prey that can be fed. Yet whether a certain eco-system will reach such extreme states is another question: that depends in this case on the initial states, the fertility rates, and other parameters. Therefore, we must distinguish between the set of all possible states and the subset of states a certain system will factually reach. This subset we call the *trajectory* of the system and the set of all possible states is called *the state space* of the system.[17] In more informal terms the trajectory can be understood as the path of the system through the space of all possible states. For visualization purposes the state space of a system is often represented as a plane and the trajectory as the according curve in this plane. We shall give examples of such visualizations below. But note that a plane is only a two-dimensional space and that state spaces are frequently defined by much more dimensions. For example, the state space of our eco-system is characterized by the size of the two different populations, by the biological gender of one prey or predator respectively, the age of the different animals, and so on. Hence, visualization usually is only possible by neglecting some dimensions and by concentrating on only two.

The two-dimensional visualization of a trajectory is not the same as the time dependent process curves one frequently can see in textbooks and other scientific publications. In this case one dimension of the plane is always the time, usually represented by the x-axis. In the case of a trajectory the dimensions of the plane are defined by certain components of the states as in the example of the eco-system. To be sure, one can add time as a third dimension but only for illustrating purposes. As an example we show in the first figure (Fig. 1.1) the time dependent curve of the behavior of a predator-prey system, programmed as a cellular automaton. The two curves represent the change of the populations of predator and prey respectively.

[17] In the natural sciences frequently the term "phase state" is used instead of state space.

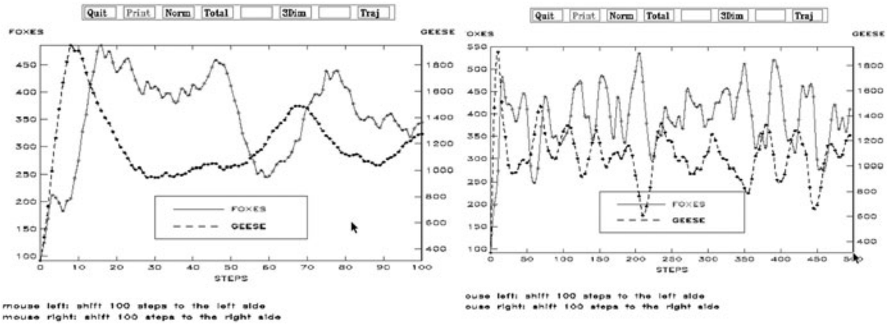


Fig. 1.1 Population variations of predator and prey

The second figure (Fig. 1.2) is the trajectory of the same system. The dimensions of the state space are defined by the size of the respective populations.

The third figure (Fig. 1.3) demonstrates the trajectory of the same system as in Fig. 1.2, but with time added as a third dimension.

Note that for visualization purposes the curve is continuous. Factually it is a discrete succession of different points in the state space, i.e., the different states of the system.

Now consider a case when the rules of the system are still operating but the trajectory reaches a certain point in the state space and stops there. This particular state, which the system does not leave anymore, is called a *point attractor*: the system is "drawn" to it by its rules of interaction. If the system reaches a certain state, generates a second one, then a third one and then reaches the first state again,

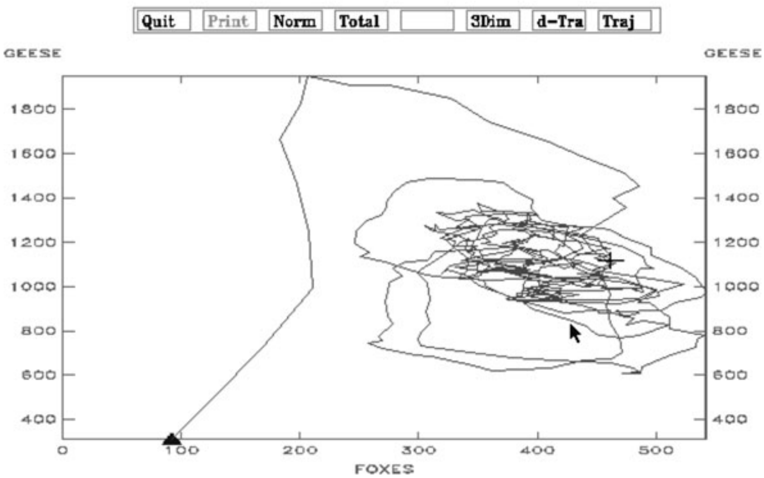


Fig. 1.2 Trajectory of the same system

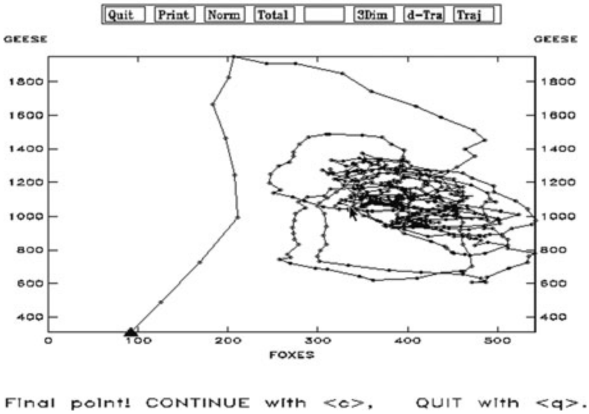


Fig. 1.3 Trajectory as path in state space and time

then this succession of states is called a *simple attractor* of period 3. Accordingly a point attractor is also called an attractor of period 1. In a more formal sense a point attractor S_A can be defined as

$$f(S_A) = S_A. \tag{2}$$

An attractor of period n can be defined the following way: Let $S_N = (S_1, S_2, \ldots, S_n)$ be a succession of n states and $S_i \in S_N$. Then S_N is an attractor of period n if

$$f^n(S_i) = S_i, \text{ for all } S_i \in S_N \tag{3}$$

An attractor of period n, n > 1, hence, is not a single state but a part of the trajectory consisting of n succeeding states. The part of the trajectory before the system reaches an attractor is called the pre period of the attractor. The following Figs. 1.4 and 1.5 show a point attractor and an attractor of period 4.

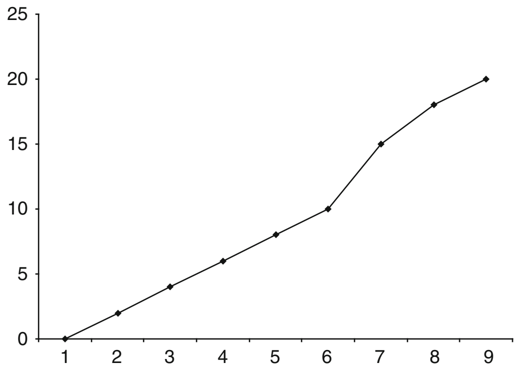


Fig. 1.4 A point attractor

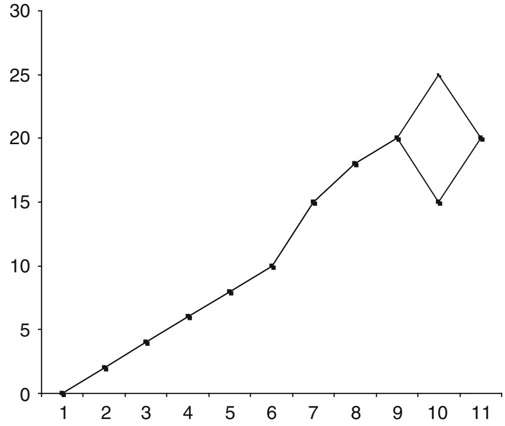


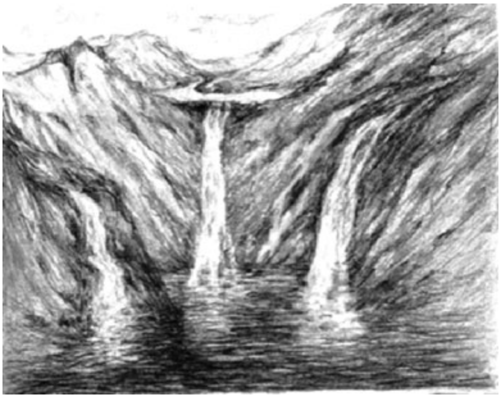
Fig. 1.5 Attractor of period 4

For theoretical purposes it is also useful to define a strange attractor. A strange attractor is, informally speaking, a certain segment of the state space, which the system enters and does not leave anymore. Inside the attractor the system often behaves in a manner that is difficult to predict – the system is *chaotic*. Chaotic systems play a role, e.g., in meteorology, yet it is a theoretically undecided question, if in reality there are in a strict mathematical sense chaotic systems at all. Indeed, the French mathematician Poincaré pointed out that finite systems always are periodic – the theorem of eternal return. For that reason we shall not deal with strange attractors and chaotic systems but always mean simple attractors of finite periods when we speak of attractors.

The theorem of eternal return, though, refers only to so-called *deterministic* systems. A deterministic system contains only deterministic rules, that is rules that always operate if the respective conditions for that rule are fulfilled. An example for such a rule is in a predator-prey system "IF a predator is hungry, and IF a prey can be caught, THEN the predator will (always) catch and eat it". In contrast to purely deterministic rules there also are *stochastic* rules, i.e., rules that operate only with a certain probability if the respective conditions are fulfilled. Many rules in social systems are stochastic, for example: "IF a young woman and a young man have married, THEN they will have children in the next years with the probability of, e.g., 0.7". Of course, the probability of that rule depends on the specific culture the young couple lives in. In Western industrial societies this probability is much lower than in agrarian societies.

Another important concept must be introduced: Whether a system reaches an attractor and if so which one depends on the rules of interaction on the one hand and the respective initial states on the other. It seems rather trivial that a system with certain rules often reaches different attractors when starting at different initial states. Yet a system frequently reaches the same attractor from different initial states. The complexity researcher Kauffman (1995) illustrates this fact with a lake (the attractor), into which different creeks flow. The creeks all start from different springs but flow into the same lake. A little Fig. 1.6 can demonstrate this.

Fig. 1.6 Different initial starting points generate the same attractor



The set of all initial states that generate the same attractor is called the *basin of attraction* of this attractor.[18] The dynamics of a certain system can *in principle* be analyzed by the study of the different basins of attraction. By the way, the set of all basins of attraction of a system is called the *basin of attraction field* of the system. In more mathematical terms one can say that a basin of attraction field divides the set of all initial states into equivalence classes with respect to the specific attractors they generate.

Comparatively large systems frequently generate not only attractor states for the whole system but also local attractors. A local attractor of a system means that the whole system has not reached a point attractor or a simple attractor with a small period but is either still in the pre period of a certain attractor or in a strange attractor. The system, so to speak, is still at unrest. Yet certain subsets of the set of all elements may have already reached a local attractor, i.e. the subset constitutes a subsystem that has generated an attractor state. Formally a local attractor is defined the following way:

Let S be a system, f the set of all rules of local interaction, S' $\subset$ S, i.e. S' is a subset of S, St(S) a state of S, and A a state of S' at the same time of St(S). Then A is a local point attractor if

$$\begin{aligned} &\mathrm{f(A) = A \text{ and}} \\ &\mathrm{f(Z(S)) \neq Z(S).} \end{aligned} \tag{4}$$

The definition of a local attractor with a period p > 1 is accordingly done.

If a sufficient large system is in a strange attractor or in a long pre period the whole system's behavior may seem to be chaotic (in the mathematical sense of the word). If certain subsystems have reached local attractors these sub-states look

[18]The picture was drawn by Magdalena Stoica. By the way, after having made many didactical experiences with this terminology we learned that the tem "basin of attraction" is a rather unfortunate one. Frequently students thought that the lake in the picture is the basin and not the set of different springs. A better term would be, e.g., "set of attractor springs" or "set of attractor initial states". But because the term "basin of attraction" has been established for rather a long time we also use this term.