

# Advances in Mathematical Modeling and Experimental Methods for Materials and Structures

# SOLID MECHANICS AND ITS APPLICATIONS

Volume 168

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## *Aims and Scope of the Series*

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

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The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

Rivka Gilat • Leslie Banks-Sills

# Advances in Mathematical Modeling and Experimental Methods for Materials and Structures

The Jacob Aboudi Volume

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# Foreword

This volume is dedicated to Jacob Aboudi, a fine scientist who has made seminal contributions in applied mechanics. The papers presented here reflect the appreciation of many of Jacob's colleagues. A publication list following this introduction provides an indication of his distinguished academic career, currently in its fifth decade, and the breadth of his knowledge. His papers consistently demonstrate originality, innovation and diligence. This list uncovers the methodical work of a dedicated researcher whose achievements established him as a leading authority in the area of mathematical modeling of the behavior of heterogeneous materials, the area which became known as homogenization theory.

Starting in 1981, Jacob established a micromechanical model known as the Method of Cells (MOC) which evolved into the Generalized Method of Cells (GMC) that predicts the macroscopic response of composite materials as a function of the properties, volume fractions, shapes, and constitutive behavior of its constituents. The versatility of the model has been demonstrated to effectively incorporate various types of constituent material behavior (i.e., both coupled and uncoupled mechanical, thermal, electrical and magnetic effects). As a result of its potential in providing an efficient tool for the emerging field of multiscale analysis, the method gained increasing attention and became a subject for further research. In 1997, NASA presented Jacob with a certificate of recognition "for the creative development of exceptional scientific and technical contributions which have been determined to be of significant value in the advancement of the aerospace technology program of NASA entitled: MICROMECHANICAL ANALYSIS CODE with GENERALIZED METHOD of CELLS (MAC/GMC)".

Subsequently, the limited accuracy of GMC which results from neglecting coupling between normal and shear stresses led to his developing the High Fidelity



Generalized Method of Cells (HFGMC). Jacob continues to extend this method with incorporation of large deformations, constitutive laws for advanced constituent materials and an improved numerical formulation.

The publication list also reflects Jacob's contributions in other areas including wave propagation, fracture mechanics, contact problems and applied numerical solutions of partial differential equations.

Of course, the publication list cannot provide any indication of Jacob's character and interests, such as his love of history. In this regard, those who have had the pleasure to collaborate with him can attest to his integrity, collegiality, sound judgment and ability to give advice on a wide spectrum of issues. He is skilled at effectively clarifying complex concepts for students, is dedicated to his graduate students, and has contributed to the academic community in numerous ways.

Jacob Aboudi was born in 1935 in Baghdad, and emigrated to Israel in 1951. After graduating in 1961 with a B.Sc. degree in Applied Mathematics from Tel Aviv University, Jacob taught in the same department while pursuing advanced studies at the Weizmann Institute of Science. The latter institution awarded him M.Sc. and Ph.D degrees in 1964 and 1968, respectively, both in Applied Mathematics. Jacob was then hired as a Lecturer at Tel Aviv University, first in the Department of Environmental Sciences and then in the Faculty of Engineering where he became a Professor in 1980. He served 8 years as the head of the Department of Solid Mechanics, Materials and Structures, 6 years as the Dean of the Faculty of Engineering and participated in many University and Faculty Committees. Jacob was the incumbent of the Diane and Arthur Belfer Chair of Mechanics and Biomechanics for 13 years.

Jacob has spent sabbatical leaves and extended visits abroad at the University of Strathclyde, UK, Northwestern University, Virginia Polytechnic Institute and State University, the University of Virginia, and at NASA Glenn Research Center, Cleveland, all in the USA.

With this volume, we wish to express our profound respect and admiration of Jacob Aboudi.

*Rivka Gilat and Leslie Banks-Sills*

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# Publications by Jacob Aboudi

## Books

1. (1991) Mechanics of composite materials – A unified micromechanical approach. Elsevier, Amsterdam
2. (1992) Random vibration and reliability of composite structures. Technomic, Lancaster, PA (with Cederbaum G, Elishakoff I, Librescu L)

## Papers

1. (1968) The common spectrum of computed seismograms. Isr J Technol 6:187–199 (with Alterman Z)
2. (1968) Pulse propagation in a laterally heterogeneous fluid sphere by finite difference methods. J Phys Earth 16:173–193 (with Alterman Z)
3. (1969) Impulsive sound propagation in a fluid sphere of variable internal friction. Isr J Technol 7:135–147 (with Alterman Z)
4. (1969) Seismic pulse in a layered sphere: normal modes and surface waves. J Geophys Res 74:2618–2626 (with Alterman Z)
5. (1969) Seismic source in a layered sphere: reflected and diffracted pulses. J Geophys Res 74:5903–5922 (with Alterman Z)
6. (1969) Point source and applied force in a fluid sphere. Isr J Technol 7:319–328 (with Alterman Z)
7. (1970) Source of finite extent, applied force and couple in an elastic half-space. Geophys J Roy Astr Soc 21:47–64 (with Alterman Z)
8. (1970) Pulse propagation in a laterally heterogeneous solid elastic sphere. Geophys J Roy Astr Soc 21:243–269 (with Alterman Z, Karal FC)
9. (1971) The motion excited by an impulsive source in an elastic half-space with surface obstacle. Bull Seismol Soc Am 61:747–763
10. (1971) Numerical simulation of seismic sources. Geophysics 36:810–821
11. (1971) Wave propagation from a spherical cavity embedded in an elastoplastic medium. J Eng Math 5:279–287
12. (1971) Propagation of elastic waves caused by an impulsive source in a half-space with corrugated surface. Geophys J Roy Astr Soc 24:59–76 (with Alterman Z)
13. (1971) Scattering of sound waves by rotating cylinders and spheres. J Sound Vib 19:437–444 (with Censor D)

14. (1972) Reflection and transmission of elastic waves by a moving slab. *Appl Sci Res* 25:313–327 (with Censor D, Neulander D)
15. (1972) Propagation of transient pulses from a spherical cavity in a viscoelastic medium. *Int J Numer Method Eng* 4:289–299
16. (1972) The response of an elastic half-space to the dynamic expansion of an embedded spherical cavity. *Bull Seismol Soc Am* 62:115–127
17. (1972) Scattering of elastic waves by moving objects. *J Acoust Soc Am* 52:203–209 (with Censor D)
18. (1972) Rayleigh wave propagation in a viscoelastic half-space. *J Eng Math* 6:313–321
19. (1972) Impact-deflection by oblique fibers in sparsely reinforced composites. *J Appl Math Phys (ZAMP)* 23:828–844 (with Weitsman Y)
20. (1973) Stress wave propagation in a laminated plate under impulsive loads. *Int J Solids Struct* 9:217–232
21. (1973) One dimensional finite amplitude wave propagation in a compressible elastic half-space. *Int J Solids Struct* 9:363–378 (with Benveniste Y)
22. (1973) The free vibrations of a thin circular finite rotating cylinder. *Int J Mech Sci* 15:269–278 (with Zohar A)
23. (1973) Elastic waves in a half-space with a thin barrier. *J Eng Mech Div* 99:69–83
24. (1973) A mixture theory for a laminated plate under impulsive loads. *J Sound Vib* 29:355–364
25. (1974) Finite amplitude one-dimensional wave propagation in a thermoelastic half-space. *Int J Solids Struct* 10:293–308 (with Benveniste Y)
26. (1974) A mixture theory for a thermoelastic laminated medium, with application to a laminated plate under impulsive loads. *J Sound Vib* 33:187–200
27. (1974) Nonlinear wave propagation in a thin viscoelastic rod. *Mecc* 11:283–290 (with Benveniste Y)
28. (1974) The nonlinear response of a fiber reinforced thin plate under dynamic loading. *Fibre Sci Technol* 7:223–236 (with Benveniste Y)
29. (1974) A numerical solution for the problem of an impacted fiber-reinforced viscoelastic half-space. *Comput Methods Appl Mech Eng* 4:349–366 (with Weitsman Y)
30. (1975) The dynamic response of a laminated plate under large deformations. *J Sound Vib* 38:425–436 (with Benveniste Y)
31. (1975) A nonlinearly thermoelastic half-space under time dependent normal and shear loading. *Int J Solids Struct* 11:709–724 (with Benveniste Y)
32. (1975) Uniaxial wave propagation in a nonlinear thermoviscoelastic medium with temperature dependent properties. *Int J Solids Struct* 11:725–740 (with Benveniste Y)
33. (1975) Stress functions for fiber-reinforced materials and the effects of fiber-inextensibility. *Isr J Technol* 13:39–45 (with Weitsman Y)
34. (1976) Wave propagation in a thermorheologically simple solid slab. *Acta Mech* 22:181–195

35. (1975) A superimposed mixture theory for wave propagation in a biaxially fiber-reinforced composite. *J Sound Vib* 41:163–175 (with Benveniste Y)
36. (1975) The nonlinear lamb problem. *Comput Methods Appl Mech Eng* 6:319–334 (with Benveniste Y)
37. (1976) Two-dimensional wave propagation in a nonlinear elastic half-space. *Comput Methods Appl Mech Eng* 9:25–46
38. (1976) A mixture theory for wave propagation in a laminated medium with debonding. *J Sound Vib* 46:473–482 (with Benveniste Y)
39. (1976) Numerical solution of dynamic stresses induced by moving cracks. *Comput Methods Appl Mech Eng* 9:301–316
40. (1977) Moving force on a nonlinearly elastic material. *Acta Mech* 27:127–144
41. (1977) The dynamic stresses induced by moving interfacial cracks. *Comput Methods Appl Mech Eng* 10:303–323
42. (1977) A nonlinear mixture theory for the dynamic response of a laminated composite under large deformations. *J Appl Math Phys (ZAMP)* 28:1067–1084 (with Benveniste Y)
43. (1977) The dynamic indentation of an elastic half-space by a rigid punch. *Int J Solids Struct* 13:995–1005
44. (1978) The dynamic contact stresses caused by the impact of a nonlinear elastic half-space by an axisymmetrical projectile. *Comput Methods Appl Mech Eng* 13:189–204
45. (1978) Crack propagation in a laminated composite material modeled by a two-dimensional mixture theory. *Acta Mech* 29:213–227 (with Benveniste Y)
46. (1978) The dynamic stress induced by the propagation of skew cracks. *Comput Methods Appl Mech Eng* 15:181–199
47. (1978) A two-dimensional mixture theory for biaxially fiber reinforced composites with application to dynamic crack problems. *Int J Eng Sci* 16:615–636 (with Benveniste Y)
48. (1978) Numerical methods in elastodynamics. In: Miklowitz J, Achenbach JD (eds) *Modern problems in elastic wave propagation*. Wiley-Interscience, Chichester, pp 45–65
49. (1978) A numerical solution to the problem of dynamic indentation of an elastic plate by a rigid punch. *J Comput Phys* 29:318–327
50. (1979) The impact-contact problem of two nonlinearly elastic bodies. *Acta Mech* 33:81–95
51. (1979) The dynamic indentation and impact of a viscoelastic half-space by an axisymmetrical rigid body. *Comput Methods Appl Mech Eng* 20:135–150
52. (1980) The dynamic contact with perfect adhesion and frictional slip between a rigid indenter and an elastic half-space. *Acta Mech* 35:147–155
53. (1980) The dynamic indentation of an elastic-viscoplastic work hardening slab by a rigid punch. *Int J Eng Sci* 18:619–629
54. (1980) Dynamic response of a slab of elastic viscoplastic material that exhibits induced plastic anisotropy. *Int J Eng Sci* 18:801–813 (with Bodner SR)

55. (1980) Hydrodynamic and transfer characteristics in free interface film due to time-dependent disturbance at the entry. *Int J Heat Mass Transf* 23:927–941 (with Maron D, Zijl W)
56. (1981) An average theory for the dynamic behaviour of a laminated elastic-viscoplastic medium under general loading. *Int J Solids Struct* 17:69–81 (with Benveniste Y)
57. (1981) Effective stiffness theory for a laminated elastic viscoplastic work-hardening composite. *Int J Solids Struct* 17:421–431
58. (1981) Rapid mode III crack propagation in a strip of viscoplastic work-hardening material. *Int J Solids Struct* 17:879–890 (with Achenbach JD)
59. (1981) An average theory for the dynamic behavior of a laminated elastic-viscoplastic work-hardening medium. *J Appl Math Mech (ZAMM)* 61: 314–324 (with Benveniste Y)
60. (1981) Generalized effective stiffness theory for non-elastic laminated composites. *Int J Eng Sci* 19:1269–1281
61. (1981) Generalized effective stiffness theory for the modeling of fiber-reinforced composites. *Int J Solids Struct* 17:1005–1018
62. (1981) Transition from brittle to ductile fracture for a rapidly propagating crack. In: *Pressure vessels and piping conference*. ASME, NY, pp 81-PVP-16 (with Achenbach JD)
63. (1981) Arrest of mode III fast fracture by a transition from elastic to viscoplastic material properties. *J Appl Mech* 48:509–514 (with Achenbach JD)
64. Fast fracture of a strip of viscoplastic work-hardening material. In: Sih GC, Mirabile M (eds) *Analytical and experimental fracture mechanics*. Noordhoff, the Netherlands, pp 773–783 (1981) (with Achenbach JD)
65. (1982) Mixture theories for modeling the dynamic response of composite materials. *Int J Eng Sci* 20:193–216 (with Benveniste Y)
66. (1982) A continuum theory for fiber-reinforced elastic viscoplastic composites. *Int J Eng Sci* 20:605–621
67. (1983) Numerical analysis of fast mode-I fracture of a strip of viscoplastic work-hardening material. *Int J Fract* 21:133–147 (with Achenbach JD)
68. (1983) Arrest of fast mode I fracture in an elastic-viscoplastic transition zone. *Eng Fract Mech* 18:109–119 (with Achenbach JD)
69. (1983) Effective constitutive equations for fiber-reinforced viscoplastic composites exhibiting anisotropic hardening. *Int J Eng Sci* 21:1081–1096
70. (1983) The effective moduli of short-fiber composites. *Int J Solids Struct* 19:693–707
71. (1983) Stress wave propagation in rods of elastic-viscoplastic material. *Int J Solids Struct* 19:305–314 (with Bodner SR)
72. (1983) Effective constitutive equations for fiber-reinforced viscoplastic composites. In: *Mechanics of composite materials: Recent advances*. Pergamon Press, Oxford, pp 57–71
73. (1984) Effective behavior of inelastic fiber-reinforced composites. *Int J Eng Sci* 22:439–449

74. (1984) The effective thermoelastic constants of short-fiber composites. *Fiber Sci Technol* 20:211–225
75. (1984) Constitutive relations for fiber reinforced inelastic laminated plates. *J Appl Mech* 51:107–113 (with Benveniste Y)
76. (1984) Elastoplasticity theory for porous materials. *Mech Mater* 3:81–94
77. (1984) Minimechanics of tri-orthogonally fiber-reinforced composites: Overall elastic and thermal properties. *Fiber Sci Tech* 21:277–293
78. (1984) A continuum model for fiber reinforced materials with debonding. *Int J Solids Struct* 20:935–951 (with Benveniste Y)
79. (1985) The effective thermomechanical behaviour of inelastic fiber-reinforced materials. *Int J Eng Sci* 23:773–787
80. (1985) The mechanical behavior of elastic plastic fiber-reinforced laminated plates. In: *Mechanical characterization of load bearing fiber composite laminates*. Elsevier, Amsterdam, pp 65–70 (with Benveniste Y)
81. (1985) Inelastic behavior of metal–matrix composites at elevated temperature. *Int J Plast* 1:359–372
82. (1985) Constitutive relations for the thermomechanical behavior of fiber-reinforced inelastic composites. *Compos Struct* 4:315–334
83. (1986) Overall finite deformation of elastic and elastoplastic composites. *Mech Mater* 5:73–86
84. (1986) Harmonic waves in composite materials. *Wave Motion* 8:289–303
85. (1986) Elastoplasticity theory for composite materials. *Solid Mech Arch* 11:141–183
86. (1987) The effective moduli of cracked bodies in plane deformations. *Eng Fract Mech* 26:171–184 (with Benveniste Y)
87. (1987) Stiffness reduction of cracked solids. *Eng Fract Mech* 26:637–650
88. (1987) Damage in composites: Modeling of imperfect bonding. *Compos Sci Technol* 28:103–128
89. (1987) Transient waves in composite materials. *Wave Motion* 9:141–156
90. (1987) Closed form constitutive equations for metal matrix composites. *Int J Eng Sci* 25:1229–1240
91. (1987) Constitutive relations for cracked metal matrix composites. *Mech Mater* 6:303–315
92. (1990) Nonlinear response of boron/aluminum under combined loading. In: Boehler JP (ed) *Proceedings of IUTAM/ICM symposium on yielding, damage and failure of anisotropic solids*. Mechanical Engineering Publications, pp 235–249 (with M-J Pindera M-J, Herakovich CT)
93. (1988) Wave propagation in damaged composite materials. *Int J Solids Struct* 24:117–138
94. (1988) Constitutive equations for elastoplastic composites with imperfect bonding. *Int J Plast* 4:103–125
95. (1988) Micromechanical analysis of yielding of metal matrix composites. *Int J Plast* 4:195–214 (with Pindera M-J)
96. (1988) Three-dimensional analysis of laminates with cross cracks. *J Appl Mech* 55:389–397 (with SW Lee, Herakovich CT)

97. (1988) Damage in composite laminates: Effects of transverse cracks. *Mech Mater* 7:91–107 (with Herakovich CT, Lee SW, Strauss EA)
98. (1988) Micromechanical analysis of the strength of unidirectional fiber composites. *Compos Sci Technol* 33:79–96
99. (1988) Nonlinear wave propagation in laminated composites. In: Mal AK, Ting TCT (eds) *Wave propagation in structural composites*. ASME, NY, AMD-Vol 90:133–140
100. (1988) 2-D and 3-D damage effects in cross-ply laminates. In: Dvorak GJ, Laws N (eds) *Mechanics of composite materials*. ASME, NY, AMD-Vol 92:143–147 (with Herakovich CT, Lee SW, Strauss EA)
101. (1989) Micromechanics prediction of fatigue failure of composite materials. *J Reinf Plast Compos* 8:150–166
102. (1989) The theory of orthotropic viscoelastic shear deformable composite flat panels and their dynamic stability. *Int J Solids Struct* 25:465–482 (with Chandiramani NK, Librescu L)
103. (1989) Micromechanical analysis of composites by the method of cells. *Appl Mech Rev* 42:193–221
104. (1989) Analysis of viscoelastic laminated composite plates. *Compos Struct* 12:243–256 (with Cederbaum G)
105. (1989) Micro-to-macro analysis of viscoelastic laminated plates. In: Marshall IH (ed) *Composite Structures*, vol 5. Elsevier, London, pp 779–739 (with Cederbaum G)
106. (1989) Dynamic response of viscoelastic laminated plates. *J Sound Vib* 132:225–238 (with Cederbaum G)
107. (1989) Viscoelastic behavior of thermorheologically complex resin matrix composites. *Compos Sci Technol* 36:351–365 (with Sadkin Y)
108. (1989) Micromechanical analysis of fibrous composites with Coulomb frictional slippage between the phases. *Mech Mater* 8:103–115
109. (1989) Micromechanical investigation of the convexity of yield surfaces of metal matrix composites. In: Khan AS, Tokuda M (eds) *Advances in plasticity*. Pergamon, Oxford, pp 129–132 (with Pindera M-J)
110. (1990) The nonlinear behavior of unidirectional and laminated composites – A micromechanical approach. *J Reinf Plast Compos* 9:13–32
111. (1990) Nonlinear response of unidirectional boron/aluminum. *J Compos Mater* 24:2–21 (with Pindera M-J, Herakovich CT, Becker W)
112. (1990) Dynamic response of composites with coulomb frictional slippage between the phases. *J Sound Vib* 138:35–46
113. (1990) Micromechanical prediction of initial and subsequent yield surfaces of metal matrix composites. *Int J Plast* 6:471–484
114. (1990) Dynamic stability analysis of viscoelastic plates by lyapunov exponents. *J Sound Vib* 139:459–467 (with Cederbaum G and Elishakoff I)
115. (1990) Micromechanical characterization of the nonlinear viscoelastic behavior of resin matrix composites. *Compos Sci Technol* 38:371–386

116. (1990) Recent developments in the micromechanics of advanced composites. In: Sih GC, Hoa SV, Pindera JT (eds) *Developments and design with advanced materials*. Elsevier, Amsterdam, pp 65–78 (with Pindera M-J)
117. (1991) A micromechanical composite yield model accounting for residual stresses. In: Dvorak GJ (ed) *Inelastic deformation of composite material*. Springer, the Netherlands, pp 375–387 (with Herakovich CT, Beuth JL)
118. (1991) Matrix mean-field and local-field approaches in the analysis of metal matrix composites. In: Dvorak GJ (ed) *Inelastic deformation of composite material*. Springer, the Netherlands, pp 761–779 (with Pindera M-J)
119. (1991) Micro-failure criteria for coated-fiber composites. *J Reinf Plast Compos* 10:146–157
120. (1991) Viscoplastic bifurcation buckling of plates. *AIAA J* 29:627–632 (with Paley M)
121. (1991) Micromechanical investigation of the convexity of yield surfaces of metal matrix composites. *Int J Plast* 7:549–566 (with Pindera M-J, Brayshaw JB)
122. (1991) Postbuckling analysis of viscoelastic laminated plates using higher-order theory. *Int J Solids Struct* 27:1747–1755 (with Shalev D)
123. (1991) Micro-failure prediction of the strength of composite materials under combined loading. *J Reinf Plast Compos* 10:495–503
124. (1991) Dynamic instability of shear deformable viscoelastic laminated plates by Lyapunov exponents. *Int J Solids Struct* 28:317–327 (with G Cederbaum, I Elishakoff)
125. (1991) Overall instantaneous properties of metal matrix composites. *Compos Sci Technol* 41:411–429 (with M Paley)
126. (1991) Plastic buckling of metal matrix laminated plates. *Int J Solids Struct* 28:1139–1154 (with M Paley)
127. (1991) Inelastic thermal buckling of metal matrix laminated plates. *J Therm Stress* 14:479–497 (with M Paley)
128. (1991) Wave propagation in angle-ply laminates. *J Sound Vib* 150:15–24 (with I Hevroni)
129. (1991) Dynamic response of pulse loaded laminated composite cylinders. *Int J Impact Eng* 11:233–248 (with D Larom, CT Herakovich)
130. (1991) Recent developments in the analysis of metal matrix composites by the method of cells. In: Boehler JP, Khan AK (eds) *Anisotropy and localization of plastic deformation*. Elsevier, London, pp 1–6
131. (1991) Reliability of composites based on micromechanically predicted strength and fatigue criteria. In: Marshall IH (ed) *Composite structures 6*. Elsevier, London, pp 75–88 (with G Cederbaum)
132. (1991) Thermo-mechanical response predictions for metal matrix composite laminates. In: Singhal SN, Jones WF, Herakovich CT (eds) *Mechanics of composites at elevated and cryogenic temperatures*. ASME, NY, AMD-Vol 118, pp 1–8. Appeared also in: *Sci Eng of Compos Mater* 2:151–169, 1993 (with JS Hidde and CT Herakovich)



133. (1991) Creep buckling of inelastic plates. In: Zyczowski M (ed) *Creep in structures*. Springer, the Netherlands, pp 595–600 (with M Paley)
134. (1992) Plastic buckling of ARALL plates. *Compos Struct* 22:217–221 (with M Paley)
135. (1992) Micromechanical analysis of composites by the generalized cells model. *Mech Mater* 14:127–139 (with M Paley)
136. (1993) Response prediction of composites composed of stiffening fibers and nonlinear resin matrix. *Compos Sci Technol* 46:51–58
137. (1993) Constitutive behavior of multiphase metal matrix composites with interfacial damage by the generalized cell model. In: Voyiadjis GZ (ed) *Damage in composite materials*. Elsevier, Amsterdam, pp 3–22
138. (1993) Postbuckling analysis and imperfection sensitivity of metal matrix laminated cylindrical panels. *Compos Struct* 25:241–248 (with E Feldman)
139. (1993) Axisymmetric response of nonlinearly elastic cylindrical shells to dynamic axial loads. *Int J Impact Eng* 13:545–554 (with R Gilat, E Feldman)
140. (1994) Postbuckling analysis and imperfection sensitivity of viscoplastic plates and cylindrical panels. *Thin-Walled Struct* 17:273–290 (with E Feldman)
141. (1994) Dynamic buckling of viscoplastic plates and shells under cylindrical bending. *J Sound Vib* 174:323–334 (with R Gilat)
142. (1994) Modeling of interfacial damage in composites. In: Talreja R (ed) *Damage mechanics of composite materials* (Composite materials series #9; Pipes RB, series ed). Elsevier, Amsterdam, pp 245–294
143. (1994) Response of functionally graded composites to thermal gradients. *Compos Eng* 4:1–18 (with SM Arnold and M-J Pindera)
144. (1994) Postbuckling analysis of metal–matrix laminated plates. *Compos Eng* 4:151–167 (with E Feldman)
145. (1994) Response of metal matrix laminates with temperature-dependent properties. *J Compos Technol Res* 16:68–76 (with F Mirzadeh and CT Herakovich)
146. (1994) Elastic response of metal matrix composites with tailored microstructures to thermal gradients. *Int J Solids Struct* 31:1393–1428 (with M-J Pindera, SM Arnold)
147. (1994) Dynamic buckling of metal matrix composite plates and shells under cylindrical bending. *Compos Struct* 28:459–469 (with R Gilat)
148. (1994) Thermo-inelastic analysis of functionally graded materials: Inapplicability of the classical micromechanics approach. In: Voyiadjis GZ, Ju JW (eds) *Inelasticity and micromechanics of metal matrix composites*. Elsevier, Amsterdam, pp 273–305 (with M-J Pindera, SM Arnold)
149. (1994) A probabilistic micromechanics model for damaged composites. In: Allen DH, Ju JW, *Damage mechanics in composites*. ASME, NY, AMD-Vol 185 (with JM Duva, CT Herakovich)
150. (1995) Limitations of the uncoupled, RVE-based micromechanical approach in the analysis of functionally graded composites. *Mech Mater* 20:77–94 (with M-J Pindera, SM Arnold)

151. (1995) Thermal postbuckling of metal matrix laminated plates. *J Ther Stress* 18:197–218 (with E Feldman)
152. (1995) Dynamic inelastic response and buckling of metal matrix composite infinitely wide plates due to thermal shocks. *Mech Compos Mater Struct* 2:257–271 (with R Gilat)
153. (1995) Dynamic buckling of nonlinear resin matrix composite structures. *Compos Struct* 32:81–88 (with R Gilat)
154. (1995) A coupled higher order theory for functionally graded composites with partial homogenization. *Compos Eng* 5:771–792 (with M-J Pindera, SM Arnold)
155. (1995) Micromechanical analysis of thermo-inelastic multiphase short-fiber composites. *Compos Eng* 5:839–850
156. (1995) Thermo-inelastic response of functionally graded composites. *Int J Solids Struct* 32:1675–1710 (with M-J Pindera, SM Arnold)
157. (1995) Recent advances in the mechanics of functionally graded composites. In: Thornton EA (ed) *Aerospace thermal structures and materials for a new era, Progress in astronautics and aeronautics, vol 168*. The American Institute of Aeronautics and Astronautics, Inc., Washington, DC, pp 181–203 (with M-J Pindera, SM Arnold)
158. (1996) Thermoelastic theory for the response of materials functionally graded in two directions. *Int J Solids Struct* 33:931–966 (with M-J Pindera, SM Arnold)
159. (1996) Coupled micro to macro analysis of a composite that hosts embedded piezoelectric actuators. *J Intell Mater Syst Struct* 7:15–24 (with D Shalev)
160. (1996) Thermomechanical coupling effects on the dynamic inelastic response and buckling of metal matrix infinitely wide plates. *Compos Struct* 35:49–63 (with R Gilat)
161. (1996) A micro-macro model for the effects of thermo-mechanical fields on optical fibers embedded in a laminated composite plate with applications to sensing. *Mech Compos Mater Struct* 3:297–320 (with D Shalev, M Tur)
162. (1996) A probabilistic micromechanics model for damaged composites. *J Eng Mater Technol* 18:548–553 (with JM Duva, CT Herakovich)
163. (1996) An interfacial damage model for titanium matrix composites. In: Voyiadjis GZ, Allen DH (eds) *Damage and interfacial debonding in composites*. Elsevier, Amsterdam, pp 149–165 (with CT Herakovich)
164. (1996) Thermoplasticity theory for bidirectionally functionally graded materials. *J Ther Stress* 19:809–861 (with M-J Pindera, SM Arnold)
165. (1996) Micromechanical analysis of composites by the method of cells – Update. *Appl Mech Rev* 49:S83–S91
166. (1997) Microstructural optimization of functionally graded composites subjected to thermal gradient via the coupled higher-order theory. *Compos Part B (Eng)* 28B:93–108 (with M-J Pindera, SM Arnold)
167. (1997) The response of shape memory alloy composites. *Smart Mater Struct* 6:1–9

168. (1997) Buckling analysis of functionally graded plates subjected to uniaxial loading. *Compos Struct* 38:29–36 (with E Feldman)
169. (1996) Microstructural effects in functionally graded thermal barrier coating. In: Shiota I, Miyamoto MY (eds) *Functionally graded materials*. Elsevier, Amsterdam, pp 113–121 (with M-J Pindera, SM Arnold)
170. (1998) Higher-order micro-macrostructural theory for the analysis of functionally graded material. In: Haddad YM (ed) *Advanced multilayered and fibre-reinforced composites*. Kluwer, the Netherlands, pp 111–132 (with M-J Pindera, SM Arnold)
171. (1998) Thermomechanical analysis of functionally graded thermal barrier coatings with different microstructural scales. *J Am Ceram Soc* 81:1525–1536 (with M-J Pindera, SM Arnold)
172. (1998) Micromechanical prediction of the effective coefficients of thermopiezoelectric multiphase composites. *J Intell Mater Syst Struct* 9:713–722
173. (1999) Effective behavior and dynamic response modeling of electro-rheological and magneto-rheological fluid composites. *Smart Mater Struct* 8:106–115
174. (1999) Thermal effects in composites. In: Hetnarski RB (ed) *Thermal stresses* 5, Chapter 1. Lastran Corporation, Rochester, NY, pp 1–142 (with Herakovich CT)
175. (1999) Micromechanical prediction of the response of electrostrictive multiphase composites. *Smart Mater Struct* 8:663–671
176. (1999) A fully thermo-mechanical micromechanical model. *J Ther Stress* 22:841–873 (with Williams TO)
177. (1999) Higher-order theory for functionally graded materials. *Compos Part B (Eng)* 30:777–832 (with Pindera M-J, Arnold SM)
178. (1999) A generalized micromechanics model with shear-coupling. *Acta Mech* 138:131–154 (with Williams TO)
179. (2000) Micromechanical modeling of the finite deformation of thermoelastic multiphase composites. *Math Mech Solids* 5:75–99 (with Arnold SM)
180. (2000) Micromechanical modeling of finite viscoelastic multiphase composites. *J Appl Math Phys (ZAMP)* 51:114–134
181. (2000) A coupled micro-macromechanical analysis of hygrothermoelastic composites. *Int J Solids Struct* 37:4149–4179 (with Williams TO)
182. (2000) The effect of interface roughness and oxide film thickness on the inelastic response of thermal barrier coatings to thermal cycling. *J Mater Sci Eng A* 284:158–175 (with Pindera M-J, Arnold SM)
183. (2000) Parametric stability of elastic composite plates by Lyapunov exponents. *J Sound Vib* 235:627–637 (with Gilat R)
184. (2001) Buckling analysis of composite plates. In: Durban D, Givoli D, Simmonds JG (eds) *Advances in the mechanics of plates and shells*. Kluwer, the Netherlands, pp 135–150 (with Gilat R)
185. (2001) Buckling of composite plates by global–local theory. *Compos Part B (Eng)* 32:229–236 (with Gilat R, Williams TO)

186. (2001) Linear thermoelastic higher-order theory for periodic multiphase materials. *J Appl Mech* 68:697–707 (with Pindera M-J, Arnold SM)
187. (2001) Micromechanical prediction of the finite thermoelastic response of rubberlike matrix composites. *J Appl Math Phys (ZAMP)* 52:823–846
188. (2001) Micromechanical analysis of fully coupled electro-magneto-thermo-elastic multiphase composites. *Smart Mater Struct* 10:867–877
189. (2002) The Lyapunov exponents as a quantitative criterion for the dynamic buckling of composite plates. *Int J Solids Struct* 39:467–481 (with Gilat R)
190. (2002) Micromechanical analysis of the fully coupled finite thermoelastic response of rubberlike matrix composites. *Int J Solids Struct* 39:2587–2612
191. (2002) Analysis of spallation mechanism in thermal barrier coatings with graded bond coats using the higher-order theory for FGM's. *Eng Fract Mech* 69:1587–1606 (with Pindera M-J, Arnold SM)
192. (2002) Improved mode model for infrared wave propagation in a troidal dielectric waveguide and applications. *Opt Eng* 41:2169–2180 (with Menachem Z, Croitoru N)
193. (2003) Higher-order theory for periodic multiphase materials with inelastic phases. *Int J Plast* 19:805–847 (with Pindera M-J, Arnold SM)
194. (2003) Micromechanical analysis of the finite elastic-viscoplastic response of multiphase composites. *Int J Solids Struct* 40:2793–2817
195. (2003) Elasto-plastic stresses in thick-walled cylinders. *J Press Vessel Technol* 125:248–252 (with Perry J)
196. (2003) Mode model for infrared propagation in a toroidal type of waveguide and applications. *J Optoelectron Adv Mater* 5:1373–1379 (with Menachem Z, Croitoru N)
197. (2003) Analysis of locally irregular composites using high-fidelity generalized method of cells. *AIAA J* 41:2331–2340 (with Pindera M-J, Arnold SM)
198. (2004) Analysis of internally cooled structures using a higher order theory. *Comput Struct* 82:659–688 (with Arnold SM, Bednarczyk BA)
199. (2004) Local field effects in titanium matrix composites subject to fiber-matrix debonding. *Int J Plast* 20:1707–1737 (with Bednarczyk BA, Arnold SM, Pindera M-J)
200. (2004) Micromechanically based constitutive equations for shape-memory fiber composites undergoing large deformations. *Smart Mater Struct* 13: 828–837
201. (2004) The generalized method of cells and high-fidelity generalized method of cells micromechanical models – A review. *Mech Adv Mater Struct* 11: 329–366
202. (2004) Micromechanics-based thermoviscoelastic constitutive equations for rubber-like matrix composites at finite strains. *Int J Solids Struct* 41: 5611–5629
203. (2004) Dynamic response of active composite plates: Shape memory alloy fibers in polymeric/metallic matrices. *Int J Solids Struct* 41:5717–5731 (with Gilat R)

204. (2004) High-fidelity micromechanical modeling of continuously reinforced elastic multiphase materials undergoing finite deformation. *Math Mech Solids* 9:599–628 (with Pindera M-J)
205. (2005) Role of material constitutive model in simulating the reusable launch vehicle thrust cell linear response. *J Aerosp Eng* 18:28–41 (with Butler DT, Pindera M-J)
206. (2005) Analysis of the spallation mechanism suppression in plasma-sprayed TBCs through the use of heterogeneous bond coat architectures. *Int J Plast* 21:1061–1096 (with Pindera M-J, Arnold SM)
207. (2005) Thermo-mechanical analysis of RLV thrust cell liners with homogeneous and graded coatings. *Mater Sci Forum* 492–493:429–434 (with Butler DT, Pindera M-J)
208. (2005) Micromechanical analysis of lattice blocks. *Int J Solids Struct* 42: 4372–4392 (with Gilat R)
209. (2005) Hysteresis behavior of ferroelectric fiber composites. *Smart Mater Struct* 14:715–726
210. (2005) Micromechanically established constitutive equations for multiphase materials with viscoelastic-viscoplastic phases. *Mech Time-Depend Mater* 9:121–145
211. (2006) Thermal buckling of activated shape memory reinforced laminated plates. *Smart Mater Struct* 15:829–838 (with Gilat R)
212. (2006) Buckling analysis of fibers in composite materials by wave propagation analogy. *Int J Solids Struct* 43:5168–5181 (with Gilat R)
213. (2006) Two-way thermomechanically coupled micromechanical analysis of shape memory alloy composites. *J Mech Mater Struct* 1:937–955 (with Freed Y)
214. (2007) The effect of a fiber loss in periodic composites. *Int J Solids Struct* 44:3497–3513 (with Rvkin M)
215. (2007) A thermomechanically micromechanical modeling of prestressed concrete reinforced with shape memory alloys fibers. *Smart Mater Struct* 16:717–727 (with Freed Y, Gilat R)
216. (2007) A new approach for optimizing the mechanical behavior of porous microstructures for porous materials by design. *Model Simul Mater Sci Eng* 15:653–674 (with Bruck HA, Gilat R, Gershon AL)
217. (2007) Micromechanical analyses of smart composite materials. In: Reece PL (ed) *Progress in smart materials and structures*. Nova Science Publishers, NY, pp 291–361
218. (2007) A continuum approach to the analysis of the stress field in a fiber reinforced composite with a transverse crack. *Int J Solids Struct* 44:6826–6841 (with Rvkin M)
219. (2007) Analysis of local thermomechanical effects in fiber-reinforced periodic composites. *Int J Fract* 145:229–236 (with Rvkin M)
220. (2008) The equivalence of the radial return and Mendelson methods for integrating the classical plasticity equations. *Comput Mech* 41:733–737 (with Bednarczyk BA, Arnold SM)

- 221. (2008) Three-dimensional continuum analysis for a unidirectional composite with a broken fiber. *Int J Solids Struct* 45:4114–4129 (with Ryvkin M)
- 222. (2008) Finite strain micromechanical analysis for thermoelastoplastic multiphase materials. *J Mech Mater Struct* 3:809–829
- 223. (2008) Thermomechanically coupled micromechanical analysis of multiphase composites. *J Eng Math* 61:111–132
- 224. (2008) Micromechanical investigation of plasticity-damage coupling of concrete reinforced by shape memory alloy fibers. *Smart Mater Struct* 17:015046 (with Freed Y)
- 225. (2008) Investigation of shape memory alloy honeycombs by means of a micromechanical analysis. *Model Simul Mater Sci Eng* 16:055002 (with Freed Y, Gilat R)
- 226. (2008) On the transformation toughening of a crack along an interface between a shape memory alloy and an isotropic medium. *J Mech Phys Solids* 56:3003–3020 (with Freed Y, Banks-Sills L)
- 227. (2008) Analysis of space shuttle spray-on-foam insulation with internal pore pressure. *J Eng Mater Tech* 130:041005 (with Bednarczyk BA, Arnold SM, Sullivan RM)
- 228. (2008) Finite strain micromechanical modeling of multiphase composites. *Int J Multiscale Comput Eng* 6:411–434
- 229. (2008) Postbuckling of layered composites by finite strain micromechanical analysis. *Int J Multiscale Comput Eng* 6:469–481 (with Gilat R)
- 230. (2009) Thermomechanically coupled micromechanical analysis of shape memory alloy composites undergoing transformation induced plasticity. *J Intell Mater Syst Struct* 20:23–38 (with Freed Y)
- 231. (2009) Finite strain micromechanical analysis of rubber-like matrix composites incorporating the Mullins damage effect. *Int J Damage Mech* 18:5–29
- 232. (2009) Micromechanical prediction of the two-way shape memory effect in shape memory composites. *Int J Solids Struct* 46:1634–1647 (with Freed Y)
- 233. (2009) Nonlinear micromechanical formulation of the high fidelity generalized method of cells. *Int J Solids Struct* 46:2577–2592 (with Haj-Ali R)

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# Aboudi's Micromechanics Theories Applied to Multiscale Analysis of Composites

Brett A. Bednarczyk and Steven M. Arnold

**Abstract** NASA Glenn Research Center in Cleveland, OH has worked with Professor Jacob Aboudi since 1992 to develop and implement his micromechanics theories into a user-friendly software suite. This effort has resulted in the publicly available Micromechanics Analysis Code with Generalized Method of Cells (MAC/GMC) software, along with the coupling of the code with finite element analysis and structural sizing software for multiscale analysis of composite structures. This chapter outlines these methods, discusses why Aboudi's methods are ideal for use in multiscale analyses, and briefly describes three recent multiscale composite analysis examples involving (i) creep of a woven ceramic matrix composite (CMC), (ii) damage/failure of a polymer matrix composite (PMC) T-stiffened panel, and (iii) damage/failure of notched PMC laminated plates.

## 1 Introduction

The use of advanced composites (PMCs, CMCs, metal matrix composites (MMCs)) provides benefits in the design of advanced lightweight, high temperature, structural systems because they provide increased specific properties (e.g., strength to density ratio) in comparison to their monolithic counterparts. To fully realize the benefits offered by these materials, however, experimentally verified, computationally efficient, multiscale design and analysis tools must be developed for the advanced multiphased materials of interest. Furthermore, in order to assist both the structural analyst in designing *with* these materials and the materials scientist in designing/developing *the* materials<sup>1</sup>, these tools must encompass the various levels of scale for composite analysis, see Fig. 1.

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<sup>1</sup> The structural engineer's perspective relates to the design of structures with given materials whereas the materials scientist's concern is how to design a material for a given application. Clearly, the two perspectives are not mutually exclusive.

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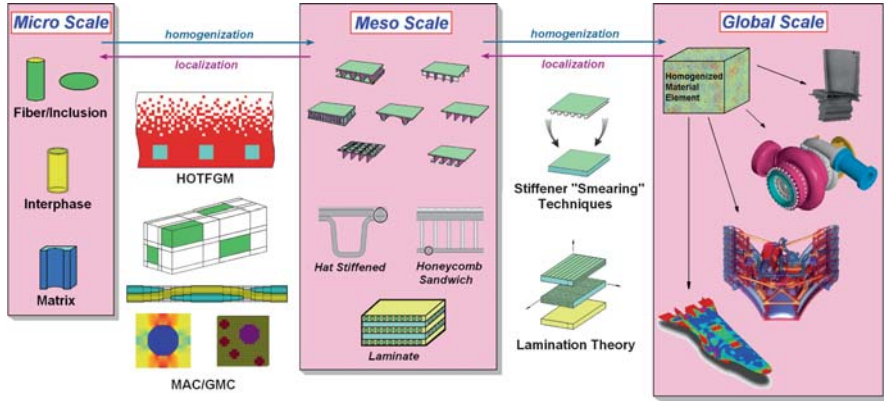


Fig. 1 Illustration of associated levels of scale for composite analysis

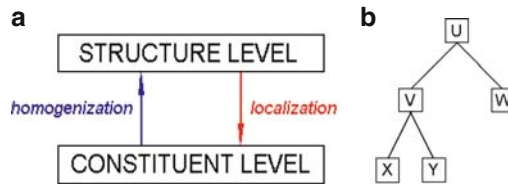


Fig. 2 (a) Homogenization provides the ability to determine structure level properties from constituent level properties while localization provides the ability to determine constituent level responses from structure level results. (b) Example tree diagram

These scales are the micro scale (constituent level), the meso scale (laminate/composite and/or stiffened panel level) and the macro scale (global/structure level), and they progress from left to right in Fig. 1. One traverses (transcends (moves right) or descends (moves left)) these scales using homogenization and localization techniques, respectively (Figs. 1 and 2a); where a homogenization technique provides the properties or response of a “structure” (higher level) given the properties or response of the structure’s “constituents” (lower scale). Conversely, localization techniques provide the response of the constituents given the response of the structure. Figure 2b illustrates the interaction of homogenization and localization techniques, in that during a multi-scale analysis, a particular stage in the analysis procedure can function on both levels simultaneously.<sup>2</sup> For example, for the process of homogenizing the stages represented by X and Y to obtain

<sup>2</sup> This is also illustrated in Fig. 1 where, for example, the global scale has subscales (components) within it (i.e., vehicle – engine – turbopump – blade) and the mesoscale has subcomponents (stiffened panel – laminate – ply).

properties for the stage represented by V, X and Y form the constituent level while V is on the structure level. However, for the process of homogenizing V and W to obtain properties for U, V is now on the constituent level (as is W). Obviously, the ability to homogenize and localize accurately requires a sophisticated theory that relates the geometric and material characteristics of structure and constituents.

Numerous homogenization techniques (micromechanical models) exist that can provide effective composite properties to a finite element package. These range from the simplest analytical approximations (i.e., Voigt/Reuss) to more accurate yet involved methods (e.g., concentric cylinder assemblage, Mori-Tanaka, Eshelby, and Aboudi's generalized method of cells) to finally, fully numerical methods that are the most general and accurate yet computationally intense (e.g., finite element, boundary element, Fourier series). Each has its realm of applicability and advantages, however, many are unable to admit general user defined deformation and damage/failure constitutive models for the various constituents (i.e., fiber or matrix) thus limiting their ultimate usefulness, especially for high temperature analysis where nonlinear, time-dependent behavior is often exhibited.

An alternative approach to micromechanics involves fully characterizing the composite material or laminate experimentally, which has the advantage of capturing the in-situ response of the constituents perfectly. However, such full characterization for all applicable temperatures and configurations (e.g., fiber volume fractions, tow spacings, etc.) can be expensive, and composites are almost always anisotropic on this scale. Thus some properties needed as input for finite element models can be virtually impossible to measure, and development of realistic models that capture the nonlinear multiaxial deformation and failure can be challenging (due to the anisotropy). Clearly, the physics of deformation and failure occur on the micro scale (and below), and, by modeling the physics at the micro scale, models for the monolithic, often isotropic, constituents can be employed.

Recently, a comprehensive and versatile micromechanics analysis computer code, known as MAC/GMC [7], has been developed at NASA Glenn Research Center based on Aboudi's well-known micromechanics theories [1–5]. FEAMAC (the coupling of MAC/GMC with the finite element analysis framework through user subroutines) and HyperMAC (the coupling of MAC/GMC with the commercial structural sizing software known as HyperSizer [10]) have begun to address the truly multiscale framework depicted in Fig. 1. This software suite, known collectively as ImMAC, provides a wide range of capabilities for modeling continuous, discontinuous, woven, and smart (piezo-electro-magnetic) composites. Libraries of nonlinear deformation, damage, failure, and fiber/matrix debonding models, continuous and discontinuous repeating unit cells, and material properties are provided, and the software is available from NASA Glenn.

## 2 Analysis Tools Based on Aboudi's Theories

### 2.1 *Micromechanics Analysis Code with Generalized Method of Cells (MAC/GMC)*

In developing an analytical tool that can serve both materials scientists and structural analysts, the employed methods must admit physics-based deformation and life models on the scale of the constituents and be capable of accurately predicting the macro composite response. This enables materials scientists to investigate the effects of deformation/damage mechanisms on the scale where they occur and make changes to develop new materials. From the structural analyst standpoint, the methods must be accurate, efficient, and compatible with structural finite element models. A number of models presently exist that can fulfill certain aspects of the aforementioned requirements. However, there are very few working models that are both computationally efficient and sufficiently accurate both at the micro and macro scales. It is the authors' position that Aboudi's micromechanics theories – the method of cells [1], the generalized method of cells (GMC) [2, 13], and high-fidelity GMC (HFGMC) [5] – are unique in this regard.

Aboudi's theories are capable of predicting the response of both continuous and discontinuous multi-phase composites with arbitrary internal microstructures and reinforcement shapes. They are continuum-based micromechanics models that provide efficient, closed-form expressions for the macroscopic composite response in terms of the properties, size, shape, distribution, and response of the individual constituents or phases that make up the material. Perhaps most importantly, Aboudi's theories admit physics-based viscoplastic deformation and arbitrary damage/life models for each constituent due to their ability to localize to the subcell level, providing full multiaxial stress and strain fields throughout the constituent materials. For these reasons, Aboudi's micromechanics theories were selected as the basis for NASA Glenn's MAC/GMC software. MAC/GMC provides industry, academia, and government engineers and materials scientists with a comprehensive, computationally efficient, user-friendly micromechanics analysis tool that can easily and accurately design/analyze multi-phase (composite) materials/structures for a given application. The distinction between HFGMC and GMC is that, through the use of an assumed higher-order local displacement field, HFGMC provides improved local field accuracy. However, HFGMC is more computationally intensive as it requires solution of a greater number of equations to fully discriminate its more accurate local fields. Two review papers documenting the application of GMC and HFGMC by various researchers were presented by Aboudi [3, 4]. MAC/GMC includes both theories and can thus be thought of as a variable-fidelity tool.

It should be noted that MAC/GMC includes capabilities for traditional constituent materials as well as thermo-electro-magnetic materials and shape memory alloys (so-called "smart" materials). The code also includes a multiscale classical lamination theory module, wherein Aboudi's micromechanics theories are employed at each integration point in each ply, see Fig. 3a. Thus, once lamination