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by

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Foreword

Early Husserl, Mathematics and Logic

Edmund Husserl's historically inalienable role as "the father of phenomenology" and the attitudes this description arouses in his friends and foes alike have led to a persistent and systematic disregard of his early work. Where notice is taken of it at all, it is generally considered as a product of apprenticeship, while he was learning his trade, before the breakthrough work of the *Logical Investigations* and the methodological turn to phenomenology with its attendant reductions and transcendental idealism. Husserl began his career as a mathematician, so the line tends to be that it was natural for him to start there but at least as natural for him to move on to the bigger (one might say, "more grown-up") issues of the foundations of logic and methodology in general. Certainly those admirers and detractors of Husserl who see his main role as a progenitor of so-called Continental philosophy are likely to be both less attuned to the interests of a philosopher who had more in common with Frege and Hilbert than with Heidegger and Derrida, and less inclined to accord that background a role in appraising Husserl's contribution to thought.

Stefania Centrone's thorough and painstaking exposition of Husserl's early work is a timely reminder that he was a philosopher of insight and stature well before he burst onto the general philosophical scene. One would hope this realisation could become universal, but given entrenched interests and attitudes it is unlikely to be heeded as widely as it should. So why should we take note of the early Husserl? What relevance does it have for his own development, and what significance for philosophy at the turn of the twentieth century and beyond?

The history of the philosophy of mathematics during the golden years of 1879–1939 hardly ever mentions Husserl. One reads about Dedekind, Cantor, Frege, Peano, Russell, Poincaré, Hilbert, Brouwer, Weyl, Gödel, Church and Turing. Husserl is effectively written out of the picture because his *Philosophie der Arithmetik* was criticised as psychologistic by Frege, and majority opinion is on Frege's side. Husserl, goes the story, saw the light, realised Frege was right, rounded on the psychologism of his earlier self and his teacher Brentano, did his penance, and then swiftly moved on to other things, turning to the more exciting and ultimately more popular forms of essentialism and idealism that stimulated two generations of students in Göttingen and Freiburg. The fourteen "lost years" in Halle are consigned to prehistory. It is my contention, based in part on the compelling evidence presented in Stefania Centrone's book, that Husserl would deserve an honourable mention in the history of the philosophy of mathematics and logic alongside the others, and that this would indeed have been more apparent had he not gone on to become the philosophical colossus with whom we are familiar.

Firstly, and for the record, Husserl was the first person outside Jena to take Frege seriously. This is despite the fact that Husserl's Halle colleague and friend Georg Cantor knew a little about Frege from an early and very sketchy review of Frege's *Begriffsschrift*. In later life, Husserl rather cruelly described Frege to Heinrich Scholz as an oddball (*Sonderling*), which was at the time an accurate reflection of the general perception of Frege's role and status. Nevertheless, unlike anyone before Russell, Husserl early on paid Frege the compliment of reading *Die Grundlagen der Arithmetik*, thinking about its theory, and criticising it in his own first book *Philosophie der Arithmetik*. And in two respects at least, Husserl's criticisms are right on the money. The first is that Frege's choice of the extension of the concept "equinumerous with the concept *F*" to be the number of the *F*s, is clearly artificial, and not what we understand by number. Secondly, Husserl's analysis and ontology of number are preferable to Frege's. We take number to be neither a property of concepts nor an abstract object but a non-distributive formal property of collections. 'Four' is not a property of the concept "evangelist", nor is it an object which is the extension of a second-order concept, but a property of the group or collection of the evangelists, the four of them. Husserl does go too far in criticising Frege in that he sees no role whatsoever for the idea of abstraction under an equivalence relation of equinumerosity. That most useful insight of Frege can and should be coupled to Husserl's multitude theory of numbers. So he is not an infallible guide to Frege. But Frege is a worse guide to Husserl. Having spotted in Husserl a number of statements that are construable psychologistically, Frege took the opportunity in a review of *Philosophie der Arithmetik* to pay Husserl back with compound interest for the temerity of having criticised him. Some of Frege's barbed and ill-tempered criticism is accurate, but a lot of it is not. So the legend was born that Husserl was converted by the strength and cogency of Frege's criticisms into being a crusading anti-psychologist himself. Certainly Husserl recognised that Frege had made some valid points, and was grateful for the criticism, but it is one-sided to suppose he was not already becoming dissatisfied with aspects of his early philosophy of arithmetic, aspects which stopped him from completing the planned second volume. On the other hand Husserl's over-zealous defenders have insisted that Frege had nothing whatsoever to teach him and that his changes of mind were wholly intrinsic and independent of the criticism. The truth lies somewhere in between. But the fact remains that Husserl never changed his view as to the nature of natural numbers as properties of collections or multitudes, so on the substantial issue of the correct ontology of arithmetic, he was unmoved, and I think rightly so.

The formative influence on Husserl's development as a philosopher was of course Franz Brentano, as Husserl was happy to acknowledge, and as he demonstrates in the dedication of *Philosophie der Arithmetik*, a dedication incidentally, which Brentano only belatedly and grudgingly acknowledged. But the Brentano who influenced Husserl was the Brentano of the lecture theatre, not the Brentano of the published works. So the impression could easily arise that all the mathematical ideas in *Philosophie der Arithmetik* were Husserl's. In fact the idea of collective combination is already present in rudimentary form in the 1884/5 Vienna lectures on logic that Husserl attended, though no acknowledgement finds its way into Husserl's text, perhaps because the source was unpublished. A balanced and objective judgement of the extent to which Husserl's ideas at this stage are indebted to those of Brentano must await a proper publication of Brentano's chaotic *Nachlass*. Husserl's work on the philosophy of mathematics from the 1890s was also largely unpublished at the time, but his literary remains have received a much more complete and favourable treatment than those of Brentano, so it is possible to examine in print the many manuscripts, notes and lecture notes from this period. They show him to have been interested in a wide variety of topics in the philosophy of mathematics and logic, and to have anticipated a number of topics that later became common currency in ensuing years. Stefania Centrone shows that some of these ideas, in particular those connected with one concept of completness, and the use of ideal or "imaginary" elements in formal mathematics, became key concepts in the Hilbertian formalism programme, without receiving from Hilbert the due they deserved. There may have been personal reasons for this: Hilbert was in favour of Husserl's appointment at Göttingen, hoping to find a philosopher to whom he could talk philosophy of mathematics, but Husserl was already broadening his interests in other directions, and they fell out over university matters, in particular the promotion of Leonard Nelson.

The most important influence on Husserl the logician was undoubtedly that of Bernard Bolzano, whose mighty *Wissenschaftslehre* (1837) Husserl chanced upon in a second-hand book shop. The *Wissenschaftslehre* anticipated or indeed forged many ideas still now widely regarded as achievements of later logicians such as Frege, Tarski, and Quine. But for Husserl's serendipitous find, Bolzano's genius might well have lain undiscovered for much longer, as Husserl is lavish with his praise in the *Logical Investigations*. Bolzano's semantic platonism was congenial to Husserl, and was turned to use as the objectivistic alternative to psychologism in logic, while being modified by Husserl to conform with Brentano's intentionality theory of the mental. Husserl was also pleased to adopt and adapt technical concepts from Bolzano's logic, explained in semantic terms, which he had not taken from Frege, whose philosophical outlook precluded any idea of semantics as a proper logical discipline. Husserl, having absorbed his formal logic initially from the logical algebraist Ernst Schröder, was no stranger to semantic matters, indeed he proposed an intensionalistic reading of Schröder's inclusion and defended this against the extensionalist logician Voigt. Husserl was never one interested in investigating logical calculi for their own sake: he was interested neither in proving theorems nor in seeing how axiomatic systems could be refined, manipulated, and simplified. Most of his remarks are metalogical rather than logical, and this goes for his remarks on mathematics as well. It is possible to see in Husserl's instinctive practice the seeds of the later concept of metamathematics of Hilbert. At the same time Husserl's lack of interest in formal manipulation stood him in poor stead when it came to gaining the respect and citation of mathematicians. Nevertheless, while he did not venture into such areas in his published writings, Husserl was quite prepared in lectures to work through logical proofs in the modern manner, as Stefania Centrone shows with regard to the logic lectures of 1896. In her chapter on Husserl the logician, Stefania Centrone steers us through the complications of Husserl's attitude to logic, his borrowings, modifications and influences.

It has long been known that during Hilbert's first phase of encounter with the foundations of mathematics, around the turn of the twentieth century, he was interested in the question whether all mathematical questions have a definite yes or no answer, and whether mathematicians can in principle show what the answer is. It has also been noted that Husserl also thought about such matters, though much of the evidence about this came from his later work *Formal and Transcendental Logic* of 1929, which represents his last published writings on the foundations of mathematics and logic. In her final chapter Stefania Centrone shows that Husserl's thinking originated much earlier, in a lecture of 1901 delivered shortly after his arrival in Göttingen, and likely to have resonated strongly with Hilbert. This lecture shows Husserl to be fully aware of and indeed himself advancing cutting-edge developments in the philosophy of mathematics: issues of formalization, algebraicization, decidability, completeness, models, and the consistent extension of mathematical concepts into new mathematical systems. If Husserl's ideas seem inchoate and unfocussed by today's standards, it is instructive to compare them with Hilbert's own writings, of this time and indeed later. To those accustomed to the limpid clarities of Frege and Russell, Hilbert's writings, despite their evident suggestiveness, and their origin in a mathematician of world ranking, are at times alarmingly unclear, and were not decisively sharpened until much later with the help of Paul Bernays and others. By those standards, Husserl's writing is equally suggestive and no less clear. A comparison with the 1929 work shows little subsequent advance. This is unsurprising, since Husserl, unlike Hilbert and others who returned again and again to the problems of logic and the foundation of mathematics, was preoccupied with many other philosophical matters after 1901, leaving it to others to gain the laurels for work on the foundations of mathematics.

Stefania Centrone's book presents us with an aspect of Husserl that, under counterfactual circumstances, might have been the familiar one: Husserl the innovative and thoughtful philosopher of mathematics and logic. It is instructive to engage in a little *epoché*: bracket the familiar Husserl of intentionality, phenomenology and transcendental idealism, and consider the colleague and contemporary of Cantor and Hilbert, writing about sets, numbers, consistency, formalization and the domains of theories. There is still much to learn about this phase and aspect of Husserl's thought, but thanks to Dr Centrone, it is now a good deal easier to engage with that enterprise.

Peter Simons
Trinity College Dublin

Preface

This book has been long in making. It partially originates from my doctoral dissertation that I wrote under the supervision of Professor Ettore Casari at the Scuola Normale Superiore in Pisa and defended in 2004. Further research on the main topics of the present book at the SNS in Pisa was supported by a 2 year post-doc grant for research on *Themes and problems of logical objectivism,* again under the supervision of Casari, followed by a 18 months research grant on *Logic and Philosophy in Husserl*, supervised by Professor Massimo Mugnai. I could finally complete this book in the hospitable environment of the Philosophy Department of Hamburg University, since Professor Wolfgang Künne, my present supervisor, kindly allowed me to use for this purpose the first 2 months of a 2 years Alexander-von-Humboldt fellowship for research on *Logical Objectivism, Inference and Foundational Proof in Bernard Bolzano's 'Wissenschaftslehre'*.

I am particularly indebted to my first teacher, Ettore Casari, who aroused my interest in logic, in mathematics and in Husserl's early writings and who made me realize that, as he used to put it, *"il mondo è vasto"*, i.e. that restriction to one single field of research can be more of a hindrance than a help for original work. Very special thanks must go to Professor Kevin Mulligan of the University of Geneva. We first met on the occasion of the defense of my PhD thesis. From then on he encouraged me more than anyone else in broadening and deepening the research I had begun in my thesis. We continued talking on many of the topics of this book over the years, and hopefully a trace of these discussions will be visible in many pages of my work. Many thanks go to Massimo Mugnai, who strongly supported my work and instilled in me an admiration for Leibniz that is bound to last, and to Burt Hopkins who very much encouraged the realization of my project. I would also like to thank Claudio Cesa and Francesco del Punta for many hours of enlightening discussions about the history of ideas. I am especially indebted to Wolfgang Künne. He commented incisively on every chapter, insisted on many clarifications and saw to it that I became more aware than ever of the importance of many issues in Bolzano's still sadly neglected Logic and in what Wittgenstein praised as "*die großartigen Werke Freges*".

Much feedback I had over the years from many scholars, in particular: Sergio Bernini, Arianna Betti, Riccardo Bruni, Andrea Cantini, Laura Crosilla, Carlo Ierna and Francesca Poggiolesi. The criticisms and suggestions made by an anonymous referee for Synthese Library who read the penultimate draft of this book were very helpful. The encouragement at a decisive moment and the friendly advice I received from Willemijn Arts, the Senior Publishing Editor, and from Ingrid van Laarhoven, the Senior Publishing Assistant of Springer Science and Business Media, were truly invaluable. Very special thanks must also go to Maja de Keijzer, Publishing Editor.

I am very grateful to my mother Nicoletta who made me love hard work and to my father Mario who aroused in me the love for philosophy. – With gratitude and affection I dedicate this book to my husband and best friend Piero.

Introduction

This book takes into account the first ten years of Edmund Husserl's work, from the publication of the *Philosophy of Arithmetic* (1891) to that of the *Logical Investigations* (1900/01), with the aim of precisely locating his early work in the field of logic and the philosophy of mathematics. This goal does seem to be worth pursuing especially in the light of the developments in formal logic during the past century. Surveying the vast growth of studies on this topic since the second world war, a tendency can be seen to emerge among the interpreters of Husserl's thought to remain within the methodological and even terminological bounds of Husserl's later phenomenology while, conversely, professional logicians fail to consider Husserl's contributions to the field of formal logic as significant for their discipline.

Our decision to focus upon Husserl's early reflections on logic and the philosophy of mathematics and to consider only selectively their elaboration in his mature work is motivated by the fact that these ideas were definitely original and surprisingly innovative at the moment of their first conception, i.e., in the years 1896–1901 when Husserl worked on the *Prolegomena*, while they no longer appear as fresh (though they are sometimes better articulated and corroborated) when they are taken up again in *Formal and Transcendental Logic* (1929). These ideas include, to mention some significant examples, the articulation of formal logic in logical levels according to a structure that is very close to what, today, is effectively used in standard logical textbooks, the unification of formal logic and mathematics in a most general mathematico-formal science that purports to be the concrete realization of the Leibnizian ideal of a *mathesis universalis*, and the explicit conception of abstract mathematics as a theory of structures.

The goal of our work is to restore the level of the real discussion between Husserl and his important early interlocutors, some of whom made definitive contributions to the development of formal logic as an autonomous discipline in the last two centuries. To this end we will consider Husserl's relationship to the algebraists of logic, in particular George Boole, as well as to Bernard Bolzano's, Gottlob Frege's and David Hilbert's contributions to logic.

With respect to the two main possibilities for textual research, philological and erudite commentary on the one hand, and comprehensive interpretative stances, on the other hand, this book opts for the second. Its contributions are almost exclusively analytical and, on the basis of a close reading of selected texts written in the indicated decade, it aims to bring to light the unity and depth of an original and comprehensive design of both a *theoretical systematization* of logic and its *philosophical foundation*.

The distinctive trait of Husserl's work during the period in question is the simultaneous presence in his logical and mathematical reflections of two different directions of research, (1) the project of a substantial mathematization of logic and (2) a conception of logic as the study of objective relations occurring among certain abstract logical entities. As regards (1), we find Husserl's interest in specifically *logico-formal* issues: he succeeds in grasping with great clarity and insight the implications of the *formal-abstract* trend in mathematics and, in particular, of its tendency toward *algebrization*, which he is able to transfer to and elaborate at the logico-theoretical level. As regards (2), we find Husserl's project to develop a philosophy of logic and mathematics focused on the systematic investigation of the properties and relations that occur among certain abstract semantical entities: a source of inspiration for this project is the theory of *Notions* (*Vorstellungen an sich*) and *Propositions* (*Sätze an sich*) in Bolzano's *Wissenschaftslehre,* and one of its more remote ancestors is the Stoic doctrine of *Sayables* (*lektá*). In this book we shall mainly focus on the research direction (1).

In **Chapter 1** we take Husserl's first major work, the *Philosophy of Arithmetic* (1891), as the starting point of our study. Dagfinn Føllesdal's conjectured in 1958 that Frege was an important factor in Husserl's conversion from the psychologism of this book to the anti-psychologism of the *Prolegomena*. This claim has been contested by Mohanty and others, but Føllesdal's defense is very convincing.[1] However, we will approach Husserl's first book from a perspective that is orthogonal to the psychologism issue. Rudolf Bernet has written that the *Philosophy of Arithmetic* "represents, not a mere youthful transgression stemming from Husserl's psychologistic period, but a highly valuable work of intrinsic and enduring importance". According to Bernet its value lies in the fact that "the *Philosophy of Arithmetic* . . . anticipates certain decisive results not only of the *Logical Investigations* but also of Husserl's later work."[2] On our view, however, the value of this text exceeds that of anticipating some claims that came to be consolidated in Husserl's phenomenology. The specific solutions that Husserl advances in his first book possess an intrinsic interest for logic and mathematics, and they are independent of the psychologistic context in which they originate.

In his *Philosophy of Arithmetic* Husserl enters into a very lively and stimulating debate about the foundational issues regarding the concepts of *number* and *set*. Moreover, this work contains many interesting insights regarding the formal and

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[1]D. Føllesdal 1958 (1994) and 1982, a reply to one of his critics.

[2]Bernet & Kern & Marbach 1989, 14.

computational aspect of theories. For instance, we find the elaboration of the concept of "number system" and investigations aiming at circumscribing the totality of all 'conceivable arithmetical operations', which bring to light how Husserl had arrived at a first delimitation of the class of number functions that today are called "partial recursive functions." In this context we discuss a part of Husserl's *Nachlass* text "On the Concept of Operation" (also from 1891) which develops a specifically logico-formal problem raised in the *Philosophy of Arithmetic* concerning the question of the formal irreducibility of the operation of multiplication to that of addition. We shall also discuss Husserl's relationship to Boole as regards the conception of the more properly formal and calculatorial aspect of theories and his relationship to Frege as regards the definition of the series of natural numbers. With respect to the latter problem we also take into consideration a *Nachlass* text "*On the Theory of Sets*" which is centered on the distinction between *finite* and *infinite* cardinals: according to Husserl himself, it was a grave "defect" of the *Philosophy of Arithmetic* not to have provided a theoretical account of this distinction. The heart of the issue is this: Husserl had defined natural numbers as the collection of all those objects that can be obtained starting from *zero* using a *finite number* of steps to *successors*, but he had not registered the fact that the crucial point of such a definition is precisely to reformulate successfully the reference to a "finite number of successor-steps" *without using the concept of a finite number* (since to use the latter concept is to fall into a vicious circle). Husserl's account is therefore at variance with what Frege had already done informally in his *Grundlagen* (1884) and then formally in the *Grundgesetze* (1893).

Chapter 2 on "the idea of a pure logic" examines selected themes belonging to the philosophy of mathematics and logic frequently raised and discussed by Husserl in the years between 1896 and 1900. The discussion pivots on various issues connected to the surprisingly innovative idea of a *stratification of formal logic* in logical levels. Roughly, (1) he outlines what was to become the modern conception of a formal language (*logical morphology*), (2) he provides a sketch of a propositional logic and a quantified logic (later in *Formal and Transcendental Logic* called "logic of consequences"), and (3) he largely anticipates the modern concept of a formal system (*theory of theories*). In this context, his attempt to unfold a concept of semi-formal enthymematic derivability and to characterize a notion of "dependency among truths," i.e. of a one-way entailment (between true propositions) of a reason-giving kind, plays a prominent role. Hence we have to consider the relation between Bolzano's notions of derivability (*Ableitbarkeit*) and consecutivity (*Abfolge*) and Husserl's notions of 'following from certain premises through correct inferences' and of 'grounding' or 'foundation' (*Begründung*).

At this point, a few words are in order about the importance of Bolzano's monumental *Wissenschaftslehre* for Husserl's early work, say from 1893–94 onwards. Husserl himself finds it important to stress in an appendix to Chapter 10 of the *Prolegomena*[3] that his investigations are not "in any sense mere commentaries upon, or critically improved expositions of, Bolzano's thought patterns", but that they "have been crucially stimulated by Bolzano …". In Husserl's eyes, Bolzano's great merit lies in his characterizing pure logic as a discipline that is concerned "with the most general conditions of *truth* itself"[4] and deals with the relations among the *contents* of our thoughts. So the emphasis is on Bolzanos' logical objectivism. He praises Bolzano's *opus magnum* as "a work which …. far surpasses everything that world-literature has to offer in the way of a systematic contribution to logic"[5]:

> Bolzano did not, of course, expressly discuss or support any independent demarcation of pure logic in our sense, but he provided one *de facto* in the first two volumes of his work, in his discussions of what underlay a *Wissenschaftslehre* or theory of science in the sense of his conception; he did so with such purity and scientific strictness, and with such a rich store of original, scientifically confirmed and ever fruitful thoughts, that we must count him as one of the greatest logicians of all time… Logic as a science must … be built upon Bolzano's work, and must learn from him its need for mathematical acuteness in distinctions, for mathematical exactness in theories. It will then reach a new standpoint for judging the mathematizing theories of logic, which mathematicians, quite unperturbed by philosophic scorn, are so successfully constructing.

However, Husserl directs at Bolzano two sorts of criticism[6] which are worth to be mentioned already in this Introduction. *Firstly*, though having circumscribed the domain of pure logic as "a closed field of independent and *a priori* abstract truths", Bolzano sees his investigations in the service of a science which sets up "the rules according to which we must proceed in the business of dividing the entire realm of truth into single sciences and in the exposition thereof in special textbooks".[7]

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[3]*PR* 224–227 (*Hinweise auf F.A. Lange und B. Bolzano*), *Pre* 222–224. The quotations that follow are all taken from this passage. Henceforth: *PR* = Husserl, *Logische Untersuchungen* I, *Prolegomena zur reinen Logik*, Tübingen 1993; *PRe* = English translation thereof, in: *Logical Investigations*, London 1970, Vol. I, 51–247. Responsibility for translations from German is mine, even when I refer to, benefit from, or simply echo published translations.

[4]Bolzano, *Wissenschaftslehre* (Sulzbach 1837), I, §16, 65. Henceforth: *WL*.

[5]In 1911 the key is a bit lower: these volumes, he now says, occupy "the highest rank in the logical world-literature of the 19th century" (quoted in Künne 2009, note 1, and commented upon in his 2008, 358).

[6]"Much as Bolzano's achievement is 'cast in one piece', it cannot be regarded (as such a deeply honest thinker would be the first to admit) as in any way final."

[7]Bolzano, *WL* I, §1, 7. In a note to Chapter 1 of the *Prolegomena* Husserl writes "The fourth volume of the *Wissenschaftslehre* is indeed especially devoted to the task which the definition expresses [The theory of science (or logic) is "the science which shows us how to present the sciences in convenient textbooks"]. But it strikes one as strange that the incomparably more important disciplines which the first three volumes treat of, should be represented merely as aids to a technology of scientific textbooks. Naturally, too, the values of this by no means as yet sufficiently valued work, which is, in fact, almost unused, rests on the researches of these earlier volumes" (*PR* 29, *PRe* 73).

According to Husserl, this relation should be inverted: pure logic should ground logic as a practical discipline. *Secondly*, Bolzano "did not quite exhaust the rich inspiration of Leibniz's logical intuitions, especially not in regard to mathematical syllogistics and to *mathesis universalis*". This is a criticism that Husserl resumes in more detail in his *Formale und transzendentale Logik*.[8] One may very well wonder whether it really hits its target, but it is of great significance because it highlights Husserl's attitude towards Bolzano's project of a unification of logic and mathematics in a most comprehensive science. In § 8 of Part I of his *Beyträge zu einer begründeteren Darstellung der Mathematik* (1810)[9] Bolzano defines mathematics as "a science which treats of the universal laws (forms) things must comply with in their existence (*eine Wissenschaft, die von den allgemeinen Gesetzen (Formen) handelt, nach welchen sich die Dinge in ihrem Daseyn richten müssen*)", where 'thing' is meant to cover "everything that can be object of our representational capacity". In I, § 9 he says of mathematics that it is "concerned with the question: what must things be like if they are to be possible at all (*wie müssen die Dinge beschaffen seyn, die möglich seyn sollen?*)" And in I, § 11 he says that the laws of what he calls "*die allgemeine Mathesis*" are "applicable to all things without any exception (*auf alle Dinge ganz ohne Ausnahme anwendbar*)". This discipline comprises, inter alia, *Logistik oder Arithmetik* and *Combinationslehre* (cp. I, § 3), whereas disciplines like geometry and chronometry are "subordinated to the whole universal mathesis as species to a genus (*der allgemeinen Mathesis insgesammt, wie Arten der Gattung, subordinirt*)".[10] Now Husserl acknowledges that Bolzano characterizes here "a universal *apriori* ontology", but he objects that Bolzano does not develop all features of *formalization*, of the transition from the material to the formal, and that he fails to keep the formal and the material aspects of ontology clearly distinct.

> When he conceives of the thing as such (*Ding überhaupt*) as the highest genus . . . it becomes clear that he did not see the difference between the empty form of the something as such as highest genus . . . and the universal realm of possible existents, of the real in the widest sense (*die universale Region des möglicherweise Daseienden, des im weitesten Sinne Realen*)), which differentiates itself in particular regions. He also did not see the difference between the subsumption of formal particularities (*Besonderungen*) under formal generalities and the subsumption of regional particularities . . . under formal generalities. . . In other words, Bolzano did not attain the proper concept of the formal,. . ., though he touched it somehow.[11]

In connection with Husserl's reflections upon the idea of a pure logic, we shall also discuss in Chapter 2 his development of a propositional calculus of the axiomatic-deductive kind, which is found in the final section of a lecture course on logic held at the university of Halle in summer 1896. This lecture course,

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[8]*Formal and Transcendental Logic* [ed. 1929] (henceforth cited as *FTL*) 74–75.

[9]See below, Ch. 2, § 2, n. 32.

[10]Cp. Casari 2004, 161.

[11]*FTL* 74–75.

generally known as *Logikvorlesung 1896*,[12] contains part of the core reflections that gave rise to the *Prolegomena* and the *Logical Investigations*.

It does *not yet* give us the "*Grundgerüst* (the basic scaffolding)" of the *Prolegomena*, in spite of Husserl's claim to the contrary in the *Preface* to the second edition of the *Logical Investigations*.[13] The editor of the critical Husserliana edition, Elisabeth Schuhmann, notes in her Introduction that there is an error in Husserl's own dating. He did not hold two complementary courses in 1896. Moreover, only a few pages of the *Prolegomena* (more exactly, §§ 4–8) coincide with material in the *Logikvorlesung*. The reason for this error is probably the fact that Husserl repeatedly re-used material from the *Logikvorlesung* of 1896 to prepare additional logic courses, for instance, the course "*Logik und Erkenntnistheorie*" (winter term 1901/02), the courses "*Logik*" and "*Allgemeine Erkenntnistheorie*" (winter 1902/03), and the series of lectures "*Alte und neue Logik*" (winter 1908/09). In particular, manuscripts from 1901/02, written after and based on the *Prolegomena*, were collected together – without indications of the date – with the *Logikvorlesung* of 1896. When preparing the new edition of the *Logical Investigations* in 1913, Husserl must have found them in the same 'convolute' as the 1901/02 lectures on "*Logik und Erkenntnistheorie*", which were also undated. Hence, in the draft for the preface to the new edition of the *Logical Investigations*, he wrote that the *Prolegomena* were, essentially, only an elaboration of the *Logikvorlesung* of the summer and winter 1896.

The issue of *imaginary numbers*, and, more precisely, of the "*logical meaning of the calculatory passage through the imaginary*," which is, without doubt, the guiding thread in Husserl's reflections on the role of the *formal* attitude in mathematics, is the specific topic of his famous "double" lecture (known as the *Doppelvortrag*) presented to the *Mathematische Gesellschaft* in Göttingen in winter 1901.

Chapter 3, the final chapter of this book, is focused on this and other thematically related texts of the *Nachlass*. In particular, we will emphasize Husserl's reflections on the notion of a *formal theory* in its double aspect of a *system of axioms* and the *manifold* underlying it. We will focus, furthermore, on the more specific reflections regarding, on the one hand, the structure of (what Husserl calls) *Universal Arithmetic* – i.e., a system of calculation rules valid in all number-systems (cardinal numbers, whole numbers, etc.), and, on the other hand, the structure of the *specific Arithmetics* or *systems of operations* – i.e. systems of calculation rules that contain those of universal arithmetic as common part plus some specific groups of rules able to characterize the behavior of arithmetical operations relating to a specific number system. Finally, we will consider Husserl's

[12]Husserl, *Logik. Vorlesung 1896*, ed. Elisabeth Schumann, Husserliana Materialienbände, Band 1, Kluwer, Dordrecht 2001. Henceforth: *LV'96*.

[13]"The *Prolegomena to Pure Logic* are, in their essential content, a simple elaboration of two complementary lecture courses held in Halle in the summer and winter of 1896."

reflections on the fundamental and closely connected notions of the *definiteness*, and the *formal extension*, of a theory.

In his many references to the *Doppelvortrag* (inter alia, in the second edition of the *Prolegomena*, in the *Ideas* and finally in *Formal and Transcendental Logic*) Husserl observes that some important ideas which he presented on that occasion were subsequently taken over, without acknowledgement, in the logical investigations of Hilbert's school.

> The concepts introduced here [Husserl means specifically the concept of a *definite* system of axioms] served me already at the beginning of the 1890s (in the "*Untersuchungen zur Theorie der formal-mathematischen Disziplinen* [Investigations pertaining to the theory of formal-mathematical disciplines]", which I intended as a continuation of my *Philosophie der Arithmetik*), to find a *fundamental* solution to the problem of the imaginary. . . . Since then I have often had occasion to develop the relevant concepts and theories in lectures and seminars, partly in complete detail; and in the winter semester of 1901/02 I dealt with them in a double lecture to the Göttingen Mathematical Society. Some parts of this train of thoughts have found their way into the literature, without mention of their original sources. – The close relationship of the concept of definiteness to the "axiom of completeness" introduced by Hilbert for the foundation of arithmetic will be immediately obvious to every mathematician.[14]

In his *Doppelvortrag* Husserl examines two notions of definiteness: "absolute definiteness," which, as he indicates, is analogous to Hilbert's axiom of completeness, and "relative definiteness," which he applies to systems of axioms and to the structures that underlie theories conceived of as deductive systems. Basically, a system of axioms that is "definite in the absolute sense" or "in the Hilbertian sense" (as Husserl puts it) is categorical, i.e. it individuates, up to isomorphism, only one model, whereas a system of axioms that is "definite in the relative sense" is not necessarily categorical, but it is such that every proposition written in the language of the theory can be decided on the basis of the axioms. Given the different implications of these two distinct notions, the aim of giving a rigorous (mathematical) definition seems to be worth pursuing. It should help to weed out some common misconceptions as regards the interpretation of these issues and to challenge some recent and well-documented contributions to this topic.

It is worth emphasizing that Husserl himself has pointed out that "the progress from vaguely formed, to mathematically exact, concepts and theories is, here as everywhere, the precondition for full insight into *a priori* connections and an inescapable demand of science".[15]

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[14] Husserl, *Ideen I*, § 72, n.1; *Ideas* 164, n. 17.
[15] Loc. cit.

Contents

Chapter 1
Philosophy of Arithmetic

1.1 Introduction

The *Philosophy of Arithmetic*,[1] Husserl's youthful work dedicated to a philosophical, or better, epistemological foundation of mathematics, shows the shift in his interests from more properly mathematical issues to those regarding the *philosophy of mathematics*. Husserl strives to understand and clarify *what* numbers and numerical relations *are*, a problem that he recasts in terms of the *subjective origin*[2] of the fundamental concepts of set theory and finite cardinal arithmetic. We will try to show that on the whole this work of Husserl's does not deserve the criticism and ensuing neglect that it suffered from, ever since Frege published his well-known *Review*.[3] Besides its hotly contested psychologism, we find ideas and conceptualizations that not only were original then, but are still interesting today, such as those concerning the autonomy of the formal-algorithmic aspect of abstract algebra and mathematics. Moreover, it is here that the Husserlian idea of a *universal arithmetic* receives its first formulation, the full elaboration of which will take at least ten more years, until his research on these topics reaches its stable form in 1901.[4]

[1] Husserl, *Philosophie der Arithmetik*. Mit ergänzenden Texten (1890–1901), Huss XII, 1–283. Henceforth cited as *PdA*. English translation cited as *PoA*.

[2] Cp. Tieszen 1996: "Husserl thinks that arithmetical knowledge is originally built up in founding acts from basic, everyday intuitions in a way that reflects our a priori cognitive involvement" (304).

[3] Frege 1894. Cp., for example: Osborn 1949; Picker 1962, 289; Beth 1966, 353. Among the interpretations that give a positive re-evaluation of some aspects of the *PdA*: Farber 1943; Føllesdal 1958; Haddock 1973 (especially Ch. VI: however, his focus is mainly on Husserl's logical theories in his later works, in particular in the *Logical Investigations* and *Formal and Transcendental Logic*); Miller 1982; Willard 1974, 97 f. & 1984; Tieszen 1990; Ortiz Hill 1994a & b.

[4] See *Das Imaginäre in der Mathematik* (December/January 1901/02), *PdA*, App. 430–451, *PoA* 409–432, and the new critical edition Schumann & Schumann 2001. Willard 1984 rightly stresses that Husserl shared "the general persuasion of mathematicians of the time that a rigorous development of higher analysis – *arithmetica universalis* in Newton's sense – would have to emanate from elementary arithmetic alone." However, few lines later he writes that "these further

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Of the two volumes of which the work, according to Husserl's initial plan, was to consist, only the first was completed and published (Halle 1891). In spite of the preliminary nature of the studies intended for the second volume that we possess,[5] we know that it was to contain two parts: one dedicated to "the justification of utilizing in calculations the quasi-numbers (*Quasizahlen*) originating out of the inverse operations: the negative, imaginary, fractional and irrational numbers,"[6] the other to the determination of the general characteristics of a universal arithmetic.

In the first and only volume the contents of the *Habilitationsschrift* "*On the Concept of Number: Psychological Analyses*" (Halle 1887)[7] appeared as Chapters I–IV without significant changes. It had as its main topic the constitution of the concept of cardinal number (*Anzahl*), and it also consists of two parts. The first part studies the fundamental concepts of mathematics – *multiplicity* (*Vielheit*), *cardinal*

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matters – intended is the foundation of the whole of mathematics on the elementary arithmetic – never received any detailed response from Husserl" (22). Though it is true that Husserl's "inquiry into the theory of number led him into general epistemological investigations that occupied him for the remainder of his life," it should not be neglected that Husserl's *Double Lecture on the Imaginary in Mathematics* is a non-trivial attempt at dealing with the reduction of other number systems (the wholes, the rationals, the reals) and of their properties to the naturals and thus to give an answer to some of the problems left unsolved in *PdA*. From Miller 1982, too, one gets the impression that Husserl did not achieve "the philosophical project he had begun under the inspiration of Weierstrass" (9). Miller argues that "one can only conjecture about Husserl's reasoning here. Perhaps his view was simply this: Since even our most elementary number concept is largely 'symbolic', there is no intrinsic mystery regarding the introduction of other 'symbolic' concepts, such as those of negative, rational, irrational and imaginary numbers. The original or 'authentic' number concept has already been broadened to include numbers not actually given to us, so why should we not broaden it further? We are perfectly justified in taking this step . . ." What Miller does not seem to pay sufficient attention to is that in the *Double Lecture* Husserl's philosophical problem is one of a conceptual kind: formally we can extend the natural number system by dropping certain restrictions to the executability of certain operations, but we cannot expand the concept at the basis of a specific numerical field (cp. our account of Husserl´s critique of Dedekind in ch. 3 below). So Husserl's reasoning seems to be just the opposite of what Miller suggests.

[5]See *PdA* App. Abhandlung I, *Zur Logik der Zeichen* (*Semiotik*), 340–373; II, *Begriff der allgemeinen Arithmetik*, 374–379; III, *Die Arithmetik als apriorische Wissenschaft*, 380–384; V, *Zum Begriff der Operation*, 408–429; IX, *Die Frage der Aufklärung des Begriffes der "natürlichen" Zahlen, als "gegebener", "individuell bestimmter"*, 489–492; X, *Zur formalen Bestimmung einer Mannigfaltigkeit*, 493–500. See Eley, Textkritischer Anhang, 521–562. A separate treatment has to be reserved for Abhandlung V, *Zur Lehre vom Inbegriff*, 385–407, see below Appendix 3.

[6]*PoA* Foreword 7; *PdA* Vorrede 7.

[7]"A part of the psychological investigations in the present volume was already included, almost word-for-word, in my *Habilitationsschrift*, from which a booklet four galley sheets in length, titled "On the Concept of Number: Psychological Analyses" was printed in the fall of 1887 but was never made available in bookstores" (*PoA* Foreword 8; *PdA* Vorrede 8). See Miller 1982, 11; Willard, 1984, 39; Ierna 2005: "Husserl's *Habilitationsschrift* was never published and the work now known as *Über den Begriff der Zahl* is in fact just the first chapter of the *Habilitationsschrift*" (8).

number (*Anzahl*)[8] and *unity* (*Einheit*) – in so far as they are presented properly (*eigentlich vorgestellt*), i.e. intuitively given. The second part tackles the study of symbolic presentations applied to mathematics.[9]

To understand the interest in a psychological foundation of arithmetic as developed in the *Philosophy of Arithmetic*, we have to take into account both the specific historical moment at which the work was written as well as Husserl's own academic background.

In 1891 a work that aimed at laying bare the psychological foundation of arithmetic was able to arouse the interest of mathematicians and of philosophers. Since a psychologistic orientation was then dominant in philosophy, it did not appear strange at all to look for the ultimate foundation of arithmetic in this science. And then, the work fitted into the general framework of the so-called '*research on the foundations*' of mathematics, and it proposed to tackle it from a philosophical and from a mathematical point of view.[10] From the former point of view, 'research on the foundations' consisted in identifying the fundamental concepts of mathematics and examining the essence of mathematical knowledge, and it is precisely on this aspect that the first volume of Husserl's work focuses. From the mathematical point of view, research on the foundations, as it had been conducted in the second half of the nineteenth century, had given prominence to elementary arithmetic, i.e. the theory of natural numbers, as the simple and secure basis on which to found the entire edifice of mathematics. And it was precisely the issue of the "reduction" of other number systems (the wholes, the rationals, the reals) and of their properties to elementary arithmetic that was to be the object of the second volume of Husserl's work, which was never completed.[11]

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[8]To be understood as '*finite* cardinal number' or 'natural number'. "*E. Schröder* introduced this term (*natürliche Zahl*) . . . it is apparently intended to mark the distinction of the cardinal numbers (*Anzahlen*) over against the other forms of number which come into play in arithmetic: the rational and irrational, the positive, negative and imaginary numbers. Moreover the term '*Anzahl*' is not totally univocal, since it has sometimes been used to designate the concepts of numbers in series. . . . Nevertheless, we have thought it most suitable in this work to adhere to the older and almost universally customary use of language" (*PdA* 114 n., *PoA* 120 n.).

[9]"In the first of its two parts, the Volume I before us deals with the questions, chiefly psychological, involved in the analysis of the concepts *multiplicity*, *unity*, and *number*, insofar as they are given to us authentically (*eigentlich*) and not through indirect symbolizations. The second part then considers the symbolic representations of multiplicity and number, and attempts to show how the fact that we are almost totally limited to symbolic concepts of numbers determines the sense and objective of number arithmetic" (*PdA* 7; *PoA* 7).

[10]See Ortiz Hill 2002, 81.

[11]As is well known, one of the traits that distinguish the mathematics of the nineteenth century from the mathematics of the preceding century, is the birth of that movement, often called the 'critical movement', characterized by the need to provide rigorous concepts and proofs for vast branches of analysis and, later on, to reconsider the foundation of mathematics. The arithmetization of analysis initiated by Weierstrass concludes with the simultaneous publication in 1872 of the foundations of the system of real numbers by Georg Cantor (1845–1918) and Richard Dedekind (1831–1916). See Kline 1972, 947–978; Casari 1973, 1 ff.

As regards Husserl's academic background, we note that while working on the *Philosophy of Arithmetic* he still was – as Brentano defined him in a letter to Stumpf of 1886, asking him to support Husserl in his attempt to obtain the status of *Privatdozent* in Halle – "a mathematician interested in philosophical questions." In fact, Husserl had studied mathematics in Berlin with mathematicians of great stature, such as Kronecker, Kummer and Weierstrass, and he had been Weierstrass' assistant, working with him until about 1883.[12] The Husserl-Archives in Leuven have the following stenographical notes (*Nachschriften*) of lectures on mathematics:

1. *Einleitung in die Theorie der analytischen Funktionen* (Weierstraß, S.S. 1878)
2. *Stenographische Nachschrift der 54 Vorlesungen über die Theorie der algebraischen Gleichungen* (Ludwig [*sic*!] Kronecker, W.S. 1878/79)
3. *Einleitung in die Theorie der elliptischen Funktionen* (Weierstraß, W.S. 1878/79)
4. *Vorlesung über die Variationsrechnung* (Weierstraß, S.S. 1879), a notebook which contains an elaboration of lectures by Weierstraß in that term, made by L. Baur. Husserl employed it to complete his elaboration of lectures by Weierstraß on the calculus of variations and mentioned it therein. The notebook has on the front page solely the mark: "Edmund Husserl 1880".
5. *Theorie der analytischen Funktionen* (Weierstraß, W.S. 1880/81).[13]

During the winter semester 1884/85 and the summer semester 1886 Husserl came under the influence of the philosophy of Franz Brentano,[14] and – as his own words testify – it was precisely in virtue of this influence that he came to dedicate himself completely to philosophy:

> In a time of growing philosophical interests and of wavering, whether I should stick with mathematics for life or dedicate myself completely to philosophy, Brentano's lectures gave the breakthrough. I attended them at first out of mere curiosity, to hear the man, who at that time was the talk of the day in Vienna, venerated and admired by some in an extreme way, by (no few) others insulted as masked Jesuit, flatterer, salesman of idle chit-chat (*Friseur*), sophist, scholastic. At the first impression I was quite affected. . . . Soon I was drawn onwards and convinced by the absolutely unique clarity and the dialectic acuity of his arguments, . . . by the cataleptic force of his way of developing problems, theories. Most of all from his lectures I got the conviction that gave me the courage of choosing philosophy as my life-long work. . . [to maintain] that philosophy, too, is an area of serious work, that it also can and hence must be treated as a rigorous science.[15]

[12]Cf. Schuhmann 1977, 7.

[13]Eley, *Einleitung des Herausgebers*, *PdA* xxi–xxii; K. Schuhmann 1977, 6–9; Miller 1982, 2–3; Ierna 2005, 5.

[14]"*me totum abdidi in studia philosophica duce Francisco Brentano*" (Schuhmann 1977, 13).

[15]Husserl in: Kraus 1919, 153–154.