Expertise in Mathematics Instruction
Expertise in Mathematics Instruction

An International Perspective
To our families –
Aimee, Andrew, Melinda, and Jianrong
&
Sarah and Thomas
– for their love and support
Contents

Part I  Introduction and Research Perspectives
Expertise in Mathematics Instruction: Advancing Research and Practice from an International Perspective  ........................................................................ 3
Yeping Li and Gabriele Kaiser

Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise  ........................................................................ 17
Michelene T. H. Chi

Images of Expertise in Mathematics Teaching  ........................................................................ 41
Rosemary S. Russ, Bruce Sherin, and Miriam Gamoran Sherin

Part II  Expertise in Mathematics Instruction in a Western Setting
Coordinating Characterizations of High Quality Mathematics Teaching: Probing the Intersection  ........................................................................ 63
Edward A. Silver and Vilma Mesa

Expertise in Swiss Mathematics Instruction  ........................................................................ 85
Christine Pauli and Kurt Reusser

Responding to Students: Enabling a Significant Role for Students in the Class Discourse  ........................................................................ 109
Ruhama Even and Orly Gottlib

Effects of a Research-Based Learning Approach in Teacher Professional Development  ........................................................................ 131
Florian H. Müller, Irina Andreitz, Konrad Krainer, and Johannes Mayr

Teacher Expertise Explored as Mathematics for Teaching  ........................................................................ 151
Elaine Simmt
Part III  Expertise in Mathematics Instruction in an Eastern Setting

Yeping Li, Rongjin Huang, and Yudong Yang

The Japanese Approach to Developing Expertise in Using the Textbook to Teach Mathematics .................................................. 197
Akihiko Takahashi

Perceptions of School Mathematics Department Heads on Effective Practices for Learning Mathematics ................................. 221
Suat Khoh Lim-Teo, Kwee Gek Chua, and Joseph Kai Kow Yeo

Exploring Korean Teacher Classroom Expertise in Sociomathematical Norms .......................................................... 243
JeongSuk Pang

Expertise of Mathematics Teaching Valued in Taiwanese Classrooms .......................................................... 263
Pi-Jen Lin and Yeping Li

Part IV  Cross-National Comparison and Reflections

Cross-Nationally Comparative Results on Teachers’ Qualification, Beliefs, and Practices .................................................. 295
Svenja Vieluf and Eckhard Klieme

Reflections on Teacher Expertise ............................................................. 327
Alan H. Schoenfeld

Reflections and Future Prospects ............................................................. 343
Gabriele Kaiser and Yeping Li

Index .................................................................................................. 355
Contributors

Irina Andreitz  Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, irina.andreitz@uni-klu.ac.at

Michelene T. H. Chi  Department of Psychology, Arizona State University, Tempe, AZ, USA, michelene.chi@asu.edu

Kwee Gek Chua  National Institute of Education, Singapore, Republic of Singapore, kweegek.chua@nie.edu.sg

Ruhama Even  Department of Science Teaching, Weizmann Institute of Science, Rehovot, Israel, ruhama.even@weizmann.ac.il

Orly Gottlib  Orot Israel College, Rehovot, Israel, paulog@netvision.net.il

Rongjin Huang  Department of Teaching, Learning and Culture, College of Education and Human Development, Texas A&M University, College Station, TX, USA, hrj318@gmail.com

Gabriele Kaiser  Faculty of Education, Psychology, Human Movement, Didactics of Mathematics, University of Hamburg, Hamburg, Germany, gabriele.kaiser@uni-hamburg.de

Eckhard Klieme  German Institute for International Educational Research, Frankfurt, Germany, klieme@dipf.de

Konrad Krainer  Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, konrad.krainer@uni-klu.ac.at

Yeping Li  Department of Teaching, Learning and Culture, College of Education and Human Development, Texas A&M University, College Station, TX, USA, yepingli@tamu.edu

Suat Khoh Lim-Teo  National Institute of Education, Singapore, Republic of Singapore, suatkhoh.teo@nie.edu.sg

Pi-Jen Lin  Graduate Institute of Mathematics and Science Education, National Hsinchu University of Education, Hsinchu, Taiwan, linpj@mail.nhcue.edu.tw
Contributors

**Johannes Mayr**  Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, johannes.mayr@uni-klu.ac.at

**Vilma Mesa**  School of Education, University of Michigan, Ann Arbor, MI, USA, vmesa@umich.edu

**Florian H. Müller**  Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, florian.mueller@uni-klu.ac.at

**JeongSuk Pang**  Department of Elementary Education (Mathematics Education), Korea National University of Education, Cheongwon-gun, Chungbuk, South Korea, jeongsuk@knue.ac.kr

**Christine Pauli**  Institute of Education, University of Zurich, Zurich, Switzerland, cpauli@ife.uzh.ch

**Kurt Reusser**  Institute of Education, University of Zurich, Zurich, Switzerland, reusser@ife.uzh.ch

**Rosemary S. Russ**  School of Education and Social Policy, Northwestern University, Evanston, IL, USA, r-russ@northwestern.edu

**Alan H. Schoenfeld**  School of Education EMST, University of California, Berkeley, CA, USA, alans@berkeley.edu

**Bruce Sherin**  School of Education and Social Policy, Northwestern University, Evanston, IL, USA, bsherin@northwestern.edu

**Miriam Gamoran Sherin**  School of Education and Social Policy, Northwestern University, Evanston, IL, USA, msherin@northwestern.edu

**Edward A. Silver**  School of Education, University of Michigan, Ann Arbor, MI 48109, easilver@umich.edu

**Elaine Simmt**  University of Alberta, Edmonton, AB, Canada, esimmt@ualberta.ca

**Akihiko Takahashi**  School of Education, DePaul University, Chicago, IL, USA, atakahas@depaul.edu

**Svenja Vieluf**  German Institute for International Educational Research, Frankfurt, Germany, vieluf@dipf.de

**Yudong Yang**  Center for Teachers’ Development, Shanghai Academy of Educational Sciences, Shanghai, China, mathedu@163.com

**Joseph Kai Kow Yeo**  National Institute of Education, Singapore, Republic of Singapore, kaikow.yeo@nie.edu.sg
About the Authors

Irina Andreitz earned a master degree in Psychology. Since 2007 she has worked as a researcher at the Institute of Instructional and School Development at the University of Klagenfurt. Her main research areas include learning motivation, teachers’ self-determined motivation, evaluation of teacher professional development and educational development programs.

Michelene Chi is a cognitive science researcher interested in issues of learning. Her research has focused on ways of optimizing students’ learning, especially in domains of sciences. Her early research investigated children’s competence as a function of knowledge and differences between novices and experts. More recently, she has focused on the origin of misconceptions, and explored approaches to teaching emergent, robustly misconceived processes. She has also published many articles on how students learn from generating self-explanations, from being tutored, from collaborating, and from observing and overhearing tutorial dialogues. Recently she introduced a framework that can differentiate students’ learning activities as active, constructive or interactive. Two of her papers have been ranked #1 and #7 most highly cited articles published by the journal *Cognitive Science*. Michelene Chi is currently a Professor in the Department of Psychology at Arizona State University, Tempe, AZ.

Kwee Gek Chua is currently a Lecturer at Mathematics and Mathematics Education Academic Group of the National Institute of Education (NIE), Singapore. She is a mathematics teacher educator involved in the training of pre-service secondary and primary mathematics teachers. Prior to joining NIE in 2009, she was in the teaching service of Ministry of Education (MOE) Singapore for 35 years, where she served in various capacities: as a primary and secondary school mathematics teacher, a mathematics head of department, a Research and Evaluation officer with MOE, and a teaching fellow at NIE. Her research interests include the impact of teachers’ behavior on students’ achievement and attitude, mathematical pedagogical content knowledge of pre-service and practicing teachers and their professional development.

Ruhama Even is a Professor at the Weizmann Institute of Science and the Rudy Bruner Chair of Science Teaching. She heads the Mathematics Group in the
Department of Science Teaching and directs the Teaching Certification program at Weizmann’s Feinberg Graduate School. Her main research interests include education and professional development for mathematics teachers and teacher educators, and the interactions among mathematics curriculum, teachers, and classrooms. She has been a member of the International Committee of the International Group of Psychology of Mathematics Education (PME) and co-chair of ICMI Study 15 on the professional education and development of teachers of mathematics. Ruhama Even earned her Ph.D. in Mathematics Education from Michigan State University in 1989.

Orly Gottlib is a Lecturer on Mathematics and Pedagogy at the Orot Israel College and PhD student at the Kreitman School of Advanced Graduate Studies, Ben Gurion University. Her research focuses on teachers’ responsiveness to utterances of students in mathematics lessons.

Rongjin Huang is a Mathematics Educator who had been working as a secondary mathematics teacher and university faculty in China for many years. He is currently continuing his further study at Texas A&M University. He is a team member of the Learner’s Perspective Study which is a well-known international classroom research project. His interests include mathematics classroom research with a focus on Chinese pedagogy of mathematics, and mathematics teacher education. He has completed several research projects funded by The University of Hong Kong and University of Macau and his work has been extensively published in articles and book chapters internationally. He has also published several books. He organized and chaired several topic study sessions at various regional and international professional conferences such as EARCOME 3 & 4, and ICME-11.

Gabriele Kaiser is a full Professor in Mathematics Education at the Faculty of Education of the University of Hamburg since 1998. She holds a master’s degree as a teacher for mathematics and humanities, which she completed at the University of Kassel in 1978. She has taught in school from 1979–2000. She completed her doctorate in mathematics education in 1986 on applications and modelling and her post-doctoral study (so-called “Habilitation”) in pedagogy on international comparative studies in 1997, both at the University of Kassel. Her post-doctoral study was supported by a grant of the German Research Society (DFG). From 1996 to 1998 Gabriele Kaiser worked as a guest professor at the University of Potsdam.

Her areas of research include modelling and applications in school, international comparative studies, gender and cultural aspects in mathematics education and empirical research on teacher education. She has received grants from the German Research Society (DFG) in order to support this research. At present she is Editor-in-Chief of the journal ZDM – The International Journal on Mathematics Education (formerly Zentralblatt fuer Didaktik der Mathematik), published by Springer and Editor of a monograph series “Advances in Mathematics Education” published by Springer too. Since July 2007 she has served as president of the International Study Group for Mathematical Modelling and Applications (ICTMA), an ICMI affiliated Study Group.
**Eckhard Klieme** is Full Professor of Educational Science at Goethe University and Director of the Center for Research on Educational Quality and Evaluation at the German Institute for International Educational Research (DIPF) in Frankfurt am Main (since 2001). His research areas include educational effectiveness and educational assessment. Eckhard Klieme received a Diploma in Mathematics and a Doctorate in Psychology from The University of Bonn, Germany. He has been the principal investigator for research projects on mathematics and science instruction (including TIMSS-Video/Germany, the Swiss-German “Pythagoras Study”, and ongoing studies on formative assessment) as well as for nation-wide school evaluation studies in Germany (citizenship education, language education, after school programs). He is also involved in international studies such as TALIS and PISA, where he currently serves as chair of the International Questionnaire Expert Group.

**Konrad Krainer** is Full Professor at the University of Klagenfurt. He started his professional career as a mathematics teacher and research assistant. Konrad Krainer took leadership in several international scientific boards, including the European Society for Research in Mathematics Education and the Education Committee of the European Mathematical Society. He was Associate Editor of the *Journal of Mathematics Teacher Education* (1998–2009) and is Co-Editor of the *International Handbook of Mathematics Teacher Education* (2008). He leads the Institute of Instructional and School Development and the Austrian National Initiative IMST (Innovations in Mathematics, Science and Technology Teaching, 2000–2012). His research interests are mathematics teacher education, school development and educational system development.

**Yeping Li** is Full Professor of Mathematics Education at Texas A&M University. He is interested in examining issues related to mathematics curriculum and teacher education in various education systems, and understanding how factors related to mathematics curriculum and teachers may come together in shaping effective classroom instruction that is valued in different cultures. His research projects were supported by various funding agencies, such as the Spencer Foundation. His work has been published extensively in many journals internationally. He has been serving as an Associate Editor for the *Journal of School Science and Mathematics*, and a guest editor for other journals including the *International Journal of Educational Research* and *ZDM – The International Journal on Mathematics Education*. He organized and chaired many group sessions at various prominent national and international professional conferences, such as ICME-10 in 2004 and ICME-11 in 2008. He received his Ph.D. in cognitive studies in education from the University of Pittsburgh, USA.

**Suat Khoh Lim-Teo** is an Associate Professor and the current Dean/Academic at the National Institute of Education, Singapore. She was formerly the Head of the Mathematics and Mathematics Education Academic Group (2000–2004) and an Associate Dean in charge of pre-service programs (2003–2006). She has been a mathematics teacher educator for more than 20 years, teaching various methods and content courses in pre-service as well as in-service programs. She has also been the
President of the Association of Mathematics Educators, Singapore (1998–2000), a Vice-President of the Singapore Mathematical Society (2001–2004) and a member of the International Program Committee for ICME10 (Denmark, 2004). While her earlier research interests were in various aspects of mathematics learning, her recent focus is in the area mathematics teacher education, particularly in the development of mathematics pedagogical content knowledge of primary school teachers.

Pi-Jen Lin is a Professor at the National Hsinchu University of Education, Taiwan. Her research focuses on mathematics instruction, pre-service and in-service teachers’ professional development. Aside from this, she has taken the responsibility of the international studies such as TIMSS2003, TIMSS2007, TIMSS2011 at the fourth grade level and TEDS-M2008 for primary level in Taiwan. Lin has authored or co-authored over 100 publications in national or international journals or book chapters and has lectures and presentations in national or international conferences or institutes. She has served on several national commissions including curriculum advisory committees at national and local levels. She is the editor of several national journals such as the Chinese Journal of Science Education. Most recently, she served as a member of the International Committee of the International Group of Psychology of Mathematics Education (PME30–PME33).

Johannes Mayr is Full Professor at the University of Klagenfurt. He started his professional career as an elementary school teacher, later on he worked as a teacher educator at the Teacher Training College of the Diocese of Linz, Austria (1975–2005). He is co-ordinator of the web-based program Career Counselling for Teachers (CCT). His main research areas are teacher education, school development, classroom climate, and classroom management.

Vilma Mesa is Assistant Professor of Education at the University of Michigan. She investigates the role that resources play in developing teaching expertise in undergraduate mathematics. Currently she is investigating the nature of mathematics teaching in community colleges, attending to how instruction is organized, delivered, and sustained. She has conducted several analyses of textbooks and has been involved in evaluation projects that study the impact of innovative teaching practices in mathematics for students in science, technology, engineering, and mathematics fields. She has a B.S. in computer sciences and a B.S. in mathematics from the University of Los Andes in Bogotá, Colombia, and a master’s and a Ph.D. in mathematics education from the University of Georgia.

Florian Müller earned a master degree in Educational Studies and Educational Psychology at the University of in Munch, Germany and a Ph.D. in Education. He worked as a lecturer and researcher from 1995 to 2006 at the University of Bundeswehr in Munich and in 2004 as a guest researcher at the University of Cape Town (South Africa), Institute of Psychology. Since 2006 he has been a member of the Faculty of Interdisciplinary Studies of the University of Klagenfurt; and Associate Professor since 2009. He is associate editor of the Journal of Educational Research – online. His main research lies in the following areas: self-determined student and teacher motivation, interest, teacher professional development, and
teaching and learning in higher education. His main practical activities are basic and advanced teacher training and further education in academic instruction.

**JeongSuk Pang** is an Associate Professor of mathematics education at Korea National University of Education (KNUE), South Korea. While her publications have been diverse covering various aspects of elementary mathematics education, she is particularly interested in the analysis of the mathematics classroom culture from a socio-cultural perspective and professional development both for pre-service and in-service teachers. With a fund from Korea Research Foundation, she recently developed case-based pedagogy for teachers to learn how to teach mathematics. In 2009, in addition to receiving the best professor award at KNUE, Pang was also awarded first place in a competition to develop “classroom-friendly” instructional methods to educate pre-service teachers. She has been actively involved in developing new mathematics curriculum and its concomitant textbook series. She has been serving as an international director of the Korea Society of Educational Studies in Mathematics and is a member of the International Committee for PME (2008–2012).

**Christine Pauli** studied education, psychology and philosophy at the University of Berne, Switzerland, after teacher training and 10 years of professional experience as an elementary school teacher. She received a Ph.D. in Educational Psychology from the University of Zurich, Switzerland. She is now a Senior Researcher at the Institute of Education, University of Zurich. Her main research interests are video-based studies on teaching and learning, classroom discourse and teacher education.

**Kurt Reusser** is Professor of Educational Psychology and Education at the University of Zurich (since 1993). After training and serving as an elementary school teacher (1966–1972), he received his M.A. (1977) and Ph.D. (1984) from the University of Berne (Hans Aebli). After 6 years educating teacher candidates at a teacher training college, he was a postdoctoral fellow (1984–1986) at the University of Colorado, Boulder, working with Walter Kintsch. He received his post-doctoral Habilitation in Educational Psychology in 1990. His research and publications have focused on cognitive and instructional psychology, problem solving, the work of Jean Piaget, general didactics, teacher learning and teacher education and video-based research on mathematics teaching and learning (principal investigator of a series of video studies including the TIMSS 1999 video study).

**Rosemary S. Russ** is a Research Assistant Professor of Learning Sciences in the School of Education and Social Policy at Northwestern University. Her research involves examining K-16 science and mathematics learning, as evidenced in classrooms and clinical interviews, to understand student and teacher epistemology and cognition. In addition, she is interested in understanding clinical interviews as discourse interactions. She draws on methodological traditions from qualitative education research, cognitive science, and conversation analysis. She has coauthored several book chapters on student and teacher cognition and recent articles appear in *Science Education* and *Issues in Teacher Education*. Russ is a 2010 National Academy of Education/Spencer Postdoctoral Fellowship finalist.
Alan Schoenfeld is the Elizabeth and Edward Conner Professor of Education and Affiliated Professor of Mathematics at the University of California at Berkeley. He is a Fellow of the American Association for the Advancement of Science and an inaugural Fellow of the American Educational Research Association, and a Laureate of the education honor society Kappa Delta Pi. He has served as President of the American Educational Research Association and as the vice president of the National Academy of Education. In 2008 he was given the Senior Scholar Award by AERA’s Special Interest Group for Research in Mathematics Education.

After obtaining his Ph.D. in mathematics from Stanford in 1973, Schoenfeld turned his attention to issues of mathematical thinking, teaching, and learning. His work has focused on problem solving (what makes people good problem solvers, and how can people get better at it?), assessment, teachers’ decision-making, and issues of equity and diversity. His most recent book, How We Think, provides detailed models of human decision making in complex situations such as teaching.

Bruce Sherin is an Associate Professor of Learning Sciences in the School of Education and Social Policy at Northwestern University. His research focuses primarily on conceptual change in science, particularly as it occurs within novel technology-based learning environments. In recent research, he has begun to investigate the use of techniques from computational linguistics for the analysis of natural language data. In addition, with Miriam Sherin, he is exploring the use of novel technologies for the study of teacher cognition. He currently serves on the editorial boards of the journals Cognition and Instruction and the Journal of the Learning Sciences. In addition, he is an executive editor of the International Journal of Computers for Mathematical Learning.

Miriam Gamoran Sherin is an Associate Professor of Learning Sciences in the School of Education and Social Policy at Northwestern University. Her interests include mathematics teaching and learning, teacher cognition, and the use of video for teacher learning. Sherin’s forthcoming book Mathematics teacher noticing: Seeing through teachers’ eyes will be published in 2011 by Routledge. Recent articles appear in Journal of Teacher Education, Teaching and Teacher Education, and Journal of Mathematics Teacher Education. In 2001, Sherin received a postdoctoral fellowship from the National Academy of Education and in 2002 she was awarded a Career Grant from the National Science Foundation. In April, 2003, Sherin received the Kappa Delta Pi/American Educational Research Association Division K Award for early career achievements in research on teaching and teacher education.

Edward A. Silver is the William A. Brownell Collegiate Professor of Education and Professor of Mathematics at the University of Michigan. His scholarly interests include the study of mathematical thinking, especially mathematical problem solving and problem posing; the design and analysis of intellectually engaging and equitable mathematics instruction for students; innovative methods of assessing and reporting mathematics achievement; and effective models for enhancing the knowledge of teachers of mathematics. He served as editor of the Journal for Research in Mathematics Education from 2000–2004, and he is currently serving as co-editor of
About the Authors xvii

The Elementary School Journal. He received his doctorate in mathematics education from Teachers College, Columbia University.

**Elaine Simmt** is Professor of Secondary Mathematics Education at the University of Alberta, Canada. She began her career as a secondary school teacher of mathematics and physical science. She completed doctoral studies in mathematics education under the supervision of Tom E. Kieren. Simmt’s research explores teaching and learning as understood through enactivism and the frame of complexity theory. Most recently that work has led to her to the study of mathematics for teaching. In that work she and Dr. Brent Davis (University of Calgary) applied organization principles from complex adaptive systems in their study of the emergent learning systems found in mathematics classrooms and professional development sessions. Site-based research has triggered Simmt’s interest in research methods and designs suitable for studying the complex and contextual phenomena of teaching and learning.

**Akihiko Takahashi** is currently the Director of the Asia-Pacific Mathematics and Science Education Collaborative and an Associate Professor of mathematics education at DePaul University in the United States. He teaches mathematics teaching and learning, and mathematics for prospective teachers. He also provides workshops and seminars for practicing teachers using ideas from the U.S. and Asian countries. He was an elementary teacher in Japan before becoming an educator of mathematics teachers. During his elementary teaching career, he was nationally active in mathematics lesson study and mentored 200 pre-service teachers. He received his Ph.D. from the University of Illinois at Urbana-Champaign; his dissertation research focused on internet use in mathematics education.

**Svenja Vieluf** received a Diploma in Psychology from University of Mainz, Germany. She is currently a research assistant at the Center for Research on Educational Quality and Evaluation at the German Institute for International Educational Research (DIPF) in Frankfurt am Main Her research interests include international comparative educational research, instructional quality, cross-cultural research methods, and migration and education. She was involved in the analysis and reporting of the OECD-Teaching and Learning International Survey and is now affiliated with the international consortium for PISA 2012.

**Yudong Yang** is currently Associate Professor and deputy director of the Center for Teachers’ Development in Shanghai Academy of Educational Sciences since 2006, and Deputy Secretariat of Shanghai Committee of School Mathematics Teaching since 2008. His major research interests are in mathematics teacher education, especially in-service teacher’s professional learning in school-based teaching environment and he is also a founding member of the WALS (World Association of Lesson Study). In his recent research project sponsored by the Ministry of Education of China, he focused on how to drive classroom teaching and learning by a series of questions originated from primitive mathematics ideas.
Joseph Kai Kow Yeo is an Assistant Professor in the Mathematics and Mathematics Education Academic Group at the National Institute of Education, Singapore. As a teacher educator, he is involved in training pre-service and in-service mathematics teachers at primary and secondary levels and has also conducted numerous professional development courses for teachers in Singapore. Before joining the National Institute of Education in 2000, he held the post of Vice Principal and Head of Mathematics Department in secondary schools. His research interests include mathematical problem solving in the primary and secondary levels, mathematics pedagogical content knowledge of teachers, mathematics teaching in primary schools and mathematics anxiety.
Part I

Introduction and Research Perspectives
Expertise in Mathematics Instruction: Advancing Research and Practice from an International Perspective

Yeping Li and Gabriele Kaiser

Abstract  Expertise in mathematics instruction, as commonly recognized, varies from one teacher to another and also affects their teaching performance. Studies on expertise in mathematics instruction are thus important, albeit long overdue, to reveal its specifics. To advance relevant research and practice for the improvement of teacher expertise in mathematics instruction, this book takes a unique approach to present new research from multiple education systems in the East and West. In this introduction chapter, we highlight the background of this book project, three important issues probed in this book, and the book’s content structure and overview.

Keywords  Eastern culture · Expert teacher · International perspective · Mathematics instruction · Teacher expertise · Western culture

Introduction

There is a general consensus on the importance of having and developing expertise in a professional field. Experts’ masterful performance in many fields, such as sports, medicine, mathematics, and music often amazes us. Efforts to pursue excellence in different fields have led not only to the better quality of work and performance, but also to the on-going quest about the nature of expertise that helps distinguish experts as they are from many others. Examining and knowing the nature of expertise also helps us understand what it may take for a novice to become an expert in that field. It is now commonly acknowledged that experts are knowledgeable about what they do and they have a more structured knowledge than non-experts (e.g., “Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise” in the chapter by Chi, this book). Yet, much still
remains to be understood about the nature of expertise, especially in those fields that often present complex and not well-structured tasks such as classroom instruction. Classroom teaching presents itself as such a task that relates to numerous factors and no well-defined algorithm is available to guarantee a successful solution. Nevertheless, the quality of classroom instruction has continually been taken as a key factor contributing to students’ learning. Examining and understanding the nature of teacher expertise in mathematics instruction is certainly not a trivial task for educational researchers, and is also imperative to those who seek ways to help teachers improve the quality of their classroom instruction.

This book reflects the ever-increasing interest and effort in improving students’ learning of mathematics through enhancing teachers’ quality and their teaching. While examining and learning about teachers’ classroom instruction and their expertise have long been the interest of educational researchers and psychologists albeit mainly in the West (e.g., Borko & Livingston, 1989; Leinhardt, 1989; Leinhardt & Greeno, 1986; Livingston & Borko, 1990; Swanson, O’Connor, & Cooney, 1990), there is a lack of systematic studies on teachers’ expertise in mathematics instruction. The initial development of this book project relates not only to the importance and needs of further research on mathematics teachers’ expertise, but also to recent international studies that documented distinct, yet often praised, teachers’ instructional performance and their knowledge in several high-achieving educational systems in East Asia. In particular, this book was initiated and motivated with the following two reasons:

First, this book presents an extension of a recent ZDM thematic issue on exemplary mathematics instruction in East Asia (Li & Shimizu, 2009). As the thematic issue of ZDM focused on exemplary mathematics instruction in six high-achieving education systems in East Asia (i.e., China, Hong Kong, Japan, Singapore, South Korea, and Taiwan), relevant studies illustrated what Asian teachers may do in carrying out their culturally valued lesson instruction but not the kind of expertise that is needed to make exemplary teaching performance possible. With limited knowledge now available about Asian teachers’ expertise in mathematics instruction, this book thus contains a collection of studies on teachers’ expertise in mathematics instruction in five out of the same six high-achieving education systems in East Asia (i.e., China, Japan, Singapore, South Korea, and Taiwan).

Second, this book was also inspired by the well-publicized Ma’s work that compared selected Chinese and US elementary teachers’ knowledge in mathematics (Ma, 1999). Ma revealed the dramatic differences in elementary teachers’ knowledge in mathematics between China and the United States, which led to further questions about the nature of expertise that may help connect or distinguish teachers’ instructional performance between the East and West. Therefore, this book is taking an international perspective to include two sets of chapters that focus on teachers’ expertise in mathematics instruction in the West and East, respectively. The international perspective should allow us to reflect on teacher expertise that is valued for developing high-quality classroom instruction in different education systems. Taken together, these intended extensions allow the book to make unique contributions to the much-needed study of teachers’ expertise in mathematics instruction.
Indeed, the book presents a new scholarship in addressing what appears to be a rather traditional topic in educational and psychological research. Although examining and understanding expertise is not a new endeavor, studying teachers’ expertise in mathematics instruction is a challenging task. First of all, the challenge lies not only in the complexity of mathematics instruction practices that do not have a commonly agreed-upon effectiveness, but also in the array of factors contributing to classroom instruction that go beyond cognition. Rather than staying away from such a challenging task, this book’s contributors undertook the challenge to develop or adopt different perspectives and methods in examining various aspects of teachers’ expertise in mathematics instruction. Second, taking an international perspective in this book is unique in that it presents not only an advantage, as mentioned above, but also a challenge. The challenge is embedded in the nature of teaching as a cultural activity (Stigler & Hiebert, 1999). What teachers do in mathematics classrooms is fundamentally influenced by specific cultural values. Examining teachers’ expertise in mathematics instruction thus calls for extra caution in understanding and interpreting teachers’ expertise that contributes to culturally-valued instructional practice in a specific system and cultural context. Therefore, what we can expect to learn from this book will differ from a typical book on expertise in many ways, and is irreplaceable due to the very nature of the task in focus.

Examining and editing of this themed book also builds upon our ongoing research interests in mathematics classroom instruction and mathematics teachers’ knowledge (e.g., Blömeke, Kaiser, Lehmann, & Schmidt, 2008; Huang & Li, 2010; Kaiser, Luna, & Huntley, 1999; Li & Shimizu, 2009). As editors of this themed book, we contribute from our own extensive experiences in mathematics education research and practices in the East and West. At the same time, we got intensive insights into the nature of expertise and its successful practices in the East and West. We are therefore convinced that the chapters in this book are valuable sources of information for international readers to learn and reflect upon possible similarities and differences in teachers’ expertise that is needed to develop culturally valued instructional practices.

Examining and Understanding Expertise in Mathematics Instruction in an International Context

As a cultural activity, mathematics teaching is situated in a specific cultural setting and also presents unique challenges to teachers in that culture. TIMSS classroom video studies (Hiebert et al., 2003; Stigler & Hiebert, 1999) have prompted further interests and studies about specific teaching practices that are formed and nurtured in a specific education system such as China (Fan, Wong, Cai, & Li, 2004), several education systems in East Asia (e.g., Li & Shimizu, 2009; Lim, White, & Kaur, 2008), or different cultural traditions around the world (Clarke, Keitel, & Shimizu, 2006). However, neither TIMSS video studies nor some other existing studies on classroom instruction aimed to analyze and discuss what teachers need to know and
be able to do in each participating education system. Much remains to be understood about the nature of teachers’ expertise that is valued in different education systems in the East and West. Included in this book, the majority of individual chapters can provide readers with its specifics about teacher expertise valued in one education system. With these chapters being put together as a collection, this book provides readers a platform to cross-examine and reflect on different aspects and issues of teacher expertise that are specified and discussed in different education systems. While readers can surely learn something beyond individual chapters through reading the book, here we would like to highlight three issues that are important for the broad readership in mathematics education and teacher education internationally.

**The Issue of Identifying and Selecting Teachers with Expertise**

In order to study teachers’ expertise, it is a common approach to examine what expert teachers know and are able to do while implementing mathematics teaching. These studies were carried out either with a comparison to novices or without such a comparison. However, identifying and selecting expert teachers is not a task that is based on a commonly-accepted approach across different studies (e.g., Berliner, 1986, 2001). While some researchers may rely on teachers’ educational background and their years of experience, others may emphasize their students’ academic performance and administrators or peers’ recommendations. In fact, there is often a lack of clear reference to teachers’ performance in classroom instruction or their knowledge when identifying expert teachers. The situation is especially acute in the West, where teachers’ instructional practices are not made public for scrutiny and discussion (e.g., Kaiser & Vollstedt, 2008). The lack of commonly used criteria in evaluating teachers and their teaching led researchers to make their own selection with different criteria in the past. Researchers’ judgement and determination of different selection criteria, as often practiced in the West, do pose an inherent difficulty when so-called or assumed expert teachers who may not possess expected expertise are selected for studying their expertise. Thus, rather than solely relying on researchers’ judgement and decisions, some contributors of this book used different approaches in identifying and selecting expert teachers. Expert teachers can be those who have been pre-identified as meeting specific certification requirements in the United States (e.g., “Coordinating Characterizations of High Quality Mathematics Teaching: Probing the Intersection” in the chapter by Silver & Mesa, this book), or those who have obtained an advanced rank as meeting specific professional requirements including classroom teaching in China (e.g., “Characterizing Expert Teaching in School Mathematics in China – A Prototype of Expertise in Teaching Mathematics” in the chapter by Li, Huang & Yang, this book). The use of such alternative approaches enables researchers to focus on those teachers who have already been identified and valued as expert teachers in a specific education system.
As it is generally acknowledged that classroom instruction is a complex and cultural activity (Stigler & Hiebert, 1999), being an expert teacher for doing what is culturally valued instructional practice is likely a culturally-related judgement in different education systems. The identification and selection of expert teachers remains to be a challenge to the fields of mathematics education and teacher education. Nevertheless, the use of alternative approaches by some researchers in this book provides us a direction for possible methodology changes in studying teacher expertise especially in an international context.

The Issue of Specifying and Analyzing Aspects of Teachers’ Expertise in Mathematics Instruction

Acknowledging the importance of teacher expertise does not provide specific suggestions or approaches for conceptualizing and studying teacher expertise. Because studies on teachers’ expertise are not a new endeavor in the realm of educational research, previous studies on teacher expertise can provide us with some hints for the aspects of teacher expertise that have typically been focused on.

In the United States, many researchers took a personal expertise perspective to examine individual expert teachers’ knowledge, their teaching practices, or teachers’ knowledge development from novice to expert (e.g., Berliner, 1986, 2001; Borko & Livingston, 1989; Leinhardt & Smith, 1985). Over the years, different approaches have been developed for examining teachers’ expertise. In particular, Sherin, Sherin, and Madanes (2000) indicated that two main different approaches have been developed to conceptualize teachers’ expertise. One is a cognitive modeling approach that focuses on classroom instruction process, and the other is a knowledge system perspective that tends to specify knowledge components of teachers’ expertise. Moreover, the development and exhibition of teachers’ expertise is also associated with their beliefs and views of what can be counted as effective/good teaching. Thus, past research has developed a repertoire of methodologies that can possibly be used in studying teachers’ expertise in mathematics instruction. Given the diverse aspects of teacher expertise that can possibly be focused on, we expect to find different aspects and approaches being taken by contributors of this book. For example, some contributors focused on teachers’ practices in classroom instruction (e.g., “Responding to Students: Enabling a Significant Role for Students in the Class Discourse” in the chapter by Even & Gottlib, this book; “Expertise of Mathematics Teaching Valued in Taiwanese Classrooms” in the chapter by Lin & Li, this book), some focused on teachers’ knowledge and/or beliefs (e.g., “Teacher Expertise Explored as Mathematics for Teaching” in the chapter by Simmt, this book), while others took a combination of both instructional practices and knowledge (e.g., “Characterizing Expert Teaching in School Mathematics in China – A Prototype of Expertise in Teaching Mathematics” in the chapter by Li et al., this book; “Cross-Nationally Comparative Results on Teachers’ Qualification, Beliefs, and Practices” in the chapter by Vieluf & Klieme, this book).
At the same time, some researchers examined the traditional teaching practices of expert teachers in the context of current educational changes (e.g., Schoenfeld, 1988). This presents a holistic perspective that takes into account the large social and cultural setting and related changes in valuing certain educational practices demonstrated with specific expertise in mathematics and pedagogy for teaching. Intuitively, results from this type of research pose a similar question and challenge in understanding what is valued as teachers’ expertise in different system and culture settings, as teaching is now commonly acknowledged as a cultural activity. Although we will further discuss the cultural issue in the next section, we want to remind readers about possible social-cultural influences on teacher expertise that are valued and examined in different chapters of this book.

The Issue of Understanding Expertise in Mathematics Instruction that is Valued in Different Cultures

Understanding and evaluating teacher expertise has been a perplexing issue in many education systems. By taking an international perspective, this book provides us with a unique opportunity to better understand the nature of teacher expertise that may be viewed and valued differently across the East and West. Indeed, taking an international perspective has helped us to learn a great deal about our own educational policy and practices in mathematics curriculum (e.g., Leung & Li, 2010; Li & Kulm, 2009), teachers’ classroom instruction (e.g., Clarke et al., 2006; Li & Shimizu, 2009; Stigler & Hiebert, 1999), and teachers’ knowledge (e.g., Ma, 1999; Sullivan & Wood, 2008). It is in the same spirit that the chapters of the book offer insight into teacher expertise that is valued in other system and cultural contexts.

At the same time, possible differences in viewing and valuing teacher expertise would also place a unique challenge for conducting cross-cultural examinations of teacher expertise. Thus, this book was not proposed as a collection of cross-cultural studies. Instead, this book contains a collection of studies of teacher expertise within individual education systems in the East and West, respectively. Correspondingly, studies on teacher expertise in individual education systems are grouped into two separate sections with one for education systems in the East and the other for the West. With this grouping we offer insight not only into teachers’ expertise of Eastern and Western cultures, but also collective differences and similarities between the East and West regions (i.e., across the two sections).

In addition, this book did not place or pre-specify any specific conception of teacher expertise. In this way, our contributors were given much flexibility in identifying and examining what is valued in teacher expertise in different education systems. The collection of individual studies from the East and West should help to provide a glimpse of the nature of teacher expertise in mathematics instruction that is also of interest to cognitive psychologists, and to explain what is valued for and in mathematics classroom instruction in the East and West. Finally, without pre-specifying a conception of teacher expertise, this book can also help raise questions
Overview of the Book

The book contains four parts. The first part provides an introduction and related research summaries. It is structured as containing three chapters, with the first chapter as this introduction chapter to this book including its organization and content overview, the second chapter to provide an overview of related theoretical perspectives and methodological approaches, and the third chapter to provide a review of related research on this topic in the field of mathematics education. The second part contains a series of five chapters that examined teacher expertise valued in a Western setting. Correspondingly, the third part contains a similar set of five chapters as Part II but with research focusing on an Eastern setting. Part IV is a part for reporting a large cross-national study related to teacher expertise and commentary chapters. Two commentary chapters are included to draw together research reported in Parts II and III. While one is to reflect on teacher expertise, the other is to reflect on what we can learn from this international collaborative effort and possible research directions for the future.

This book structure allows readers to get relevant information about the three issues highlighted in the above section “Examining and Understanding Expertise in Mathematics Instruction in an International Context”. In particular, because each chapter in Parts II and III tends to focus on those (expert) teachers and their expertise valued in a specific education system in the East or West, we expect that readers can gain much information about the first and second issues (see section “Examining and Understanding Expertise in Mathematics Instruction in an International Context”) from reading individual chapters. However, the third issue of understanding teacher expertise in mathematics instruction that is valued in different cultures is not always stated explicitly in each chapter. Thus, the book’s structure of separate parts for the Western and Eastern regions (i.e., Parts II and III) should assist readers when reading and reflecting on possible similarities and differences on teacher expertise both within and across the Western and Eastern regions.

Part I: Introduction and Related Research Summaries

Three chapters included in this part aim to provide a general background about this book and relevant research on expertise. The chapter written by Michelene Chi provides an overview of psychological studies of expertise. With a focus on the changes in theoretical perspectives and methodological approaches, Chi outlines some major developments in psychological studies over the years. Although the concept of expertise has always been related to knowledge, it took years of research development to learn the importance of knowledge especially structured knowledge.
Now, psychological researchers pay close attention to the acquisition of expertise. Some current constructs include deliberate practice, adaptive expertise, and team expertise. A new idea about the acquisition of expertise is also proposed as the construct of a perspective shift. Nevertheless, Chi indicates that many questions still remain to be explored about expertise and its acquisition.

Different from Chi’s overview of psychological studies on expertise, Russ, Sherin, and Sherin focus on the concept of teaching expertise that are emerged in the study of mathematics teaching. Thus, these researchers take a historical perspective to trace the study of mathematics teaching in an attempt to capture emerged images of teachers in mathematics teaching. In particular, four images have been identified: mathematics teachers as diagnosticians of students’ thinking, conductors of classroom discourse, architects of curriculum, or river guides who are flexible in the moments of teaching. The identification of these images helps us not only understand specific expertise that may be required behind different images, but also guide further efforts in identifying and positioning possible new images in mathematics teaching.

**Part II: Understanding and Examining Teacher Expertise in a Western Setting**

There are five chapters that report on the study of teacher expertise in a Western setting. These five chapters present diverse perspectives and approaches employed to examine various aspects of teacher expertise in different education systems. While the first three chapters make a close connection with teachers’ classroom instruction in studying teacher expertise, the remaining two chapters conceptualize teacher expertise more in terms of knowledge or structured components.

In the first chapter, Silver and Mesa probed different approaches and their intersections in characterizing high quality mathematics teaching. By taking three different views of exceptional mathematics teaching, the researchers examined empirically how lesson instruction and teachers’ commentaries on lessons submitted by a group of teachers obtained the NBPTS (US National Board for Professional Teaching Standards) certification may be similar or different from those by a group of teachers who were not awarded the NBPTS certification. Their analyses identified some strong interactions between the NBPTS view of accomplished teaching and the effective use of cognitively demanding tasks in the mathematics classroom, but not with the expected use of innovative pedagogical strategies to engage students. Through these three different views, the study certainly enables us to develop a better understanding about those NBPTS certified teachers’ instructional performance. At the same time, the results also illustrate those teachers’ strengths and weakness in selected aspects of teacher expertise in mathematics instruction.

Pauli and Reusser developed the chapter, expertise in Swiss mathematics instruction, through drawing together data and findings from several video studies on mathematics teaching in Switzerland. The researchers proposed a profile of teacher
Expertise in Mathematics Instruction

expertise that is associated with different components of a didactic triangle (i.e., content, teacher, and students). With the model of the didactic triangle, the researchers aimed to identify possible strengths and weaknesses in expertise that Swiss mathematics teachers in general (not just expert teachers) may have. In particular, the researchers pointed out that Swiss mathematics teachers have particular strengths in the culture of communication, support, and relationships that mainly connects teacher and students in the didactic triangle, and positive but less strong in connecting content and students. They further suggested that Swiss teachers need to improve their didactics of mathematics that connects teacher and content.

The chapter written by Even and Gottlib focuses on an experienced high school mathematics teacher’s classroom practices in responding to students. Different from other chapters in the second part, the researchers provided a detailed analysis of the teacher’s classroom instruction. The identification and selection of this experienced teacher was due to her reputation of involving and engaging students in the class discourse. The teacher’s extensive involvement in some curriculum committees at the national and local levels also suggests her extensive knowledge in mathematics curriculum and instruction. The detailed analyses of her teaching in both a lower-achieving 9th grade class and a high-achieving 10th grade class revealed how the teacher developed her instruction as building upon students’ talk. Developing communication and relationships with students is apparently taken as an important component for making effective instruction possible in this classroom. Behind the teacher’s sensitivity about students’ talk and her skills in identifying and developing learning opportunities for students, the researchers illustrated some important aspects of teacher expertise that are valued in a mathematics classroom in Israel.

Four researchers from Austria, Müller, Andreitz, Krainer and Mayr, contributed this chapter to document the effects of a research-based learning approach (a four-semester program of “Pedagogy and Subject Didactics for Teachers”) on teachers’ professional development. In addition to surveying teacher participants’ motivation, learning strategies, and satisfaction with course, the researchers employed multiple scales to capture possible changes in the program participants’ interests, competencies, and knowledge. The substantial changes in the multiple scales of competence and knowledge show not only the effectiveness of the program, but also several aspects that are valued in Austria as teaching job-related expertise. In particular, the use of a video task for teaching related analysis in the program confirms the idea that teachers’ active participation and practices are essential for their professional development.

In her chapter, Simmt focused on the nature of mathematics that teachers need to work with to conceptualize teacher’s expertise as mathematics for teaching (MFT). The model of MFT is further specified as a multi-layered and nested knowledge. In this way, Simmt highlights the knowledge nature of mathematics teacher’s expertise and its structure. The model is then used to illustrate teachers’ MFT and its changes through analyzing the actions and interactions of a group of mathematics teachers in a professional development session.
Part III: Understanding and Examining Teacher Expertise in an Eastern Setting

Similar to Part II, Part III also includes five chapters that individually present different studies on teacher expertise in five different education systems in East Asia. Different from Part II, all five chapters in this part tend to connect with teachers’ instructional practices when addressing the issue of teacher expertise. Yet, the diversity is evident in terms of their selection of focal aspects and use of different perspectives across these five chapters.

The chapter, written by Li, Huang and Yang, aimed to characterize teacher’s expertise through analyzing teachers’ lesson instruction, their lesson design and reflections. The researchers focused on five selected expert teachers who are officially recognized with the teacher ranking system in China. A prototype view of teaching expertise was used to identify six similarity-based central tendencies in mathematics instruction that are shared among these expert teachers. The content of teacher expertise is thus not pre-defined but revealed through teachers’ instructional practices in this study. Moreover, the researchers included a case analysis of one expert teacher’s lesson instruction to provide rich descriptions and illustrations of the prototype of these teachers’ teaching expertise. The findings help not only to illustrate the complexity of mathematics teaching expertise, but also to inform of the aspects of teacher’s expertise that are important for developing culturally valued mathematics instruction in China.

Takahashi pointed out that “teaching the textbook” is taken as different from “using the textbook to teach mathematics” in Japan. The distinction has been used in Japan to differentiate and classify teachers into three levels in terms of their extents of using textbooks for teaching. To take a closer look at possible knowledge and expertise requirements behind these distinctions, Takahashi surveyed a small group of teachers who were pre-classified as belonging to these three different levels. The results reveal the differences in knowledge and expertise among these teachers in three levels, and also provide a glimpse of the type of expertise in “using textbooks to teach mathematics” that is valued and practiced in Japan.

Three researchers from Singapore took an alternative approach to examine what is valued in Singapore’s mathematics instruction. Other than examining teachers’ lesson instruction directly, Lim-Teo, Chua and Yeo conducted a survey and interviews of primary schools’ mathematics department heads on their perceptions of effective practices for learning mathematics. The results reveal that mathematics department heads value those instructional practices that enhance conceptual learning and pupil motivation to learn. Although the results are not necessarily aligned with the general perception of typical instructional practices in Singapore, the study revealed much expected changes in what is valued in mathematics instruction in Singapore.

Taking students’ engagement in meaningful discourse as an important instructional practice, Pang compared and contrasted more successful and less successful teachers in carrying out such practice in Korea. The comparison focused on the ways two selected teachers lead to the development of unequally successful mathematics classrooms. As the two classes established similar social
participation patterns but a different quality of mathematical discourse, the results suggest important differences in teacher expertise that can contribute to the establishment of different sociomathematical norms in these two classrooms.

Focusing on the case of Taiwan, Lin and Li also adopted the prototype view of teaching expertise to explore similarity-based family resemblance in expert teachers’ mathematics instruction. Three expert teachers were identified and selected using a set of criteria that are valued in Taiwan. Those expert teachers’ lesson instruction was analyzed in terms of: selecting and sequencing problems for and in classroom instruction, selecting and sequencing students’ solutions for the whole-class discussion, asking questions and responding to students during the class discussion, and transitioning from one activity to another. The common features of these expert teachers’ lesson instruction were thus revealed and further illustrated with an expert teacher’s instructional practices.

**Part IV: Researching and Reflecting on Teacher Expertise in an International Context**

This last part includes three chapters that do not place a focus on teacher expertise solely in the East or West. The chapter contributed by Vieluf and Klieme draws on data from the OECD-Teaching and Learning International Survey (TALIS) collected from 23 countries. The researchers aimed to gain an overview of cross-national similarities and differences in selected measures on teacher qualification, beliefs, and practices. Their results reveal some global similarities in broad terms, a finding consistent with the understanding of global similarities in schools and instructional organization. At the same time, their results suggest the importance of examining and understanding cross-national differences in profiles and constructs of teacher quality. The researchers pointed out that more cross-cultural research on teacher expertise and teacher quality, both qualitative and quantitative, is needed.

The last two chapters are reflections on teacher expertise and related research. The chapter contributed by Schoenfeld addresses two important and related issues: value-based variations in conceptualizing and measuring teacher expertise and the development of teacher expertise itself. He first highlights possible variations in teachers’ and researchers’ beliefs and values about what are “important” in the act of teaching. Such differences directly relate to different ways to conceptualize and consequently measure teacher expertise. Schoenfeld argues that it is ultimately important to link teacher expertise with students’ enhanced performance, although this has been a serious challenge to all researchers. He then shifts the focus to teachers’ lesson instruction itself, and hypothesizes that the development of teacher expertise should bear a direct connection to those aspects (i.e., teacher’s resources, goals, and orientations) that will lead to the improvement of teacher’s instructional performance.

In the concluding chapter by Kaiser and Li it is summarized what we can learn from this book concerning the concept and nature of expertise, how it is theoretically described and empirically measured. The chapter summarizes the
differences between Eastern and Western perspectives on expertise and exemplifies their different orientation towards the teaching subject mathematics or the individual students. Furthermore the paper analyses our knowledge of the factual situation referring to current studies and describes possible research directions for the future.

**Significance and Limitations**

By taking an international perspective, this book aims to provide a unique platform for mathematics educators and teacher educators worldwide to develop a better understanding about teacher expertise in mathematics instruction. The significance of this scholarly work lies in its timely importance of developing and promoting research on mathematics teachers’ quality and instruction. Similar to the case of mathematics teaching, it is not surprising for us to learn some similarities as well as differences in aspects of teacher expertise that are valued in different system and cultural contexts. Gaining such knowledge from this book is necessary not only for understanding the development of culturally-valued teaching performance in different education systems, but also for identifying aspects of teacher expertise for improvement.

At the same time, we realize that it is impossible for the book to address all the questions related to teacher expertise. In fact, we sincerely hope that this book can stimulate further study and discussion of teacher expertise and its development in different education systems. For example, although the book contains chapters about teacher expertise from the East and West respectively, it is not clear whether different aspects of teacher expertise valued in the East or West may link to students’ enhanced learning (“Reflections on Teacher Expertise” in the chapter by Schoenfeld, this book). Nor is it clear which aspect of teacher expertise may be more important than others. Equally, if not more important, mathematics educators and teacher educators would also be interested in learning about effective approaches and practices used for developing teacher expertise in different education systems (“Reflections and Future Prospects” in the chapter by Kaiser & Li, this book). Indeed, the richness of the topic itself suggests that the book can well be a starting point for developing the much needed research on this topic in the future.

**References**


