Ahmad A. Kamal

1000 Solved Problems in Classical Physics

An Exercise Book

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Dedicated to my Parents

Preface

This book complements the book *1000 Solved Problems in Modern Physics* by the same author and published by Springer-Verlag so that bulk of the courses for undergraduate curriculum are covered. It is targeted mainly at the undergraduate students of USA, UK and other European countries and the M.Sc. students of Asian countries, but will be found useful for the graduate students, students preparing for graduate record examination (GRE), teachers and tutors. This is a by-product of lectures given at the Osmania University, University of Ottawa and University of Tebriz over several years and is intended to assist the students in their assignments and examinations. The book covers a wide spectrum of disciplines in classical physics and is mainly based on the actual examination papers of UK and the Indian universities. The selected problems display a large variety and conform to syllabi which are currently being used in various countries.

The book is divided into 15 chapters. Each chapter begins with basic concepts and a set of formulae used for solving problems for quick reference, followed by a number of problems and their solutions.

The problems are judiciously selected and are arranged section-wise. The solutions are neither pedantic nor terse. The approach is straightforward and step-by-step solutions are elaborately provided. There are approximately 450 line diagrams, one-fourth of them in colour for illustration. A subject index and a problem index are provided at the end of the book.

Elementary calculus, vector calculus and algebra are the prerequisites. The areas of mechanics and electromagnetism are emphasized. No book on problems can claim to exhaust the variety in the limited space. An attempt is made to include the important types of problems at the undergraduate level.

It is a pleasure to thank Javid, Suraiya and Techastra Solutions (P) Ltd. for typesetting and Maryam for her patience. I am grateful to the universities of UK and India for permitting me to use their question papers; to R.W. Norris and W. Seymour, *Mechanics via Calculus*, Longmans, Green and Co., 1923; to Robert A. Becker, *Introduction to Theoretical Mechanics*, McGraw-Hill Book Co. Inc, 1954, for one problem; and Google Images for the cover page. My thanks are to Springer-Verlag, in particular Claus Ascheron, Adelheid Duhm and Elke Sauer, for constant encouragement.

Murphy, Texas *Ahmad A. Kamal*
November 2010

Contents

Chapter 1
Kinematics and Statics

Abstract *Chapter 1* is devoted to problems based on one and two dimensions. The use of various kinematical formulae and the sign convention are pointed out. Problems in statics involve force and torque, centre of mass of various systems and equilibrium.

1.1 Basic Concepts and Formulae

Motion in One Dimension

The notation used is as follows: u = initial velocity, v = final velocity, a = acceleration, s = displacement, t = time (Table 1.1).

Table 1.1 Kinematical equations

	U	V	A	S	t
(i) $v = u + at$	✓	✓	✓	X	✓
(ii) $s = ut + 1/2at^2$	✓	X	✓	✓	✓
(iii) $v^2 = u^2 + 2as$	✓	✓	✓	✓	X
(iv) $s = \frac{1}{2}(u + v)t$	✓	✓	X	✓	✓

In each of the equations u is present. Out of the remaining four quantities only three are required. The initial direction of motion is taken as positive. Along this direction u and s and a are taken as positive, t is always positive, v can be positive or negative. As an example, an object is dropped from a rising balloon. Here, the parameters for the object will be as follows:

u = initial velocity of the balloon (as seen from the ground)

u = +ve, $a = -g$. t = +ve, v = +ve or −ve depending on the value of t, s = +ve or −ve, if s = −ve, then the object is found below the point it was released.

Note that (ii) and (iii) are quadratic. Depending on the value of u, both the roots may be real or only one may be real or both may be imaginary and therefore unphysical.

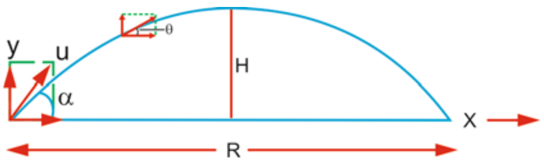
v–t and *a–t* Graphs

The area under the *v–t* graph gives the displacement (see prob. 1.11) and the area under the *a–t* graph gives the velocity.

Motion in Two Dimensions – Projectile Motion

$$\text{Equation: } y = x \tan \alpha - \frac{1}{2}\frac{gx^2}{u^2 \cos^2 \alpha} \tag{1.1}$$

Fig. 1.1 Projectile Motion



$$\text{Time of flight: } T = \frac{2u \sin \alpha}{g} \tag{1.2}$$

$$\text{Range: } R = \frac{u^2 \sin 2\alpha}{g} \tag{1.3}$$

$$\text{Maximum height: } H = \frac{u^2 \sin^2 \alpha}{2g} \tag{1.4}$$

$$\text{Velocity: } v = \sqrt{g^2t^2 - 2ug \sin \alpha . t + u^2} \tag{1.5}$$

$$\text{Angle: } \tan \theta = \frac{u \sin \alpha - gt}{u \ \cos \alpha} \tag{1.6}$$

Relative Velocity

If v_A is the velocity of A and v_B that of B, then the relative velocity of A with respect to B will be

$$v_{AB} = v_A - v_B \tag{1.7}$$

Motion in Resisting Medium

In the absence of air the initial speed of a particle thrown upward is equal to that of final speed, and the time of ascent is equal to that of descent. However, in the presence of air resistance the final speed is less than the initial speed and the time of descent is greater than that of ascent (see prob. 1.21).

Equation of motion of a body in air whose resistance varies as the velocity of the body (see prob. 1.22).

Centre of mass is defined as

$$\boldsymbol{r}_{\mathrm{cm}} = \frac{\Sigma m_i r_i}{\Sigma m_i} = \frac{1}{M}\Sigma m_i \boldsymbol{r}_i \tag{1.8}$$

Centre of mass velocity is defined as

$$\boldsymbol{V}_{\mathrm{c}} = \frac{1}{M}\Sigma m_i \dot{r}_i \tag{1.9}$$

The centre of mass moves as if the mass of various particles is concentrated at the location of the centre of mass.

Equilibrium

A system will be in translational equilibrium if $\Sigma \boldsymbol{F} = 0$. In terms of potential $\dfrac{\partial V}{\partial x} = 0$, where V is the potential. The equilibrium will be stable if $\dfrac{\partial^2 V}{\partial x^2} < 0$. A system will be in rotational equilibrium if the sum of the external torques is zero, i.e. $\Sigma \tau_i = 0$

1.2 Problems

1.2.1 Motion in One Dimension

1.1 A car starts from rest at constant acceleration of $2.0\,\mathrm{m/s^2}$. At the same instant a truck travelling with a constant speed of 10 m/s overtakes and passes the car.

(a) How far beyond the starting point will the car overtake the truck?
(b) After what time will this happen?
(c) At that instant what will be the speed of the car?

1.2 From an elevated point A, a stone is projected vertically upward. When the stone reaches a distance h below A, its velocity is double of what it was at a height h above A. Show that the greatest height obtained by the stone above A is $5h/3$.

[Adelaide University]

1.3 A stone is dropped from a height of 19.6 m, above the ground while a second stone is simultaneously projected from the ground with sufficient velocity to enable it to ascend 19.6 m. When and where the stones would meet.

1.4 A particle moves according to the law $x = A \sin \pi t$, where x is the displacement and t is time. Find the distance traversed by the particle in 3.0 s.

1.5 A man of height 1.8 m walks away from a lamp at a height of 6 m. If the man's speed is 7 m/s, find the speed in m/s at which the tip of the shadow moves.

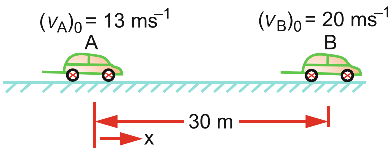
1.6 The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction, where x is in metres and t in seconds. Find the displacement when the velocity is zero.

1.7 A particle projected up passes the same height h at 2 and 10 s. Find h if $g = 9.8\,\text{m/s}^2$.

1.8 Cars A and B are travelling in adjacent lanes along a straight road (Fig. 1.2). At time, $t = 0$ their positions and speeds are as shown in the diagram. If car A has a constant acceleration of $0.6\,\text{m/s}^2$ and car B has a constant deceleration of $0.46\,\text{m/s}^2$, determine when A will overtake B.

[University of Manchester 2007]

Fig. 1.2



1.9 A boy stands at A in a field at a distance 600 m from the road BC. In the field he can walk at 1 m/s while on the road at 2 m/s. He can walk in the field along AD and on the road along DC so as to reach the destination C (Fig. 1.3). What should be his route so that he can reach the destination in the least time and determine the time.

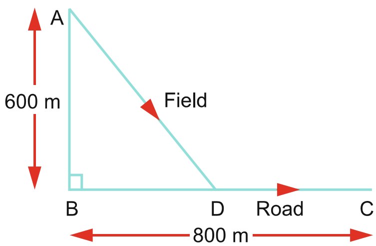
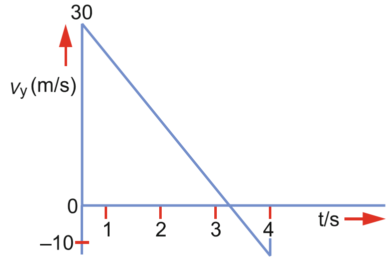


Fig. 1.3

1.10 Water drips from the nozzle of a shower onto the floor 2.45 m below. The drops fall at regular interval of time, the first drop striking the floor at the instant the third drop begins to fall. Locate the second drop when the first drop strikes the floor.

1.11 The velocity–time graph for the vertical component of the velocity of an object thrown upward from the ground which reaches the roof of a building and returns to the ground is shown in Fig. 1.4. Calculate the height of the building.

Fig. 1.4



1.12 A ball is dropped into a lake from a diving board 4.9 m above the water. It hits the water with velocity v and then sinks to the bottom with the constant velocity v. It reaches the bottom of the lake 5.0 s after it is dropped. Find

(a) the average velocity of the ball and
(b) the depth of the lake.

1.13 A stone is dropped into the water from a tower 44.1 m above the ground. Another stone is thrown vertically down 1.0 s after the first one is dropped. Both the stones strike the ground at the same time. What was the initial velocity of the second stone?

1.14 A boy observes a cricket ball move up and down past a window 2 m high. If the total time the ball is in sight is 1.0 s, find the height above the window that the ball rises.

1.15 In the last second of a free fall, a body covered three-fourth of its total path:

(a) For what time did the body fall?
(b) From what height did the body fall?

1.16 A man travelling west at 4 km/h finds that the wind appears to blow from the south. On doubling his speed he finds that it appears to blow from the southwest. Find the magnitude and direction of the wind's velocity.

1.17 An elevator of height h ascends with constant acceleration a. When it crosses a platform, it has acquired a velocity u. At this instant a bolt drops from the top of the elevator. Find the time for the bolt to hit the floor of the elevator.

1.18 A car and a truck are both travelling with a constant speed of 20 m/s. The car is 10 m behind the truck. The truck driver suddenly applies his brakes, causing the truck to decelerate at the constant rate of 2 m/s^2. Two seconds later the driver of the car applies his brakes and just manages to avoid a rear-end collision. Determine the constant rate at which the car decelerated.

1.19 Ship A is 10 km due west of ship B. Ship A is heading directly north at a speed of 30 km/h, while ship B is heading in a direction 60° west of north at a speed of 20 km/h.

(i) Determine the magnitude and direction of the velocity of ship B relative to ship A.
(ii) What will be their distance of closest approach?

[University of Manchester 2008]

1.20 A balloon is ascending at the rate of 9.8 m/s at a height of 98 m above the ground when a packet is dropped. How long does it take the packet to reach the ground?

1.2.2 Motion in Resisting Medium

1.21 An object of mass m is thrown vertically up. In the presence of heavy air resistance the time of ascent (t_1) is no longer equal to the time of descent (t_2). Similarly the initial speed (u) with which the body is thrown is not equal to the final speed (v) with which the object returns. Assuming that the air resistance F is constant show that

$$\frac{t_2}{t_1} = \sqrt{\frac{g + F/m}{g - F/m}}; \quad \frac{v}{u} = \sqrt{\frac{g - F/m}{g + F/m}}$$

1.22 Determine the motion of a body falling under gravity, the resistance of air being assumed proportional to the velocity.

1.23 Determine the motion of a body falling under gravity, the resistance of air being assumed proportional to the square of the velocity.

1.24 A body is projected upward with initial velocity u against air resistance which is assumed to be proportional to the square of velocity. Determine the height to which the body will rise.

1.25 Under the assumption of the air resistance being proportional to the square of velocity, find the loss in kinetic energy when the body has been projected upward with velocity u and return to the point of projection.

1.2.3 Motion in Two Dimensions

1.26 A particle moving in the xy-plane has velocity components $\mathrm{d}x/\mathrm{d}t = 6 + 2t$ and $\mathrm{d}y/\mathrm{d}t = 4 + t$
where x and y are measured in metres and t in seconds.

(i) Integrate the above equation to obtain x and y as functions of time, given that the particle was initially at the origin.
(ii) Write the velocity $\boldsymbol{v}$ of the particle in terms of the unit vectors $\hat{i}$ and $\hat{j}$.

(iii) Show that the acceleration of the particle may be written as $a = 2\hat{i} + \hat{j}$.
(iv) Find the magnitude of the acceleration and its direction with respect to the x-axis.

[University of Aberystwyth Wales 2000]

1.27 Two objects are projected horizontally in opposite directions from the top of a tower with velocities u_1 and u_2. Find the time when the velocity vectors are perpendicular to each other and the distance of separation at that instant.

1.28 From the ground an object is projected upward with sufficient velocity so that it crosses the top of a tower in time t_1 and reaches the maximum height. It then comes down and recrosses the top of the tower in time t_2, time being measured from the instant the object was projected up. A second object released from the top of the tower reaches the ground in time t_3. Show that $t_3 = \sqrt{t_1 t_2}$.

1.29 A shell is fired at an angle θ with the horizontal up a plane inclined at an angle α. Show that for maximum range, $\theta = \frac{\alpha}{2} + \frac{\pi}{4}$.

1.30 A stone is thrown from ground level over horizontal ground. It just clears three walls, the successive distances between them being r and $2r$. The inner wall is 15/7 times as high as the outer walls which are equal in height. The total horizontal range is nr, where n is an integer. Find n.

[University of Dublin]

1.31 A boy wishes to throw a ball through a house via two small openings, one in the front and the other in the back window, the second window being directly behind the first. If the boy stands at a distance of 5 m in front of the house and the house is 6 m deep and if the opening in the front window is 5 m above him and that in the back window 2 m higher, calculate the velocity and the angle of projection of the ball that will enable him to accomplish his desire.

[University of Dublin]

1.32 A hunter directs his uncalibrated rifle toward a monkey sitting on a tree, at a height h above the ground and at distance d. The instant the monkey observes the flash of the fire of the rifle, it drops from the tree. Will the bullet hit the monkey?

1.33 If α is the angle of projection, R the range, h the maximum height, T the time of flight then show that
(a) $\tan\alpha = 4h/R$ and **(b)** $h = gT^2/8$

1.34 A projectile is fired at an angle of 60° to the horizontal with an initial velocity of 800 m/s:

(i) Find the time of flight of the projectile before it hits the ground
(ii) Find the distance it travels before it hits the ground (range)
(iii) Find the time of flight for the projectile to reach its maximum height

(iv) Show that the shape of its flight is in the form of a parabola $y = bx + cx^2$, where b and c are constants [acceleration due to gravity $g = 9.8\,\mathrm{m/s^2}$].
[University of Aberystwyth, Wales 2004]

1.35 A projectile of mass 20.0 kg is fired at an angle of 55.0° to the horizontal with an initial velocity of 350 m/s. At the highest point of the trajectory the projectile explodes into two equal fragments, one of which falls vertically downwards with no initial velocity immediately after the explosion. Neglect the effect of air resistance:

(i) How long after firing does the explosion occur?
(ii) Relative to the firing point, where do the two fragments hit the ground?
(iii) How much energy is released in the explosion?
[University of Manchester 2008]

1.36 An object is projected horizontally with velocity 10 m/s. Find the radius of curvature of its trajectory in 3 s after the motion has begun.

1.37 A and B are points on opposite banks of a river of breadth a and AB is at right angles to the flow of the river (Fig. 1.4). A boat leaves B and is rowed with constant velocity with the bow always directed toward A. If the velocity of the river is equal to this velocity, find the path of the boat (Fig. 1.5).

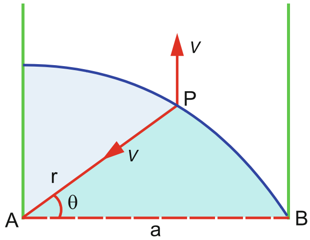


Fig. 1.5

1.38 A ball is thrown from a height h above the ground. The ball leaves the point located at distance d from the wall, at 45° to the horizontal with velocity u. How far from the wall does the ball hit the ground (Fig. 1.6)?

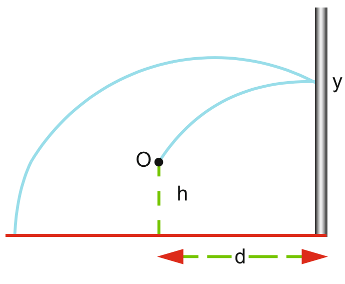


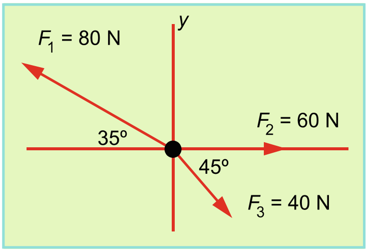
Fig. 1.6

1.2.4 Force and Torque

1.39 Three vector forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$ act on a particle of mass $m = 3.80$ kg as shown in Fig. 1.7:

(i) Calculate the magnitude and direction of the net force acting on the particle.
(ii) Calculate the particle's acceleration.
(iii) If an additional stabilizing force $\boldsymbol{F}_4$ is applied to create an equilibrium condition with a resultant net force of zero, what would be the magnitude and direction of $\boldsymbol{F}_4$?

Fig. 1.7



1.40 **(a)** A thin cylindrical wheel of radius $r = 40$ cm is allowed to spin on a frictionless axle. The wheel, which is initially at rest, has a tangential force applied at right angles to its radius of magnitude 50 N as shown in Fig. 1.8a. The wheel has a moment of inertia equal to 20 kg m^2.

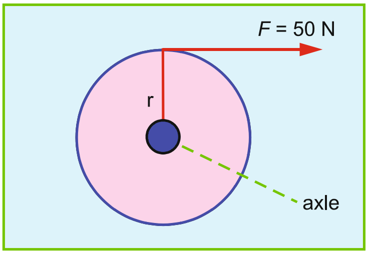


Fig. 1.8a

Calculate

(i) The torque applied to the wheel
(ii) The angular acceleration of the wheel
(iii) The angular velocity of the wheel after 3 s
(iv) The total angle swept out in this time

(b) The same wheel now has the same force applied but inclined at an angle of 20° to the tangent as shown in Fig. 1.8b. Calculate

(i) The torque applied to the wheel
(ii) The angular acceleration of the wheel

[University of Aberystwyth, Wales 2005]

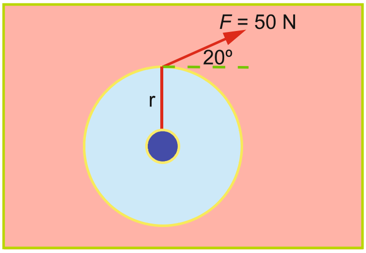


Fig. 1.8b

1.41 A container of mass 200 kg rests on the back of an open truck. If the truck accelerates at $1.5\,\text{m/s}^2$, what is the minimum coefficient of static friction between the container and the bed of the truck required to prevent the container from sliding off the back of the truck?

[University of Manchester 2007]

1.42 A wheel of radius r and weight W is to be raised over an obstacle of height h by a horizontal force F applied to the centre. Find the minimum value of F (Fig. 1.9).

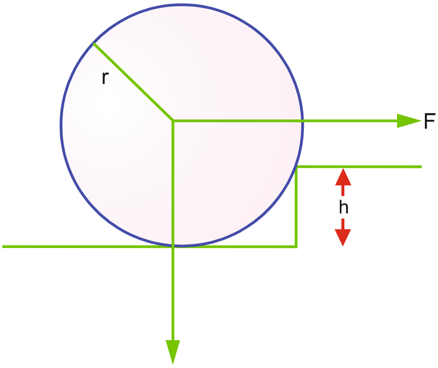


Fig. 1.9

1.2.5 Centre of Mass

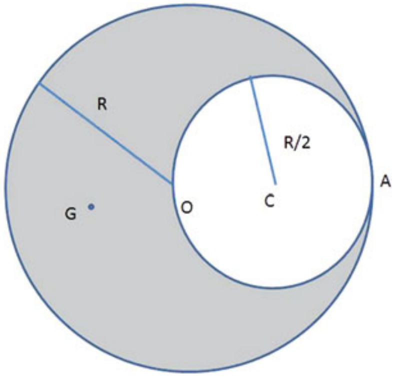
1.43 A thin uniform wire is bent into a semicircle of radius R. Locate the centre of mass from the diameter of the semicircle.

1.44 Find the centre of mass of a semicircular disc of radius R and of uniform density.

1.45 Locate the centre of mass of a uniform solid hemisphere of radius R from the centre of the base of the hemisphere along the axis of symmetry.

1.46 A thin circular disc of uniform density is of radius R. A circular hole of radius ½R is cut from the disc and touching the disc's circumference as in Fig. 1.10. Find the centre of mass.

Fig. 1.10



1.47 The mass of the earth is 81% the mass of the moon. The distance between the centres of the earth and the moon is 60 times the radius of earth $R = 6400\,\text{km}$. Find the centre of mass of the earth–moon system.

1.48 The distance between the centre of carbon and oxygen atoms in CO molecule is 1.13 Å. Locate the centre of mass of the molecule relative to the carbon atom.

1.49 The ammonia molecule NH_3 is in the form of a pyramid with the three H atoms at the corners of an equilateral triangle base and the N atom at the apex of the pyramid. The H–H distance = 1.014 Å and N–H distance = 1.628 Å. Locate the centre of mass of the NH_3 molecule relative to the N atom.

1.50 A boat of mass 100 kg and length 3 m is at rest in still water. A boy of mass 50 kg walks from the bow to the stern. Find the distance through which the boat moves.

1.51 At one end of the rod of length L, a body whose mass is twice that of the rod is attached. If the rod is to move with pure translation, at what fractional length from the loaded end should it be struck?

1.52 Find the centre of mass of a solid cone of height h.

1.53 Find the centre of mass of a wire in the form of an arc of a circle of radius R which subtends an angle 2α symmetrically at the centre of curvature.

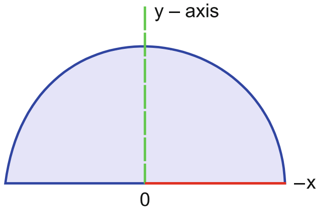
1.54 Five identical pigeons are flying together northward with speed v_0. One of the pigeons is shot dead by a hunter and the other four continue to fly with the same speed. Find the centre of mass speed of the rest of the pigeons which continue to fly with the same speed after the dead pigeon has hit the ground.

1.55 The linear density of a rod of length L is directly proportional to the distance from one end. Locate the centre of mass from the same end.

1.56 Particles of masses m, $2m$, $3m \ldots nm$ are collinear at distances L, $2L$, $3L \ldots nL$, respectively, from a fixed point. Locate the centre of mass from the fixed point.

1.57 A semicircular disc of radius R has density ρ which varies as $\rho = cr^2$, where r is the distance from the centre of the base and $c$is a constant. The centre of mass will lie along the y-axis for reasons of symmetry (Fig. 1.11). Locate the centre of mass from O, the centre of the base.

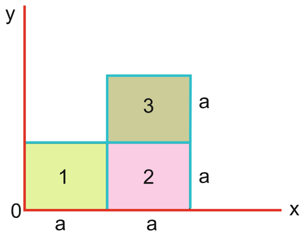
Fig. 1.11



1.58 Locate the centre of mass of a water molecule, given that the OH bond has length 1.77 Å and angle HOH is 105°.

1.59 Three uniform square laminas are placed as in Fig. 1.12. Each lamina measures 'a' on side and has mass m. Locate the CM of the combined structure.

Fig. 1.12



1.2.6 Equilibrium

1.60 Consider a particle of mass m moving in one dimension under a force with the potential $U(x) = k(2x^3 - 5x^2 + 4x)$, where the constant $k > 0$. Show that the point $x = 1$ corresponds to a stable equilibrium position of the particle.

[University of Manchester 2007]

1.61 Consider a particle of mass m moving in one dimension under a force with the potential $U(x) = k(x^2 - 4xl)$, where the constant $k > 0$. Show that the point $x = 2l$ corresponds to a stable equilibrium position of the particle.
Find the frequency of a small amplitude oscillation of the particle about the equilibrium position.

[University of Manchester 2006]

1.62 A cube rests on a rough horizontal plane. A tension parallel to the plane is applied by a thread attached to the upper surface. Show that the cube will slide or topple according to the coefficient of friction is less or greater than 0.5.

1.63 A ladder leaning against a smooth wall makes an angle α with the horizontal when in a position of limiting equilibrium. Show that the coefficient of friction between the ladder and the ground is $\frac{1}{2}\cot\alpha$.

1.3 Solutions

1.3.1 Motion in One Dimension

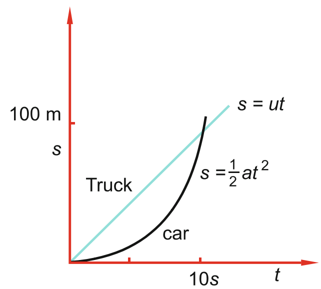
1.1 **(a)** Equation of motion for the truck: $s = ut$ (1)

Equation of motion for the car: $s = \dfrac{1}{2}at^2$ (2)

The graphs for (1) and (2) are shown in Fig. 1.13. Eliminating t between the two equations

$$s\left(1 - \frac{1}{2}\frac{as}{u^2}\right) = 0 \quad (3)$$

Fig. 1.13



Either $s = 0$ or $1 - \dfrac{1}{2}\dfrac{as}{u^2} = 0$. The first solution corresponds to the result that the truck overtakes the car at $s = 0$ and therefore at $t = 0$.

The second solution gives $s = \dfrac{2u^2}{a} = \dfrac{2 \times 10^2}{2} = 100\,\text{m}$

(b) $t = \dfrac{s}{u} = \dfrac{100}{10} = 10\,\text{s}$

(c) $v = at = 2 \times 10 = 20\,\text{m/s}$

1.2 When the stone reaches a height h above A

$$v_1^2 = u^2 - 2gh \tag{1}$$

and when it reaches a distance h below A

$$v_2^2 = u^2 + 2gh \tag{2}$$

since the velocity of the stone while crossing A on its return journey is again u vertically down.

Also, $v_2 = 2v_1$ (by problem) (3)

Combining (1), (2) and (3) $u^2 = \frac{10}{3}gh$ (4)

Maximum height

$$H = \frac{u^2}{2g} = \frac{10}{3}\frac{gh}{2g} = \frac{5h}{3}$$

1.3 Let the stones meet at a height s m from the earth after t s. Distance covered by the first stone

$$h - s = \frac{1}{2}gt^2 \tag{1}$$

where $h = 19.6$ m. For the second stone

$$s = ut = \frac{1}{2}gt^2 \tag{2}$$

$$v^2 = 0 = u^2 - 2gh$$

$$u = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 19.6} = 19.6\,\text{m/s} \tag{3}$$

Adding (1) and (2)

$$h = ut,\ t = \frac{h}{u} = \frac{19.6}{19.6} = 1\,\text{s}$$

From (2),

$$s = 19.6 \times 1 - \frac{1}{2} \times 9.8 \times 1^2 = 14.7\,\text{m}$$

1.4 $x = A \sin \pi t = A \sin \omega t$

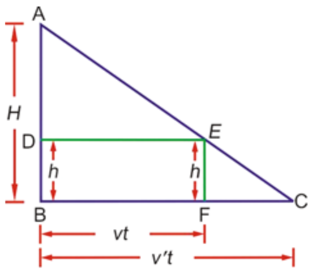
where ω is the angular velocity, $\omega = \pi$

Time period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\pi} = 2\,\text{s}$

In $\frac{1}{2}$s (a quarter of the cycle) the distance covered is A. Therefore in 3 s the distance covered will be $6A$.

1.5 Let the lamp be at A at height H from the ground, that is AB $= H$, Fig. 1.14. Let the man be initially at B, below the lamp, his height being equal to BD $= h$, so that the tip of his shadow is at B. Let the man walk from B to F in time t with speed v, the shadow will go up to C in the same time t with speed v':

Fig. 1.14



$\text{BF} = vt;\;\; \text{BC} = v't$

From similar triangles EFC and ABC

$$\frac{\text{FC}}{\text{BC}} = \frac{\text{EF}}{\text{AB}} = \frac{h}{H}$$

$$\frac{\text{FC}}{\text{BC}} = \frac{\text{EF}}{\text{AB}} = \frac{h}{H} \rightarrow \frac{v't - vt}{v't} = \frac{h}{H}$$

or

$$v' = \frac{Hv}{H-h} = \frac{6 \times 7}{(6-1.8)} = 10\,\text{m/s}$$

1.6 $\sqrt{3x} = 3t - 6$ (1)

Squaring and simplifying $x = 3t^2 - 12t + 12$ (2)

$$v = \frac{dx}{dt} = 6t - 12$$

$$v = 0 \text{ gives } t = 2\,\text{s} \tag{3}$$

Using (3) in (2) gives displacement $x = 0$

1.7 $$s = ut + \frac{1}{2}at^2 \tag{1}$$

$$\therefore \quad h = u \times 2 - \frac{1}{2}g \times 2^2 \tag{2}$$

$$h = u \times 10 - \frac{1}{2}g \times 10^2 \tag{3}$$

Solving (2) and (3) $h = 10g = 10 \times 9.8 = 98\,\text{m}$.

1.8 Take the origin at the position of A at $t = 0$. Let the car A overtake B in time t after travelling a distance s. In the same time t, B travels a distance $(s - 30)$ m:

$$s = ut + \frac{1}{2}at^2 \tag{1}$$

$$s = 13t + \frac{1}{2} \times 0.6\,t^2 \quad \text{(Car A)} \tag{2}$$

$$s - 30 = 20t - \frac{1}{2} \times 0.46\,t^2 \quad \text{(Car B)} \tag{3}$$

Eliminating s between (2) and (3), we find $t = 0.9$ s.

1.9 Let BD $= x$. Time t_1 for crossing the field along AD is

$$t_1 = \frac{\text{AD}}{v_1} = \frac{\sqrt{x^2 + (600)^2}}{1.0} \tag{1}$$

Time t_2 for walking on the road, a distance DC, is

$$t_2 = \frac{\text{DC}}{v_2} = \frac{800 - x}{2.0} \tag{2}$$

$$\text{Total time } t = t_1 + t_2 = \sqrt{x^2 + (600)^2} + \frac{800 - x}{2} \tag{3}$$

Minimum time is obtained by setting $dt/dx = 0$. This gives us $x = 346.4$ m. Thus the boy must head toward D on the round, which is 800–346.4 or 453.6 m away from the destination on the road.
The total time t is obtained by using $x = 346.4$ in (3). We find $t = 920$ s.

1.10 Time taken for the first drop to reach the floor is

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2.45}{9.8}} = \frac{1}{\sqrt{2}}\,\text{s}$$

As the time interval between the first and second drop is equal to that of the second and the third drop (drops dripping at regular intervals), time taken by the second drop is $t_2 = \frac{1}{2\sqrt{2}}$ s; therefore, distance travelled by the second drop is

$$S = \frac{1}{2}gt_2^2 = \frac{1}{2} \times 9.8 \times \left(\frac{1}{2\sqrt{2}}\right)^2 = 0.6125\,\text{m}$$

1.11 Height h = area under the $\upsilon - t$ graph. Area above the t-axis is taken positive and below the t-axis is taken negative. h = area of bigger triangle minus area of smaller triangle.
Now the area of a triangle = base × altitude

$$h = \frac{1}{2} \times 3 \times 30 - \frac{1}{2} \times 1 \times 10 = 40\,\text{m}$$

1.12 **(a)** Time for the ball to reach water $t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 4.9}{9.8}} = 1.0\,\text{s}$
Velocity of the ball acquired at that instant $\upsilon = gt_1 = 9.8 \times 1.0 = 9.8\,\text{m/s}$.
Time taken to reach the bottom of the lake from the water surface

$$t_2 = 5.0 - 1.0 = 4.0\,\text{s}.$$

As the velocity of the ball in water is constant, depth of the lake,

$$d = \upsilon t_2 = 9.8 \times 4 = 39.2\,\text{m}.$$

(b) $< \upsilon > = \frac{\text{total displacement}}{\text{total time}} = \frac{4.9 + 39.2}{5.0} = 8.82\,\text{m/s}$

1.13 For the first stone time $t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 44.1}{9.8}} = 3.0\,\text{s}$.
Second stone takes $t_2 = 3.0 - 1.0 = 2.0$ s to strike the water

$$h = ut_2 + \frac{1}{2}gt_2^2$$

Using $h = 44.1$ m, $t_2 = 2.0$ s and $g = 9.8\,\text{m/s}^2$, we find $u = 12.25$ m/s

1.14 Transit time for the single journey = 0.5 s.
When the ball moves up, let υ_0 be its velocity at the bottom of the window, υ_1 at the top of the window and $\upsilon_2 = 0$ at height h above the top of the window (Fig. 1.15)