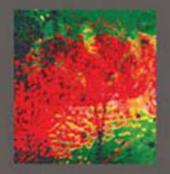
Applied Quantitative Finance



Discounting, Libor, CVA and Funding

Interest Rate and Credit Pricing

Chris Kenyon Roland Stamm

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To Anabel and Leon To Jeanne

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Preface

NYM [Sheathing his sword] I shall have my noble? PISTOL In cash most justly paid. NYM [Shaking hands] Well, then, that's the humour of't. Shakespeare, *Henry V*

One of the most basic yet also most important questions in the financial world is the worth of one currency unit in the future versus one unit now. The answer to this question provides the *discount curve* allowing payments at all future times to be compared and brought back to the present. Obviously, the answer will depend on the credit quality or riskiness of the person or company that promises you to pay you the EUR 1; you might trust a family member more to make good on the promise than a total stranger, or you trust your government more than your bank, and trust a car salesperson even less. Or maybe not.

Ideological debates aside, financial instruments traded on an exchange or through clearing houses are the least risky – from a credit perspective. This is because all participants have to post cash collateral to a margin account, and have to replenish this account on a daily basis if their position's value moves against them. The curve that is used to discount the outstanding future payments of such collateralized trades could therefore be considered a risk-free curve. The definition of a risk-free curve is important because it will be the benchmark against which the credit riskiness of future cash flows will be measured.

London, 17 June 2010

LCH.Clearnet Ltd (LCH.Clearnet), which operates the world's leading interest rate swap (IRS) clearing service, SwapClear, is to begin using the overnight index swap (OIS) rate curves to discount its \$218 trillion IRS portfolio.

This quote demonstrates a fundamental change to the discount curve by a major clearing house for a vast quantity of derivatives, \$0.2 *quadrillion*. Just as an example, as of November 29, 2010, the discount factor from an EONIA-based curve for December 01, 2020, was 2.75% as opposed to 3.104% from the standard 6 month EURIBOR curve. That's 35 basis points difference (p.a.!). Note that one basis point on a notional of \$218 trillion is \$21.8 billion.

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How is it possible to disagree about such a fundamental building block in finance as the discount curve? What is this curve? What was the market using before? Will it change its mind again? Does this mean that all banks use this (and only this) curve for all purposes? We shall see that the answers to these questions usher in a new beginning for mathematical finance and the pricing of interest rate and credit derivatives.

The no-arbitrage principle ("there is no such thing as a free lunch") was and still is one of the foundations of financial pricing models. This principle is accompanied by two fundamental theorems:

- 1. A market is arbitrage free if and only if there is an equivalent martingale measure.
- 2. A market is complete if and only if this equivalent martingale measure is unique.

The famous Black–Scholes–Merton model is a simple result of this; so simple, in fact, that there were several major changes since its inception in 1973:

- 1. **emergence of the volatility smile** in 1987 with the Wall Street Crash;
- 2. **implicit disregard for no-arbitrage** with the widespread adoption of stochastic volatility models to fit observed volatility smiles around 1997–2002 (these models are incomplete-market models; e.g., typically the volatility of the volatility cannot be hedged with liquid instruments);
- 3. **complete abandonment of no-arbitrage** in the structured credit markets with the adoption of copula-based Collateraized Debt Obligation (CDO) pricing around 2001. Also compounded by using CDO pricing models depending on unobservable parameters with **no** tradable market quotes at all, e.g. recovery rates and correlations. This is even before considering stochastic or dynamic models.
- 4. **liquidity and credit become significant** this is the current state, where LIBOR fixings can no longer be predicted from discount curves, and the market is changing from quotes-on-spreads to quote-by-price (e.g. the ISDA (International Swaps and Derivatives Association) CDS Big Bang in April 2009) for many instruments because institutions cannot agree on which discount curve to use.
- 5. **Basel III** a direct consequence of the previous point, regulation now has a significant impact on trading starting even before its implementation date of 1st January, 2013.

We mention one common misunderstanding at this point. The fact that hedging produces unique prices, even under incomplete markets, makes some people think that incompleteness is only a minor distraction. The hedging statement is true but applies only to *attainable* claims (instruments that can be priced by hedging). The problem is that even very simple products, such as European options, may be un-attainable (see chapter 11 of [Shreve (2004)] for a nice exposition and [Cont and Tankov (2003)] for more depth). No-arbitrage then only gives bounds on prices, and these can be too wide to be useful. Although this book is not about incomplete markets, the fact that real markets are generally incomplete is a significant practical limitation.

At the extremes, many of the Black–Scholes–Merton assumptions have always been broken – but before 2007 the general view was that this could be safely ignored. The financial and sovereign debt crises of 2007–2012 did away with the assumption that market players could borrow and lend freely at the riskless rate. Nowadays, the question of where to get the funding from and whether one has to post collateral in order to get it is paramount. The new mantra therefore is "funding, funding, funding".

This text aims to rebuild interest rate product pricing and credit product pricing for the new world in which bankers, and everyone else, now live. Funding and default risk are no longer quiet niches but central to pricing. We shall also see that there is no longer a Law of One Price: depending on what it costs an institution to borrow, or which currency it runs its business in, it will see prices that are different from its peers'. Collateralization, once considered the end of all pricing arguments, has been broken by the simple question "but where do you get the collateral from?" This has been reinforced by the US Dodd–Frank Wall Street Reform and Consumer Protection Act of 2010 which even discourages rehypothecation, i.e. using received collateral from one party as collateral for a second party. We provide the state of the art, in practice, from practicing bankers and theoreticians who have to answer these questions every day.

Readers should be aware that Mathematical Finance in the post-crisis world has changed at every level of complexity. For yield curve building (now discount and LIBOR curves) the complexity is limited as the techniques are relatively straightforward (at least in common cases). When we start to price with the new curves we immediately get into measure changes and FX analogies. When calculating the credit value adjustment (CVA), we find that now even linear instruments, like vanilla

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swaps, acquire option-like characteristics (a swap with an uncollateralized counterparty is a sold option on the right to default). Taking one's own credit riskiness into account by calculating the debit value adjustment (DVA) as well, one allows both parties to have options. One redeeming feature of CVA and DVA is that default is not exercised like an American option, i.e. intelligently or economically, to maximise the value of individual derivatives, but simply as a contingent event. Then we see that careful consideration of closeout and funding means that pricing becomes recursive. If all this were not enough, when we look at the firm-level we see effects that involve the whole balance sheet.

Because of the above, the required level of mathematical background varies in this text. For the first six chapters, a good basic knowledge of mathematics is sufficient; the few differential equations shining up here and there can be safely ignored. Chapters 7 through 11, however, require a thorough understanding of interest rate models such as the Hull–White model and LIBOR Market Models as described in [Brigo and Mercurio (2006)], and the mathematics underlying them, at least at the level of [Shreve (2004)].

Here is a brief outline of the book. Chapter 1 is a reminder of the basic terminology and details of the interest rate markets, as well as the "old school" approach of pricing simple interest rate products. Chapter 2 explains how to build a zero/discount curve from market data.

After the introduction to credit instruments and associated terminology, Chapter 3 presents the most relevant definitions of credit spreads and the instruments associated with them, their market risk, and the connections between them.

Leaving the credit spreads behind, Chapter 4 introduces the problems with the old way of pricing that arose with the credit crisis in 2008. The basis spreads, formerly almost irrelevant phenomena in the interest rate markets, became a new source of market risk as well as pricing complexity. Their impact on curve building and pricing is investigated in Chapter 5, which describes what happens in a one-currency world, and Chapter 6 which makes the jump to foreign currencies. To close the topic of basis spreads, Chapter 7 presents what needs to be done for non-linear products, i.e. products that contain optional components.

The next two chapters are concerned with CVA, and its cousins DVA and funding value adjustment (FVA). While Chapter 8 gives an overview of the current state of research in this area, Chapter 9 presents strong evidence that the common view of DVA giving rise to profits if one's own credit quality deteriorates is false when the bank's balance sheet as a whole is considered. Firm-level effects may appear to be only

paper or accounting realities, but they have real effects. If your balance sheet says that your equity is below its regulatory threshold then there are firm-level consequences. Furthermore, regulators decided to introduce the CVA volatility capital charge as a direct consequence of CVA losses in the 2008 crisis. The fact that trading realities are not accounting realities creates tensions for trading desks.

Moving on to Basel III, Chapter 10, the cry is not "funding, funding, funding", but "capital, capital, capital". Basel III brings in a host of innovations, the first of which to hit significantly is the capital charge for CVA volatility. Note that Basel III does not require the pricing in of CVA (or DVA or FVA), but *changes* in CVA. The choice facing traders is now whether to pay a capital charge for their position or buy credit protection (e.g. via not very liquid contingent credit default swaps, or CCDS). This is already reshaping the trading business of banks *as it was intended to*. Quants cannot ignore the effect, whether or not they only see it as a change in their funding costs.

Finally, Chapter 11 is on backtesting of risk factor equations (RFEs). Capital charges for trading desks are calculated from counterparty exposure profiles per legally enforceable netting set. Basel III insists that banks may only use one exposure profile for both default charge and the CVA volatility charge. These, traditionally, *have been* related to fundamentally different concepts: historical default versus marketimplied credit valuation adjustment (which *must* be calculated using market CDS spreads). Exposure profiles depend on the future development of underlying risk factors (e.g. discount curves, market-implied swaption volatilities, etc.). Basel III explicitly permits calibration of RFEs to either historical data, or market-implied. However these RFEs are calibrated they must pass *historical* backtesting. We present a Basel III inspired framework for this historical backtesting that can be applied however RFEs are calibrated, paying special attention to the case where all available models fail.

A lot of the material in this book is not new. However, there are a few exceptions: details of hedge accounting with respect to multiple curves, in Chapter 6; the derivation of a two-factor short rate model in a discount + spread curve environment within Chapter 7; most of the results for CVA at the firm level in Chapter 9; and the backtesting framework in Chapter 11.

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of the German government for more than \in 100 billion of funding: to the bank – not the authors.

London, April 2012 Mainz, April 2012 Dr Chris Kenyon Dr Roland Stamm

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- WBS Training: The CVA Conference: Implementation, Trading, Liquidity, Modelling & Funding, March 2012, London

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1

Back to the Basics

SHYLOCK Three thousand ducats, well. BASSANIO Ay, sir, for three months. SHYLOCK For three months, well. Shakespeare, *The Merchant of Venice*.

This chapter is meant to refresh the reader's memory about the precrisis valuation standards. For an excellent introduction to the topic, see, for instance, [Hull (2009)]. As this text is meant to be used in practice, a lot of emphasis is put on the nitty-gritty details that are often ignored in other books.

1.1 Interest rates

1.1.1 LIBOR

Interest is the price someone has to pay for borrowing money. The interest usually depends on the maturity of the loan as well as the creditworthiness of the borrower. The latter is influenced by the borrower's (projected) liquidity situation and her available assets. Creditworthiness is improved if collateral is posted to the lender. This collateral can be a lien in the case of a mortgage, bonds or shares, commodities like precious metals, or even cash.

One particularly important market in the financial world is the money market, where banks lend each other cash or borrow it from the central bank. Maturities in the money market are typically up to one year. Although the rate at which two banks enter into a loan is always negotiated individually and changes arbitrarily over time, there is the concept of an average rate which is fixed once on a daily basis. This Interbank Offer Rate is called EURIBOR in the EUR market, LIBOR for all currencies that are traded on the money market in London, and by some similar name for local rates, like STIBOR for the SEK rate fixed