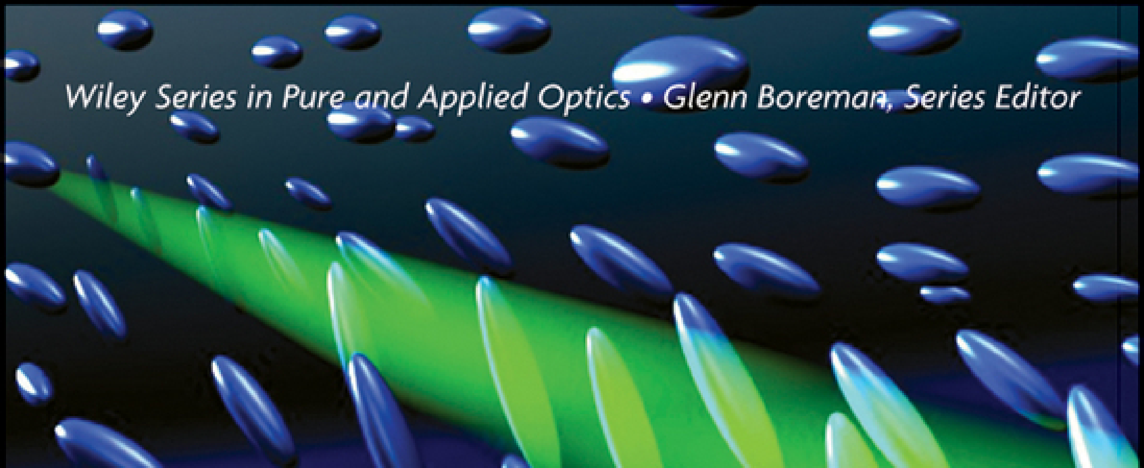

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NEMATICONS

Spatial Optical Solitons in Nematic Liquid Crystals

GAETANO ASSANTO

WILEY

Nematicons

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To my parents

Contents

Preface

Solitons in physics and solitons in optics are well-established contemporary topics, addressed in a large number of scientific papers and several books. Spatial optical solitons form a specific class, as optics in space is characterized by diffraction rather than dispersion, beam size rather than pulse duration, one or two transverse dimensions rather than one in the temporal domain. For a long time, the available experimental observations of optical solitons in space were limited by the magnitude of the material nonlinearities, until molecular and photorefractive media allowed investigating them at low power and with continuous-wave sources, including incoherent ones. Among the well-known molecular dielectrics exhibiting a large optically nonlinear response were liquid crystals, typically employed in thin samples. It was realized in the early days of both nonlinear optics and liquid crystals that the reorientational response of nematic liquid crystals could lead to quite impressive effects, both in the electro-optic and all-optical domains. Later on, beam propagation over extended distances in nematic liquid crystals was exploited to demonstrate self-focusing and related phenomena, until it became clear that optical spatial solitons could be supported by such a response at the molecular level. I came across light self-localization in nematic liquid crystals during international meetings, where I attended the inspiring presentations by Prof. M. Karpierz (Poland) and Prof. M. Warenghem (France) on light self-confinement in nematic liquid crystals, and decided to get involved in research on nematicons. The discussions with Prof. I. C. Khoo were enlightening and the collaboration with Prof. C. Umeton allowed the program to get started on the right foot. The term "nematicon" was actually coined during a car trip in Poland as I was having a conversation on the topic with M. Karpierz and G. I. Stegeman. The Greek root $\nu\epsilon\mu\alpha\tau\iota\kappa o\sigma$ means "filament-like" or "spaghetti-like," appropriate to both the topic and the culinary culture of someone like me, of Italian birth and upbringing.

This is the first book specifically dealing with spatial optical solitons in nematic liquid crystals. It is a multiauthor contribution to the field and contains review as well as original (previously unpublished) material, from theoretical models to advanced numerical simulations and from experimental observations to applications. The various contributors and chapters have been selected and invited in order to cover most of the relevant activities in this field over the past 12 years.

G. Assanto

Italy
February 2012

Acknowledgments

Prof. Glenn Boreman and his wife, Maggie, friends since my PhD studies at the University of Arizona in Tucson, Arizona, encouraged me to consider preparing a Wiley book on nematicons. George Telecki soon joined them in keeping up the necessary pressure. Thanks a lot. I hope you were right and that readers will enjoy this book.

I thank all the authors who kindly accepted my invitation to contribute one or more chapters, and to subject themselves to a number of requests concerning contents, style, mode of presentation, and deadlines. I express my gratitute to all the students and colleagues who do not appear as book contributors but are coauthors of papers and precious actors inspiring various portions of the scientific activities. They include R. Asquini, R. Barboza, I. Burgess, O. Buchnev, G. Coschignano, D. Christodoulides, A. d'Alessandro, A. de Luca, R. Dabrowski, A. Dyadyusha, A. Fratalocchi, M. Kaczmarek, I. C. Khoo, M. Kwasny, L. Lucchetti, R. Morandotti, E. Nowinowski-Kruszelnicki, A. Pasquazi, K. A. Rutkowska, S. V. Serak, F. Simoni, G. I. Stegeman, N. Tabiryan, M. Trotta, and C. Umeton.

Finally, I pay a special tribute to Alessandro Alberucci and Armando Piccardi for greatly supporting me in the no less important task of arranging, organizing, managing, and editing the manuscript.

GA

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Nematicons

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1.1 INTRODUCTION

The term *nematicon* was coined to denote the material, nematic liquid crystals (NLC), supporting the existence of optical spatial solitons via a molecular response to light, a *reorientational* nonlinearity. *Nematicons* was first used in the title of Reference 1, after three years since the first publication on reorientational spatial optical solitons in NLC [2]. Since then, a large number of results, including experimental, theoretical, and numerical, have been presented in papers and conferences and formed a body of literature on the subject. In this chapter we attempt to summarize the most important among them, leaving the details to the specific articles but trying to provide a feeling of the amount of work carried out in slightly more than a decade.

1.1.1 Nematic Liquid Crystals

Liquid crystals are organic mesophases featuring various degrees of spatial order while retaining the basic properties of a fluid. In the absence of absorbing dopants, they are excellent dielectrics, transparent from the ultraviolet to the mid-infrared, with highly damaged thresholds, relatively low electronic susceptibilities, and significant birefringence at the molecular level and in the nematic phase. In the latter phase, their elongated molecules have the same average angular orientation, although their individual location is randomly distributed as they are free to move (Fig. 1.1a). NLC exhibit a molecular nonlinearity; when an electric field is present, the electrons in the molecular orbitals tend to oscillate with it and give rise to dipoles which, in turn, react to and tend to align with the field in order to minimize the resulting Coulombian torque [3–5] (Fig. 1.1b–c). This torque is

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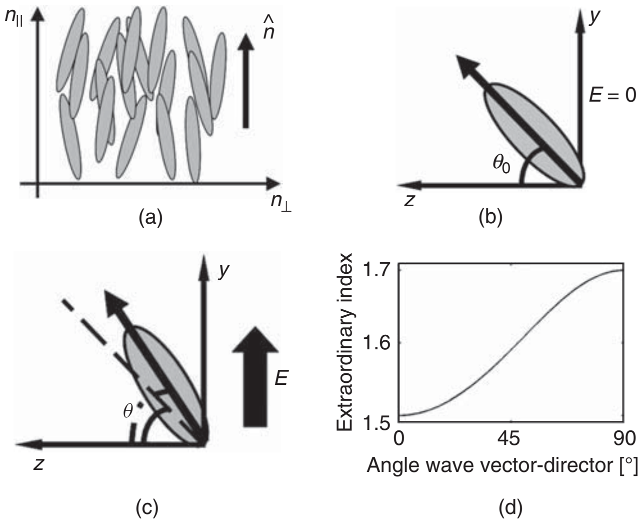

FIGURE 1.1 (a) Sketch of molecular distribution in the nematic phase and definition of director $\hat{n}$; the ellipses represent NLC molecules. (b) Director orientation in the absence of electric field: the angle θ_0 is determined by anchoring at the boundaries. (c) In a positive uniaxial NLC, a linearly polarized electric field can induce dipoles and rotate the molecular director towards its vector; the resulting stationary angle θ is determined by the equilibrium between the electric torque and the elastic intemolecular links. (d) Extraordinary refractive index versus angle between wave vector and director for a positive uniaxial NLC with $n_{\|} = 1.7$ and $n_{\perp} = 1.5$.

counteracted by the elastic forces stemming from intermolecular links: equilibrium is established when the free energy of the system is minimized, as modeled by a set of Euler–Lagrange equations. Because the polarizability of the molecules is higher along their major axes, their reorientation toward the field will increase the optical density, both at the microscopic and macroscopic levels. It is noteworthy that an initial orthogonality between the field and the induced molecular dipoles corresponds to a threshold effect known as *Freedericksz transition* [3]. For static or low frequency fields, reorientation leads to a large electro-optic response with a positive refractive index variation for light polarized in the same plane of the field lines and the long molecular axes [3]. For fields at optical frequencies, the average angular orientation or molecular director in the nematic phase corresponds to the optic axis of the equivalent uniaxial crystal; hence, the refractive index for extraordinarily polarized electric fields (i.e., with field vector coplanar with both optic axis and wave-vector) will increase with the orientation angle θ (Fig. 1.1c–d for wave-vectors along z).

The reorientational mechanism described above is neither instantaneous nor fast (see Chapter 13), but can be very large, with effective Kerr coefficients n_2 of about 10^{-4} cm/W^2 [6], that is, eight to twelve orders of magnitude larger than that in CS_2 and in electronic media, respectively [7]. Therefore, nonlinear effects can be observed in NLC even with continuous wave lasers, at variance with many other nonlinear dielectrics often requiring pulsed excitations.

Nevertheless, the reorientational response is not the only available response in NLC. Owing to their fluidic nature, a high electric field can change the portion of molecules aligned to the director, that is, can affect the order parameter [8], particularly in the presence of dye dopants [9]. Doped NLC also features an enhanced reorientational nonlinearity because of the Janossy effect [10]. As a result of thermo-optic effect, a nonlinear response also stems from temperature changes, modifying the refractive indices mainly via the order parameter in phase transitions [6] (see Chapter 9). Moreover, NLC can show the photorefractive effect [4] and fast electronic nonlinearities (see Chapter 14).

1.1.2 Nonlinear Optics and Solitons

In nonlinear optics, the basic example of an intensity-dependent refractive index is the Kerr response $n(I) = n_0 + n_2 I$. When n_2 is positive, the index increases with the light intensity and, in the case of a finite beam, it gives rise to a lens-like refractive distribution, which is capable of self-focusing the excitation. Such a mechanism can actually compensate for the natural diffraction of the beam, resulting (in the simplest case) in a size/profile-invariant spatial soliton. Otherwise stated, the excitation beam deforms the refractive index distribution of the nonlinear (initially uniform) dielectric, generating a transverse graded-index profile that acts as a waveguide, that is, confines the field into a guided mode. The fundamental soliton in space is the lowest order mode guided by the self-induced dielectric waveguide. Spatial solitons of a Kerr nonlinearity, the so-called Townes solitons [11], tend to be unstable in two transverse dimensions because the exact balance of diffraction and self-focusing is achieved at a critical power [12, 13]. They are stable in one dimension (e.g., in planar waveguides [14]) or in the presence of higher order effects as compared to the Kerr law, such as saturation of the nonlinear change in index [15, 16], multiphoton absorption [17], discreteness [18, 19], and nonlocality [20]. In most cases they are observable in actual media although, being no longer exact solutions of an integrable differential system, they should be rigorously referred to as *spatial solitary waves* [21]. The terms soliton and solitary wave are interchangeably used throughout this chapter.

1.1.3 Initial Results on Light Self-Focusing in Liquid Crystals

As discussed in Section 1.1.1, several terms can contribute to the nonlinear response of NLC. Experiments conducted in the early 1980s demonstrated that, in undoped NLC, the dominant contribution is the reorientational nonlinearity [6, 22, 23]. An equivalent Kerr response was measured with light beams passing through the thickness of a planar cell, the latter behaving as a lens, the focus of which is dependent on the input power. For Rayleigh distances much smaller than the NLC layer thickness, rings could be observed in the diffraction pattern [24].

An experiment on self-focusing in the bulk of a dye-doped NLC layer was carried out in 1993 by Braun et al. [25], who imaged the scattered light from a beam propagating in a cylindrical geometry with NLC subject to Freedericksz

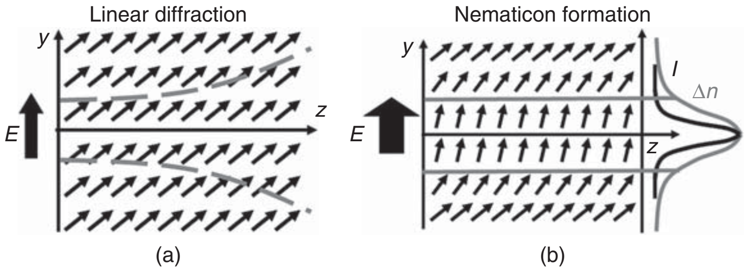

FIGURE 1.2 Basic physics of nematicons. An extraordinarily polarized bell-shaped beam with wave-vector along $\hat{z}$ is launched in an NLC layer with director lying in the plane yz. The major axes of the molecules are at an angle $\neq 90^\circ$ with the wave-vector, thanks to a pretilt (the arrows indicate the molecular director). (a) In the linear regime light does not affect the angular distribution of the director: the beam diffracts as in homogeneous media. (b) Conversely, at high powers the director is perturbed and reorientates toward $\hat{y}$, increasing θ and thus the refractive index (Fig. 1.1d). The perturbation is stronger where the intensity I is higher; hence, an index well is created by the light beam itself, leading to the formation of a waveguide and a self-trapped nematicon. Noticeably, the perturbation extends far beyond the beam profile owing to the elastic links between molecules. For the sake of simplicity, in this illustration the role of walk-off is ignored (Section 1.2.1).

threshold. Various phenomena were observed, including undulation, filamentation, and nonstationary evolution along the capillary; they were interpreted and modeled with joint reorientational and nonlinear Schrödinger equations [26–27]. After such a pioneering work, self-localization of light as a consequence of thermo-optic effects in capillaries was reported by Derrien et al. [28]; the interplay between thermal and reorientational responses was addressed by Warenghem et al. [29] (see Chapter 9). The use of suitably built planar cells with the director tilted by an external bias to avoid the Freedericksz threshold allowed Peccianti et al. to observe the profile-invariant spatial solitons at a few milliWatts [2]. Unbiased planar cells with pretilt determined by rubbing permitted the detailed study of walk-off [30] (see Chapter 6). Figure 1.2 sketches the basic mechanism of nematicon formation via a purely reorientational response.

Finally, nematicons were also reported in slab waveguides with homeotropically aligned NLC [31], in one-dimensional arrays of coupled waveguides [18, 32] (see Chapter 10) and in twisted/chiral NLC [33, 34] (see Chapter 12).

1.2 MODELS

In this section, we review the main theoretical results concerning nonlinear light propagation in NLC cells, with specific reference to a reorientational response supporting optical spatial solitons as well as modulational instability. We first discuss scalar geometries (voltage-biased cells), that is, those in which the role of birefringent walk-off can be left aside. Afterward, we consider the most general case of cells where the walk-off has a substantial effect.

The director distribution can be described by the two polar angles ξ (tilt from the plane yz) and ζ (in the plane yz) (Fig. 1.3a). In addition, θ is the angle

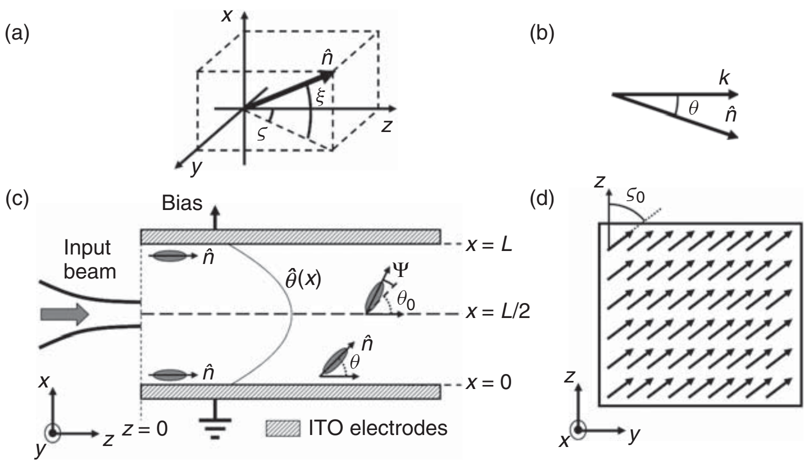

FIGURE 1.3 (a) Definition of polar angles describing the director in the xyz space. (b) Definition of the angle θ between the wave-vector $\boldsymbol{k}$ and the director $\hat{n}$. (c) Side view sketch of a biased planar NLC cell with anchoring condition at the interfaces such that $\hat{n}\|\hat{z}$ (i.e., $\theta = \xi$) and a focused light beam launched along z. The structure is assumed to be infinitely extended along y. (d) Top view of a planar cell showing the rubbing angle ζ_0 in the plane yz; the arrows represent the director distribution in the absence of external excitations (neither bias nor illumination).

between the beam wave-vector $\boldsymbol{k}$ and the molecular director $\hat{n}$ (Fig. 1.3b). In scalar geometries $\zeta = 0$, the latter implying $\theta = \xi$ (Fig. 1.3c). In general, $\zeta \neq 0$ owing to the anchoring at the (glass/NLC) interfaces parallel to the plane yz; at rest the director $\hat{n}$ lies in the plane yz at an angle ζ_0 with z (Fig. 1.4d).

We stress that the equations and the results shown hereby hold valid in the limit of small optical perturbations; the highly nonlinear case is dealt with in Chapter 11.

1.2.1 Scalar Perturbative Model

We consider the configuration of Fig. 1.3c: a finite light beam is launched in the planar NLC cell with wave-vector along the z axis and the field linearly polarized along the x axis. Two parallel glass plates contain the NLC, with molecular director $\hat{n}$ lying in the plane xz (i.e., $\hat{n} \cdot \hat{y} = 0$) at an angle θ with $\hat{z}$ (i.e., $\hat{n} \cdot \hat{z} = \cos\theta$). A low frequency electric field E_{LF} is applied (via transparent electrodes on the plates) across $\hat{x}$ to overcome the Fréedericksz threshold and pretilt the molecules in the plane xz via the electro-optic response, creating a potential $\hat{\theta}(x)$ in the absence of illumination; $\hat{\theta}$ depends only on x due to the symmetry of the problem.

In this configuration the beam excites only the extraordinary component, generally at the walk-off δ with respect to $\hat{x}$, owing to birefringence. Hereby, we scalarize the problem and assume the electric field E_{opt} of the beam to be linearly polarized along $\hat{x}$, leaving the vectorial case to Section 1.2.2. The use of a scalar model also implies neglecting the tilt between Poynting and wave-vectors. Let us define A, the slowly varying envelope of E_{opt}, that is,

$$E_{\mathrm{opt}} = A\,(x, y, z) \exp\left[ik_0 n_{\mathrm{e}} \left(\theta_0\right) z\right]$$

with θ_0 the orientation without light and $n_\mathrm{e}(\theta) = \left(\frac{\cos^2\theta}{\epsilon_\perp} + \frac{\sin^2\theta}{\epsilon_\parallel}\right)^{-1/2}$ the extraordinary wave refractive index, where $\epsilon_\perp$ ($\epsilon_\parallel$) is the electronic susceptibility perpendicular (along) to $\hat{n}$. In the paraxial approximation, light propagation is ruled by [2]

$$2\mathrm{i}k\frac{\partial A}{\partial z} + \nabla_{xy}^2 A + k_0^2\epsilon_a \sin(2\theta_0)\Psi A = 0, \tag{1.1}$$

where $k = k_0 n_\mathrm{e}(\theta_0)$ and we set $\theta = \theta_0 + \Psi$, with Ψ being the light-induced perturbation on θ.

As we are interested in the reorientational nonlinearity, we need a further equation describing how the angle θ varies under the application of both E_opt and E_LF. To this extent, minimization of the NLC free energy that assumes a single constant to describe the elastic (intermolecular) forces, yields the Euler–Lagrange equation [3, 35]

$$K\frac{\hat{\theta}}{\theta_0}\left[\nabla^2\Psi + \frac{\cos(2\theta)}{K}\left(\Delta\epsilon_\mathrm{LF}E_\mathrm{LF}^2 + \frac{\epsilon_0\epsilon_a|A|^2}{2}\right)\Psi\right] + K\frac{\Psi}{\theta_0}\frac{\mathrm{d}^2\hat{\theta}}{\mathrm{d}x^2} + \frac{2K}{\theta_0}\frac{\mathrm{d}\Psi}{\mathrm{d}x}\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}x} + \frac{\epsilon_0\epsilon_a}{4}|A|^2\sin(2\hat{\theta}) = 0, \tag{1.2}$$

with $\Delta\epsilon_\mathrm{LF}$ being the anisotropy and K the Frank elastic constant, and $\theta(x,y,z) = \hat{\theta}(x) + \frac{\hat{\theta}(x)}{\theta_0}\Psi(x,y,z)$ being $\mathrm{d}^2\hat{\theta}/\mathrm{d}x^2 + (\Delta\epsilon_\mathrm{LF}E_\mathrm{LF}^2/2)\sin(2\hat{\theta}) = 0$ [35]. Equation 1.2 is obtained when $(\hat{\theta}/\theta_0)\Psi \ll \hat{\theta}$, that is, in the perturbative limit.

For straight beam trajectories (i.e., homogeneous medium, uniform director distribution, no walk-off) we can set $\hat{\theta} \approx \theta_0$. When $\epsilon_0\epsilon_a|A|^2 \ll \Delta\epsilon_\mathrm{LF}E_\mathrm{LF}^2$ and the beam axis is in the cell mid-plane $x = 0$ with $\mathrm{d}\hat{\theta}/\mathrm{d}x = 0$, the light-induced reorientation is governed by [35]

$$K\nabla^2\Psi - \frac{\sin(2\theta_0)}{2\theta_0}\left[1 - 2\theta_0\frac{\cos(2\theta_0)}{\sin(2\theta_0)}\right]\Delta\epsilon_\mathrm{LF}E^2\Psi + \frac{\epsilon_0\epsilon_a}{4}\sin(2\theta_0)|A|^2 = 0, \tag{1.3}$$

that is, by a Yukawa (or screened Poisson) equation, with forcing term given by the light intensity and screening length l equal to

$$l = \sqrt{\frac{2\theta_0}{\sin(2\theta_0) - 2\theta_0\cos(2\theta_0)}\frac{K}{\Delta\epsilon_\mathrm{LF}E^2}} = c_l(\theta_0)l_\mathrm{LC}, \tag{1.4}$$

where we set $c_l(\theta_0) = \sqrt{\frac{2\theta_0}{\sin(2\theta_0)-2\theta_0\cos(2\theta_0)}}$ and $l_\mathrm{LC} = \sqrt{\frac{K}{\Delta\epsilon_\mathrm{LF}E^2}}$. It is straightforward to obtain $\lim_{\theta_0\to 0} c_l = \infty$ and $c_l(\theta_0 = \pi/2) = 1$; between these two extrema c_l decreases monotonically. We note that θ_0 depends only on the applied voltage

$V \approx E_{LF}L$; hence, Equation 1.4 provides $l = c_l[\theta_0(V)]\sqrt{K/\Delta\epsilon_{LF}}L/V$ and for a given bias V, the spatial width of the nonlinear response is proportional to the cell thickness L.

The system formed by Equations 1.1 and 1.3 governs nonlinear light propagation in biased NLC cells; for any NLC and cell size (i.e., thickness L), the parameters depend on the bias via the applied electric field and θ_0, that is, on low frequency reorientation bias, including pretilt. Such a feature allows for electrically tuning both nonlinearity and nonlocality of the medium [36].

To quantify the nonlinearity, let us define the material-dependent parameter $\gamma = \epsilon_0\epsilon_a/(4K)$; using Equation 1.3, Ψ can be expressed via the Green formalism as

$$\Psi = \gamma \sin(2\theta_0) \int G(\boldsymbol{r} - \boldsymbol{r}')|A(\boldsymbol{r}')|^2 \mathrm{d}^3\boldsymbol{r}', \tag{1.5}$$

where $G(\boldsymbol{r} - \boldsymbol{r}')$ is the Green function of the Yukawa equation (Eq. 1.3).

Using Equation 1.5 the *photonic potential* [defined as $V_{ph} = k_0^2\epsilon_a \sin(2\theta_0)\Psi$ and corresponding to the potential with a change in sign] reads $V_{ph} = k_0^2\epsilon_a\gamma \sin^2(2\theta_0) \int G(\boldsymbol{r} - \boldsymbol{r}')|A(\boldsymbol{r}')|^2\mathrm{d}^3\boldsymbol{r}'$. We can thus write the effective (nonlocal) Kerr coefficient as

$$n_2^{\text{eff}} = \frac{\epsilon_0\epsilon_a^2}{4K} l^2(\theta_0) \sin^2(2\theta_0) = \frac{\epsilon_0\epsilon_a^2}{4K} l_{LC}^2 c_l^2(\theta_0) \sin^2(2\theta_0). \tag{1.6}$$

The square dependence on the screening length l^2 stems from the integral in Equation 1.5: for intensity distributions maintaining their transverse size with respect to l (i.e., $|A(\boldsymbol{r}/l)|^2$ invariant), the perturbation Ψ scales with l^2. Conversely, the magnitude of the nonlocality, that is, the ratio between the widths of the photonic potential and the intensity profile, is determined by l. In fact, in the limit $l \to \infty$, Equation 1.3 becomes a Poisson equation, with degree of nonlocality fixed by the boundaries (see Chapter 11 and references therein). After setting $|A|^2 = |B|^2/l^2$, for $l \to 0$ we get $\Psi \propto |B|^2$: in this regime NLC resemble local Kerr media.

1.2.1.1 Solitary Waves Let us define the normalized coordinates $X = \sqrt{2k}x$, $Y = \sqrt{2k}y$, and $Z = z$; we also introduce the normalized quantities $\psi = k_0^2 \sin(2\theta_0)/(2K)\Psi$ and $a = \sqrt{1/(k_0^2 n_2^{\text{eff}})}A$, with $\alpha = 1/(2kl^2)$. The parameter α is inversely proportional to nonlocality, that is, α is equal to zero if the nonlocality range is infinite, whereas it tends to ∞ in the local (Kerr) case. Equations 1.1 and 1.3 now read [35]

$$\frac{1}{2k}\frac{\partial^2\psi}{\partial Z^2} + \nabla_{XY}^2\psi - \alpha\psi + |a|^2 = 0, \tag{1.7}$$

$$i\frac{\partial a}{\partial Z} + \nabla_{XY}^2 a + \psi a = 0. \tag{1.8}$$