

Han-Xiong Li
Chenkun Qi

Intelligent Systems, Control and
Automation: Science and Engineering

Spatio-Temporal Modeling of Nonlinear Distributed Parameter Systems

A Time/Space Separation Based Approach

 Springer

Spatio-Temporal Modeling of Nonlinear Distributed Parameter Systems

International Series on
INTELLIGENT SYSTEMS, CONTROL, AND AUTOMATION:
SCIENCE AND ENGINEERING

VOLUME 50

Editor:

Professor S.G. Tzafestas, National Technical University of Athens, Athens, Greece

Editorial Advisory Board

Professor P. Antsaklis, University of Notre Dame, Notre Dame, IN, USA

Professor P. Borne, Ecole Centrale de Lille, Lille, France

Professor D.G. Caldwell, University of Salford, Salford, UK

Professor C.S. Chen, University of Akron, Akron, Ohio, USA

Professor T. Fukuda, Nagoya University, Nagoya, Japan

Professor S. Monaco, University La Sapienza, Rome, Italy

Professor G. Schmidt, Technical University of Munich, Munich, Germany

Professor S.G. Tzafestas, National Technical University of Athens, Athens, Greece

Professor F. Harashima, University of Tokyo, Tokyo, Japan

Professor N.K. Sinha, McMaster University, Hamilton, Ontario, Canada

Professor D. Tabak, George Mason University, Fairfax, Virginia, USA

Professor K. Valavanis, University of Denver, Denver, USA

For other titles published in this series, go to
www.springer.com/series/6259

Han-Xiong Li • Chenkun Qi

Spatio-Temporal Modeling of Nonlinear Distributed Parameter Systems

A Time/Space Separation Based Approach

Han-Xiong Li
City University of Hong Kong
Dept of Manufacturing
Engineering and
Engineering Management
Hong Kong
China, People's Republic

Chenkun Qi
Shanghai Jiao Tong University
School of Mechanical Engineering
Shanghai
China, People's Republic
E-mail: chenkqi@sjtu.edu.cn

and

Central South University
School of Mechanical and
Electrical Engineering
Changsha
China, People's Republic
E-mail: mehxli@cityu.edu.hk

ISBN 978-94-007-0740-5

e-ISBN 978-94-007-0741-2

DOI 10.1007/978-94-007-0741-2

Springer Dordrecht Heidelberg London New York

© Springer Science+Business Media B.V. 2011

No part of this work may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission from the Publisher, with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work.

Typesetting & Cover design: Scientific Publishing Services Pvt. Ltd., Chennai, India

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Distributed parameter systems (DPS) widely exist in many industrial processes, e.g., thermal process, fluid process and transport-reaction process. These processes are described in partial differential equations (PDE), and possess complex spatio-temporal coupled, infinite-dimensional and nonlinear dynamics. Modeling of DPS is essential for process control, prediction and analysis. Due to its infinite-dimensionality, the model of PDE can not be directly used for implementations. In fact, the approximate models in finite-dimension are often required for applications. When the PDEs are known, the modeling actually becomes a *model reduction* problem. However, there are often some unknown uncertainties (e.g., unknown parameters, nonlinearity and model structures) due to incomplete process knowledge. Thus the *data-based modeling* (i.e. *system identification*) is necessary to estimate the models from the process data. The model identification of DPS is an important area in the field of system identification. However, compared with traditional lumped parameter systems (LPS), the system identification of DPS is more complicated and difficult. In the last few decades, there are many studies on the system identification of DPS. The purpose of this book is to provide a brief review of the previous work on model reduction and identification of DPS, and develop new spatio-temporal models and their relevant identification approaches. All these work will be presented in a unified view from time/space separation. The book also illustrates their applications to thermal processes in the electronics packaging and chemical industry.

In the book, a systematic overview and classification on the modeling of DPS is presented first, which includes model reduction, parameter estimation and system identification. Next, a class of block-oriented nonlinear systems in traditional LPS is extended to DPS, which results in the spatio-temporal Wiener and Hammerstein systems and their identification methods. Then, the traditional Volterra model is extended to DPS, which results in the spatio-temporal Volterra model and its identification algorithm. All these methods are based on linear time/space separation. Sometimes, the nonlinear time/space separation can play a better role in modeling of very complex process. Thus, a nonlinear time/space separation based neural modeling is also presented for a class of DPS with more complicated dynamics. Finally, all these modeling approaches are successfully applied to industrial thermal processes, including a catalytic rod, a packed-bed reactor and a snap curing oven.

The book assumes a basic knowledge about distributed parameter systems, system modeling and identification. It is intended for researchers, graduate students and engineers interested in distributed parameter systems, nonlinear systems, and process modeling and control.

Authors are grateful to students, colleagues and visitors in our research group for their support and contributions, and also would like to thank the Research Grant Council of Hong Kong and National Natural Science Foundation of China for their financial support to our research. Last, but not least, we would like to express our deepest gratitude to our wives, children and parents for their love, understanding and support.

Han-Xiong Li
City University of Hong Kong
Central South University
Chenkun Qi
Shanghai Jiao Tong University

Contents

Preface	V
List of Figures	XI
List of Tables	XV
Abbreviations	XVII
1 Introduction.....	1
1.1 Background.....	1
1.1.1 Examples of Distributed Parameter Processes	1
1.1.2 Motivation.....	5
1.2 Contributions and Organization of the Book	7
References	10
2 Modeling of Distributed Parameter Systems: Overview and Classification.....	13
2.1 Introduction	13
2.2 White-Box Modeling: Model Reduction for Known DPS.....	16
2.2.1 Eigenfunction Method.....	16
2.2.2 Green's Function Method.....	17
2.2.3 Finite Difference Method	17
2.2.4 Weighted Residual Method.....	18
2.2.4.1 Classification Based on Weighting Functions	21
2.2.4.2 Classification Based on Basis Functions	23
2.2.5 Comparison Studies of Spectral and KL Method.....	29
2.3 Grey-Box Modeling: Parameter Estimation for Partly Known DPS	31
2.3.1 FDM Based Estimation	31
2.3.2 FEM Based Estimation.....	32
2.3.3 Spectral Based Estimation.....	33
2.3.4 KL Based Estimation	33
2.4 Black-Box Modeling: System Identification for Unknown DPS.....	33
2.4.1 Green's Function Based Identification.....	34
2.4.2 FDM Based Identification	34
2.4.3 FEM Based Identification	35
2.4.4 Spectral Based Identification	36
2.4.5 KL Based Identification	38

2.4.6	Comparison Studies of Neural Spectral and Neural KL Method	38
2.5	Concluding Remarks	41
	References	42
3	Spatio-Temporal Modeling for Wiener Distributed Parameter Systems.....	51
3.1	Introduction	51
3.2	Wiener Distributed Parameter System.....	52
3.3	Spatio-Temporal Wiener Modeling Methodology	54
3.4	Karhunen-Loève Decomposition	54
3.5	Wiener Model Identification.....	57
3.5.1	Model Parameterization	58
3.5.2	Parameter Estimation	59
3.6	Simulation and Experiment	61
3.6.1	Catalytic Rod.....	62
3.6.2	Snap Curing Oven	65
3.7	Summary.....	70
	References	70
4	Spatio-Temporal Modeling for Hammerstein Distributed Parameter Systems.....	73
4.1	Introduction	73
4.2	Hammerstein Distributed Parameter System	75
4.3	Spatio-Temporal Hammerstein Modeling Methodology	76
4.4	Karhunen-Loève Decomposition	76
4.5	Hammerstein Model Identification	77
4.5.1	Model Parameterization	78
4.5.2	Structure Selection	79
4.5.3	Parameter Estimation	83
4.6	Simulation and Experiment	85
4.6.1	Catalytic Rod.....	86
4.6.2	Snap Curing Oven	89
4.7	Summary.....	93
	References	93
5	Multi-channel Spatio-Temporal Modeling for Hammerstein Distributed Parameter Systems.....	95
5.1	Introduction	95
5.2	Hammerstein Distributed Parameter System	97
5.3	Basic Identification Approach	97
5.3.1	Basis Function Expansion	97
5.3.2	Temporal Modeling Problem	100
5.3.3	Least-Squares Estimation	101
5.3.4	Singular Value Decomposition	101
5.4	Multi-channel Identification Approach.....	103

5.4.1	Motivation	103
5.4.2	Multi-channel Identification.....	103
5.4.3	Convergence Analysis.....	106
5.5	Simulation and Experiment	112
5.5.1	Packed-Bed Reactor	113
5.5.2	Snap Curing Oven	116
5.6	Summary.....	119
	References	119
6	Spatio-Temporal Volterra Modeling for a Class of Nonlinear DPS.....	123
6.1	Introduction	123
6.2	Spatio-Temporal Volterra Model.....	124
6.3	Spatio-Temporal Modeling Approach	126
6.3.1	Time/Space Separation.....	127
6.3.2	Temporal Modeling Problem	129
6.3.3	Parameter Estimation	130
6.4	State Space Realization.....	131
6.5	Convergence Analysis	133
6.6	Simulation and Experiment	138
6.6.1	Catalytic Rod.....	138
6.6.2	Snap Curing Oven	141
6.7	Summary.....	145
	References	145
7	Nonlinear Dimension Reduction Based Neural Modeling for Nonlinear Complex DPS.....	149
7.1	Introduction	149
7.2	Nonlinear PCA Based Spatio-Temporal Modeling Framework	150
7.2.1	Modeling Methodology.....	150
7.2.2	Principal Component Analysis.....	151
7.2.3	Nonlinear PCA for Projection and Reconstruction	153
7.2.4	Dynamic Modeling.....	153
7.3	Nonlinear PCA Based Spatio-Temporal Modeling in Neural System ...	154
7.3.1	Neural Network for Nonlinear PCA.....	154
7.3.2	Neural Network for Dynamic Modeling	156
7.4	Simulation and Experiment	157
7.4.1	Catalytic Rod.....	157
7.4.2	Snap Curing Oven	160
7.5	Summary.....	163
	References	164
8	Conclusions.....	167
8.1	Conclusions	167
	References	170
Index		173

List of Figures

Fig.1.1 Snap curing oven system.....	2
Fig.1.2 A catalytic rod.....	3
Fig.1.3 A catalytic packed-bed reactor.....	4
Fig.1.4 Spatio-temporal models for nonlinear DPS.....	7
Fig.1.5 Spatial information processing for DPS modeling.....	8
Fig.2.1 Geometric interpretations of finite difference and method of lines.....	18
Fig.2.2 Geometric interpretation of time-space separation for $n=3$	19
Fig.2.3 Framework of weighted residual method.....	20
Fig.2.4 Geometric interpretation of weighted residual method.....	20
Fig.2.5 Piecewise linear polynomials in one dimension.....	24
Fig.2.6 Eigenfunctions of Case 1.....	26
Fig.2.7 Separation of eigenspectrum.....	26
Fig.2.8 Empirical eigenfunctions of Case 1.....	28
Fig.2.9 KL and spectral method for Case 1.....	30
Fig.2.10 KL and spectral method for Case 2.....	30
Fig.2.11 Output error approach.....	32
Fig.2.12 Geometric interpretation of FDM based identification.....	35
Fig.2.13 Neural spectral method.....	37
Fig.2.14 Neural observer spectral method.....	37
Fig.2.15 Neural KL method.....	38
Fig.2.16 Neural spectral and neural KL methods for Case 1.....	39
Fig.2.17 Neural spectral and neural observer spectral methods for Case 1.....	39
Fig.2.18 Neural spectral method for Case 2.....	40
Fig.2.19 Neural KL method for Case 2.....	40
Fig.3.1 Wiener distributed parameter system.....	53
Fig.3.2 Time/space separation of Wiener distributed parameter system.....	53
Fig.3.3 KL based modeling methodology for Wiener distributed parameter system.....	54
Fig.3.4 Wiener model.....	57
Fig.3.5 Catalytic rod: Measured output for Wiener modeling.....	63
Fig.3.6 Catalytic rod: KL basis functions for KL-Wiener modeling.....	64
Fig.3.7 Catalytic rod: KL-Wiener model output.....	64
Fig.3.8 Catalytic rod: Prediction error of KL-Wiener model.....	64
Fig.3.9 Catalytic rod: Spline basis functions for SP-Wiener modeling.....	65
Fig.3.10 Catalytic rod: $SNAE(t)$ of SP-Wiener and KL-Wiener models.....	65
Fig.3.11 Sensors placement for modeling of snap curing oven.....	66

Fig.3.12 Snap curing oven: Input signals of heater 1 in the experiment.....	67
Fig.3.13 Snap curing oven: KL basis functions ($i=1$) for KL-Wiener modeling..	67
Fig.3.14 Snap curing oven: KL basis functions ($i=2$) for KL-Wiener modeling..	67
Fig.3.15 Snap curing oven: Performance of KL-Wiener model at sensor s1	68
Fig.3.16 Snap curing oven: Performance of KL-Wiener model at sensor s6	68
Fig.3.17 Snap curing oven: Predicted temperature distribution of KL-Wiener model at $t=10000s$	68
Fig.3.18 Snap curing oven: Spline basis functions ($i=1$) for SP-Wiener modeling	69
Fig.3.19 Snap curing oven: Spline basis functions ($i=2$) for SP-Wiener modeling	69
Fig.3.20 Snap curing oven: Predicted temperature distribution of SP-Wiener model at $t=10000s$	69
Fig.4.1 Hammerstein distributed parameter system	75
Fig.4.2 Time/space separation of Hammerstein distributed parameter system	75
Fig.4.3 KL based modeling methodology for Hammerstein distributed parameter system	76
Fig.4.4 Hammerstein model	78
Fig.4.5 Structure design of Hammerstein model	82
Fig.4.6 Catalytic rod: Measured output for Hammerstein modeling	87
Fig.4.7 Catalytic rod: KL basis functions for KL-Hammerstein modeling	87
Fig.4.8 Catalytic rod: KL-Hammerstein model output	88
Fig.4.9 Catalytic rod: Prediction error of KL-Hammerstein model.....	88
Fig.4.10 Catalytic rod: Spline basis functions for SP-Hammerstein modeling	88
Fig.4.11 Catalytic rod: Comparison of SP- and KL-Hammerstein models	89
Fig.4.12 Catalytic rod: Comparison of OFR-LSE-SVD and LSE-SVD for algorithms KL-Hammerstein model	89
Fig.4.13 Snap curing oven: KL basis functions ($i=1$) for KL-Hammerstein modeling	90
Fig.4.14 Snap curing oven: KL basis functions ($i=2$) for KL-Hammerstein modeling	90
Fig.4.15 Snap curing oven: Performance of KL-Hammerstein model at sensor s1	91
Fig.4.16 Snap curing oven: Performance of KL-Hammerstein model at sensor s6	91
Fig.4.17 Snap curing oven: Predicted temperature distribution of KL-Hammerstein model at $t=10000s$	91
Fig.4.18 Snap curing oven: Spline basis functions ($i=1$) for SP-Hammerstein modeling	92
Fig.4.19 Snap curing oven: Spline basis functions ($i=2$) for SP-Hammerstein modeling	92
Fig.5.1 Hammerstein distributed parameter system	97
Fig.5.2 Multi-channel identification of spatio-temporal Hammerstein model ...	104
Fig.5.3 Multi-channel spatio-temporal Hammerstein model.....	105

Fig.5.4 Spatio-temporal Laguerre model of the c^{th} channel.....	106
Fig.5.5 Packed-bed reactor: Process output for multi-channel Hammerstein modeling	114
Fig.5.6 Packed-bed reactor: KL basis functions for multi-channel Hammerstein modeling.....	114
Fig.5.7 Packed-bed reactor: Prediction output of 3-channel Hammerstein model.....	115
Fig.5.8 Packed-bed reactor: Prediction error of 3-channel Hammerstein model	115
Fig.5.9 Packed-bed reactor: $TNAE(x)$ of Hammerstein models.....	115
Fig.5.10 Packed-bed reactor: $SNAE(t)$ of Hammerstein models.....	116
Fig.5.11 Packed-bed reactor: $RMSE$ of 3-channel Hammerstein model.....	116
Fig.5.12 Snap curing oven: KL basis functions ($i=1$) for multi-channel Hammerstein modeling.....	117
Fig.5.13 Snap curing oven: KL basis functions ($i=2$) for multi-channel Hammerstein modeling.....	117
Fig.5.14 Snap curing oven: Performance of 3-channel Hammerstein model at sensor s1	118
Fig.5.15 Snap curing oven: Performance of 3-channel Hammerstein model at sensor s6	118
Fig.5.16 Snap curing oven: Predicted temperature distribution of 3-channel Hammerstein model at $t=10000s$	118
Fig.6.1 Spatio-temporal Volterra modeling approach	126
Fig.6.2 Laguerre network for state space realization of Volterra model	132
Fig.6.3 Catalytic rod: Measured output for Volterra modeling	139
Fig.6.4 Catalytic rod: KL basis functions for Volterra modeling.....	139
Fig.6.5 Catalytic rod: Predicted output of 2 nd -order Volterra model	140
Fig.6.6 Catalytic rod: Prediction error of 2 nd -order Volterra model	140
Fig.6.7 Catalytic rod: $SNAE(t)$ of 1 st and 2 nd -order Volterra models	140
Fig.6.8 Catalytic rod: $TNAE(x)$ of 1 st and 2 nd -order Volterra models	141
Fig.6.9 Catalytic rod: $RMSE$ of 2 nd -order Volterra model	141
Fig.6.10 Snap curing oven: KL basis functions ($i=1$) for Volterra modeling	142
Fig.6.11 Snap curing oven: KL basis functions ($i=2$) for Volterra modeling	142
Fig.6.12 Snap curing oven: Performance of 2 nd -order Volterra model at sensor s1	143
Fig.6.13 Snap curing oven: Performance of 2 nd -order Volterra model at sensor s6	143
Fig.6.14 Snap curing oven: Predicted temperature distribution of 2 nd -order Volterra model at $t=10000s$	143
Fig.6.15 Snap curing oven: $SNAE(t)$ of 1 st -order Volterra model.....	144
Fig.6.16 Snap curing oven: $SNAE(t)$ of 2 nd -order Volterra model.....	144
Fig.6.17 Snap curing oven: $RMSE$ of 2 nd -order Volterra model.....	145

Fig.7.1 NL-PCA based spatio-temporal modeling methodology	151
Fig.7.2 NL-PCA network	155
Fig.7.3 Catalytic rod: Measured output for neural modeling.....	158
Fig.7.4 Catalytic rod: NL-PCA reconstruction error	159
Fig.7.5 Catalytic rod: NL-PCA-RBF model prediction - $\hat{y}_1(t)$	159
Fig.7.6 Catalytic rod: NL-PCA-RBF model prediction - $\hat{y}_2(t)$	159
Fig.7.7 Catalytic rod: NL-PCA-RBF model prediction error.....	160
Fig.7.8 Catalytic rod: $SNAE(t)$ of NL-PCA-RBF and PCA-RBF models	160
Fig.7.9 Snap curing oven: Performance of NL-PCA-RBF model at sensor s1...	161
Fig.7.10 Snap curing oven: Performance of NL-PCA-RBF model at sensor s2	161
Fig.7.11 Snap curing oven: Predicted temperature distribution of NL-PCA-RBF model at $t=10000s$	162

List of Tables

Table 2.1 Dimensionless parameters for Case 1	15
Table 2.2 Dimensionless parameters for Case 2	16
Table 2.3 Classification of spatial basis functions	23
Table 3.1 Snap curing oven: $TNAE(x)$ of KL-Wiener and SP-Wiener models	70
Table 4.1 Snap curing oven: $TNAE(x)$ of KL-Hammerstein and SP-Hammerstein models.....	93
Table 4.2 Snap curing oven: $TNAE(x)$ of OFR-LSE-SVD and LSE-SVD algorithms for KL-Hammerstein model.....	93
Table 5.1 Snap curing oven: $TNAE(x)$ of Hammerstein models	119
Table 6.1 Snap curing oven: $TNAE(x)$ of 1st and 2nd-order Volterra models ...	144
Table 7.1 Catalytic rod: Comparison of PCA and NL-PCA for modeling.....	160
Table 7.2 Snap curing oven: $TNAE(x)$ of PCA-RBF and NL-PCA-RBF models.....	162
Table 7.3 Snap curing oven: accuracy comparison for all models.....	162
Table 7.4 Snap curing oven: computation comparison for all modeling approaches (seconds)	163

Abbreviations

AE	Algebraic Equation
AIM	Approximated Inertial Manifold
BF	Basis Function
DE	Difference Equation
DPS	Distributed Parameter System
EEF	Empirical Eigenfunction
EF	Eigenfunction
ERR	Error Reduction Ratio
FDM	Finite Difference Method
FEM	Finite Element Method
FMNS	Fading Memory Nonlinear System
IC	Integrated Circuit
IM	Inertial Manifold
IV	Instrumental Variables Method
KL	Karhunen-Loève Decomposition
KL-Hammerstein	Karhunen-Loève based Hammerstein Model
KL-Wiener	Karhunen-Loève based Wiener Model
LDS	Lattice Dynamical System
LPS	Lumped Parameter System
LSE	Least-Squares Estimation
LTI	Linear Time Invariant
MARE	Mean of Absolute Relative Error
MIMO	Multi-Input-Multi-Output
MO	Multi-Output
MOL	Method of Lines

NARX	Nonlinear Autoregressive with Exogenous Input
NL-PCA	Nonlinear PCA
NL-PCA-RBF	NL-PCA based RBF model
ODE	Ordinary Differential Equation
OFR	Orthogonal Forward Regression
PCA	Principal Component Analysis
PCA-RBF	PCA based RBF model
PDE	Partial Differential Equation
POD	Proper Orthogonal Decomposition
RBF	Radial Basis Function
RMSE	Root of Mean Squared Error
SISO	Single-Input-Single-Output
SNAE	Spatial Normalized Absolute Error
SNR	Signal-to-Noise Ratio
SO	Single-Output
SP-Hammerstein	Spline Functions based Hammerstein Model
SP-Wiener	Spline Functions based Wiener Model
SVD	Singular Value Decomposition
TNAE	Temporal Normalized Absolute Error
WRM	Weighted Residual Method

1 Introduction

Abstract. This chapter is an introduction of the book. Starting from typical examples of distributed parameter systems (DPS) encountered in the real-world, it briefly introduces the background and the motivation of the research, and finally the contributions and organization of the book.

1.1 Background

Advanced technological needs, such as, semiconductor manufacturing, nanotechnology, biotechnology, material engineering and chemical engineering, have motivated control of material microstructure, fluid flows, spatial profiles (e.g., temperature field) and product size distributions (Christofides, 2001a). These physical, chemical or biological processes all lead to so called distributed parameter systems (DPS) because their inputs and outputs vary both temporally and spatially. As the significant progress in the sensor, actuator and computing technology, the studies of distributed parameter processes become more and more active and practical in science and engineering. Recently several special issues for control of DPS have been organized by Dochain *et al.* (2003), Christofides (2002, 2004), Christofides & Armaou (2005), and Christofides & Wang (2008). Modeling is the first step for many applications such as prediction, control and optimization. This book will focus on the modeling problem of nonlinear DPS with application examples chosen as industrial thermal processes. In general, the modeling approaches presented are applicable to a wide range of distributed parameter processes.

Next, we will introduce some typical thermal process in integrated circuit (IC) packaging and chemical industry, which will be used as examples in the rest of chapters.

1.1.1 Examples of Distributed Parameter Processes

a) Thermal Process in IC Packaging Industry

One important thermal process in the semiconductor back-end packaging industry considered in this book is the curing process (Deng, Li & Chen, 2005). After the required amount of epoxy is dispensed on the leadframe from the dispenser, and a die is moved from the wafer to attach on the leadframe by the bond arm, then the bonded leadframe is moved into the snap curing oven to cure at a specified temperature. As shown in Figure 1.1, the snap curing oven is an important equipment to provide the required curing temperature distribution. The oven has four heaters for heating and four thermocouples for temperature sensing in the operation. The parts to be cured will be moved in and out from inlet and outlet, respectively.

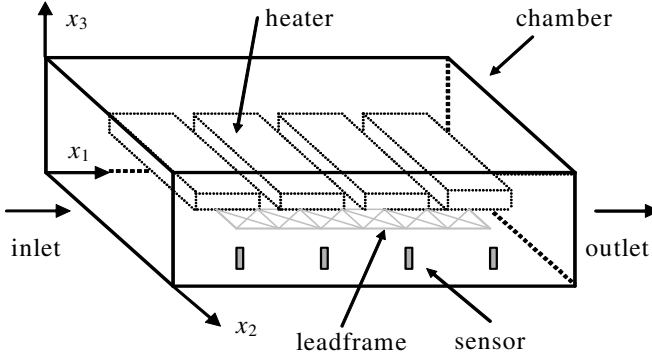


Fig. 1.1 Snap curing oven system

The temperature distribution inside the chamber is often needed for a quality curing control as well as the fundamental analysis for oven design. In practice, it is difficult to place many sensors to measure the temperature distribution during the curing. Thus it motivates us to build a model of the oven and use it to estimate the temperature distribution of the curing process.

This thermal process can be simplified for the easy modeling. The volume of the epoxy between a die and the leadframe is much smaller as compared to the volume of the leadframe and the volume of the oven chamber. Also, the volume of the leadframe is much smaller as compared to the volume of the oven chamber. Thus, the effects of the epoxy and the leadframe on the temperature in the oven chamber are usually neglected in modeling of the curing process. These effects can be considered as disturbances and could be compensated in the later control process.

This thermal process will follow the basic principles of the heat transfer (conduction, radiation and convection). The fundamental heat transfer equation of the oven can be expressed as a nonlinear parabolic **partial differential equation (PDE)** with some unknown parameters, unknown nonlinearities and unknown boundary conditions:

$$\rho(T)c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_1} \left(k(T) \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(k(T) \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(k(T) \frac{\partial T}{\partial x_3} \right) + f_c(T) + f_r(T) + bu(t), \quad (1.1)$$

where

$T = T(x_1, x_2, x_3, t)$ is the temperature at time t and location (x_1, x_2, x_3) ,

$x_1 \in [0, x_{10}]$, $x_2 \in [0, x_{20}]$ and $x_3 \in [0, x_{30}]$ are spatial coordinates,

$k(T)$ is the thermal conductivity, which is usually inaccurately given,

$\rho(T)$ is the density, which is usually inaccurately given,

c is the specific heat,