Soheil Mohammadi

XFEM

Fracture Analysis of Composites

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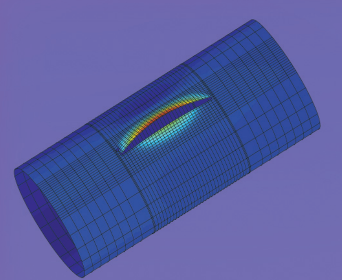

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Preface

A decade after its introduction, the extended finite element method (XFEM) has now become the primary numerical approach for analysis of a wide range of discontinuity applications, including crack propagation problems. The simplicity of the idea of enrichment for reproducing a singular/discontinuous nature of the field variable, the flexibility in handling several cracks and crack propagation patterns on a fixed mesh, and the level of accuracy with minimum additional degrees of freedom (DOFs) have transformed XFEM into the most efficient computational approach for handling various complex discontinuous problems. Concepts of XFEM are now even taught in a number of postgraduate courses, for instance advanced fracture mechanics and meshless methods, in major engineering departments, such as Civil, Mechanical, Material, Aerospace and so on, all over the world.

On the other hand, the highly flexible design of composites allows attractive prescribed tailoring of material properties, fitted to the engineering requirements for a wide range of engineering and industrial applications; from advanced aerospace and defence systems to traditional structural strengthening techniques, and from large scale turbines and power plants to nanoscale carbon nanotubes (CNTs) applications. Despite excellent characteristics, composites suffer from a number of shortcomings, mainly in the form of unstable cracking which can be initiated and propagated under different production imperfections and service circumstances. Therefore, the study of the crack stability and load bearing capacity of these types of structures, which directly affect the safety and economics of many important industries, has become one of the important topics of research for the computational mechanics community.

This text is dedicated to discussing various aspects of the application of the extended finite element method for fracture analysis of composites on the macroscopic scale. Nevertheless, most of the discussed subjects can be similarly used for fracture analysis of other materials, even on microscopic scales. The book is designed as a textbook, which provides all the necessary theoretical bases before discussing the numerical issues.

The book can be classified into four parts. The first part is dedicated to the basics. The introduction chapter provides a general overview of the problem in hand and summarily reviews available analytical and numerical techniques for fracture analysis of composites. The second chapter deals with the basics of the theory of elasticity, and is followed by discussions on asymptotic solutions for displacement and stress fields in different fracture modes, basic concepts of stress intensity factors, energy release rate, various forms of the J contour integral and mixed mode fracture criteria.

The second part, Chapter 3, is a redesigned and completed edition of the same chapter in my previous book, and presents a detailed discussion on the extended finite element method.

After presenting the basic formulation, the chapter continues with three sections on available options for strong discontinuity enrichment functions, weak discontinuity enrichments for material interfaces, and a collection of several crack-tip enrichments for various engineering applications. It concludes with sample simulations of a wide range of problems, including classical in-plane mixed mode fracture mechanics, cracking in plates and shells, simulation of shear band creation and propagation, self-similar fault rupture, sliding contact, hydraulic fracture, and dislocation dynamics, to assess the accuracy, performance, robustness and efficiency of XFEM.

The main part of the book includes four comprehensive chapters dealing with various aspects of fracture in composite structures. Static crack analysis in orthotropic materials, dynamic fracture mechanics for stationary and moving cracks, inhomogneous functionally graded materials and bimaterial delamination analysis are discussed in detail. After a review of anisotropic and orthotropic elasticity, all chapters begin with a complete discussion of available analytical solutions for near crack-tip fields in the corresponding orthotropic problem, followed by orthotropic mixed mode fracture mechanics and associated forms of the interaction integral. XFEM enrichment functions for each class of orthotropic materials are obtained and numerical issues related to XFEM discretization are addressed. A number of illustrative numerical simulations are presented and discussed at the end of each chapter to assess the performance of XFEM compared to alternative analytical and numerical techniques.

The final part reviews a number of ongoing research topics for orthotropic materials. First, the orthotropic version of the extended isogeometric analysis (XIGA) is presented by briefly explaining the basic concepts of NURBS and IGA methodology and discussing a number of simple isotropic and orthotropic simulations. Then, the newly developed idea of plane strain anisotropic dislocation dynamics is briefly presented and related XFEM formulation and necessary enrichment functions for the self-stress state of edge dislocations are explained. The book concludes with two brief introductory sections on orthotropic biomaterial applications of XFEM and the piezoelectric problems.

I would like to express my sincere gratitude to Prof. T. Belytschko, for his valuable friendly comments and encouraging message after the publication of the first book on XFEM, and to Prof. A.R. Khoei, as a friend, a colleague and a referee with excellent comments and discussions on various subjects of computational mechanics, especially XFEM. In preparing the present book, particularly the first two parts, I have used the available contributions from brilliant research works by many others, and I have done my best to properly and explicitly acknowledge their achievements within the text, relevant figures, tables and formulae. I am much indebted to their outstanding works, and I apologize sincerely for any unintentional failure in appropriately acknowledging them.

My special thanks to many of my former and present M.Sc. and Ph.D. students who have endeavored many aspects of XFEM over the past decade. First, Dr A. Asadpoure, with whom we started to explore XFEM and new orthotropic enrichment functions in 2002. The results for the dynamic fracture of stationary and moving cracks were obtained by Dr D. Motamedi, and Ms S. Esna Ashari developed the orthotropic bimaterial enrichment functions and simulated all the delamination and interlaminar crack problems. Most of the results for FGM problems were prepared by Mr H. Bayesteh, who actively contributed in many other parts of the book. Mr S.S. Ghorashi and Mr N. Valizadeh skillfully developed the XIGA methodology and Mr S. Malekafzali implemented XFEM for anisotropic dislocation dynamics. Other results were obtained by my students: S.H. Ebrahimi, A. Daneshyar, M. Parchei, M. Goodarzi, S.N. Rezaei, S.N. Mahmoudi and M.M.R. Kabiri.

My acknowledgement is extended to John Wiley & Sons, Ltd., Publication for the excellent professional work that facilitated the whole process of publication of the book; in particular to Dr E.F. Kirkwood and E. Willner, D. Cox, R. Davies, L. Wingett, A. Hunt, C. Lim, S. Sharma and Dr R. Whitelock.

The inspiration and power for completing this work have been the love and understanding of my family, as they had to comply with all my commitments. After a life-time engagement in mathematics, physics and engineering, satisfaction is not obtained just in academic or professional progress, novelty and innovation; it should perhaps be sought in ethics, responsibility, love and freedom. This book has been completed at the twilight of a long hard winter, with a hope for a bright flourishing spring of prosperity and freedom to come. I would like to proudly dedicate this work to all spirited noble Iranian students who accomplish academic achievements while challenging for more DOFs!

Soheil Mohammadi
Spring 2012
Tehran, IRAN

Nomenclature

Parameters not shown in this nomenclature are temporary variables or known constants, defined immediately when cited in the text.

α	Curvilinear coordinate
α	First Dundurs parameter
α, β	Newmark parameters
α, β, γ	FGM constants
α_0	Curvilinear coordinate α of an ellipse
α_{lk}	Components of coordinate transformation tensor
β	Curvilinear coordinate
β	Second Dundurs parameter
β_0	Curvilinear coordinate β of an ellipse
β_{d}, β_{s}	Dilatational and shear wave functions
γ	Wedge angle
γ_{s}	Surface energy density
γ_{d}, γ_{s}	Dilatational and shear wave functions
γ_{ij}	Engineering shear strain
δ	Plastic crack tip zone
δ	Variation of a function
$\delta(\xi)$	Dirac delta function
δ_{ij}	Kronecker delta function
δ_{I}, δ_{II}	Local displacements of crack edges
$\boldsymbol{\varepsilon}$	Strain tensor
ε	Oscillation index
ε_{ij}, $\boldsymbol{\varepsilon}_i$	Strain components
$\bar{\varepsilon}_{ij}$	Dimensionless angular geometric function
$\varepsilon_{ij}^{\mathrm{aux}}$	Auxiliary strain components
ε_{o}	Applied displacement loading
$\varepsilon_{\mathrm{yld}}$	Yield strain
ξ	Local curvilinear (mapping) coordinate system

ξ_i	Knot i
ξ_{tip}	Crack-tip position
$\xi(\mathbf{x})$	Distance function
$\xi_{\text{g}}, \eta_{\text{g}}$	Gauss point position along the contour J
η	Local curvilinear (mapping) coordinate system
η	Equivalent inelastic strain
θ	Crack propagation angle with respect to initial crack
θ	Angular polar coordinate
θ_0	Crack angle
$\theta_k, \bar{\theta}_k$	Orthotropic angular functions
$\theta_{\text{d}}, \theta_{\text{s}}$	Dynamic distance functions
κ, κ'	Material parameters
κ_0	Effective material parameter
λ	Lame modulus
λ	Power of radial enrichment
λ	Ratio of orthotropic Young modules E_2/E_1
λ, λ_n	Roots of the characteristic equation
μ, μ_{ij}	Isotropic and orthotropic shear modulus
ν, ν_{ij}	Isotropic and orthotropic Poisson's ratios
$\bar{\nu}$	Average orthotropic Poisson's ratios
ρ	Radius of curvature
ρ	Density
$\boldsymbol{\sigma}$	Stress tensor
σ_0	Applied normal traction
σ_{cr}	Critical stress for cracking
σ_{eff}	von Mises effective stress
σ_{ij}, σ_i	Stress components
$\bar{\sigma}_{ij}$	Dimensionless angular geometric function
σ_{ij}^{aux}	Auxiliary stress components
σ_{yld}	Yield stress
$\sigma_{\theta\theta}$	Hoop stress
τ_0	Applied tangential traction
τ_n	Decohesive shear stress
$\phi(\mathbf{x})$	Level set function
$\phi(z)$	Complex stress function
φ	Angle of orthotropic axes
φ	Crack angle
φ	Ramp function for transition domain
φ	Electric potential

$\chi_m(\mathbf{x})$	Enrichment function for weak discontinuities
$\chi(z)$	Complex stress function
ψ	Friction coefficient
ψ	Phase angle
$\psi(\mathbf{x})$	Enrichment function
$\psi(\mathbf{x}, t)$	Level set function
$\psi(z)$	Complex stress function
Γ	Boundary
Γ_1	Infinitesimally small internal contour
Γ_c	Crack boundary
Γ_t	Traction (natural) boundary
Γ_u	Displacement (essential) boundary
Δ	Finite variation of a function
Δt	Time-step
Δa	Crack length increment
$\Lambda_i(t)$	Time interval shape functions
Ξ	Knot vector
Π	Potential energy
Φ	Airy stress function
$\Phi_j(\mathbf{x})$	MLS shape functions
$\Phi_i(z_i)$	Complex functions
Ω	Domain
Ω_1, Ω_2	Non-overlapping subdomains
Ω_{pu}	Domain associated with the partition of unity
Ψ_H^α	Dislocation glide enrichment
$(1, 2)$	Material axes
a	Crack length/half length
a	Semi-major axis of ellipse
$\bar{a}$	Effective crack length
$\mathbf{a}(\mathbf{x})$	Vector of unknown coefficients
$\mathbf{a}, \mathbf{a}_h$	Heaviside enrichment degrees of freedom
$\mathbf{a}_i, \mathbf{a}_k$	Enrichment degrees of freedom
$\mathbf{a}^{enr}$	Enrichment degrees of freedom
A	Area associated with the domain J integral
A_1	Area inside the infinitesimally small internal contour Γ_1
A^+, A^-	Area of the influence domain above and below the crack
A_i, A_{ij}	Coefficients
b	Width of a plate
b	Semi-minor axis of ellipse

$\mathbf{b}_k$, $\mathbf{b}_k^l$	Crack tip enrichment degrees of freedom
$\mathbf{b}^\alpha$	Burgers vector for dislocation α
b_α	Magnitude of the Burgers vector for dislocation α
b_n	Series coefficients
$\mathbf{B}$	Matrix of derivatives of shape functions
B_{12}, B_{66}	Coefficients of characteristic equation
$B_{i,p}(\xi)$	B-spline basis function of order p
$\mathbf{B}^h$	Matrix of derivatives of final shape functions
$\mathbf{B}_i^r$	Strain-displacement matrix (derivatives of shape functions)
$\mathbf{B}_i^{\mathbf{u}}$	Matrix of derivatives of classical FE shape functions
$\mathbf{B}_i^{\mathbf{a}}$	Matrix of derivatives of Heaviside enrichment shape functions
$\mathbf{B}_i^{\mathbf{b}}$	Matrix of derivatives of crack tip enrichment shape functions
$\mathbf{B}_i^{\mathbf{c}}$	Matrix of derivatives of weak discontinuity enrichment shape functions
$\mathbf{B}_i^{\mathbf{c}}$	Matrix of derivatives of transition shape functions
c	Dugdale effective crack length
c_J	Size of crack tip contour for J integral
c_{ij}	Material compliance constants
$\mathbf{c}_{\mathrm{m}}$	Degrees of freedom for weak discontinuity enrichment
$\mathbf{c}_{\mathrm{m}}$	Degrees of freedom for transitional enrichment
c_{d}	Dilatational wave speed
c_{L}	Wave speed along the loading axis
c_{R}	Rayleigh speed
c_{s}	Shear wave speed
C	Material constitutive matrix
C	4$^{\text{th}}$ order material compliance tensor
C_{ijkl}	Cartesian components of C
C_n	Coefficient
$\mathbf{C}(\xi)$	NURBS curve
d_{ij}	Material modulus constants
d_{ij}^d	Dynamic material modulus constants
$\mathbf{d}_{\mathrm{m}}$	Degrees of freedom for transitional enrichment
D	Dynamic function
$\mathbf{D}$	Two dimensional Material modulus matrix
D	4$^{\text{th}}$ order material elasticity modulus tensor
D_{ijkl}	Cartesian components of D
$\mathbf{D}_{\alpha\beta}$	Components of $\mathbf{D}$
D_i, D_x, D_y	Elastic displacement vector
E, E_i	Isotropic and orthotropic Young's modules
E_i, E_x, E_y	Electric field

E'	Effective material parameter
E^0	Reference Young modulus
E^{12}	Equivalent bimaterial elastic modulus
$\bar{E}$	Average orthotropic Young modules
$f_k(\mathbf{x})$	Set of PU functions
$\mathbf{f}$	Nodal force vector
$\mathbf{f}_i^r$	Nodal force components (classic and enriched)
$\mathbf{f}^{\mathrm{b}}$	Body force vector
$\mathbf{f}^{\mathrm{t}}$	External traction vector
$\mathbf{f}^{\mathrm{c}}$	Cohesive crack traction vector
$\mathbf{f}^{\mathrm{ext}}$	External force vector
$f_{\mathrm{I}}, f_{\mathrm{II}}$	Functions of the crack-tip speed
$f_{\mathrm{I}}^{\mathrm{d}}, f_{\mathrm{II}}^{\mathrm{d}}$	Universal functions of the crack-tip speed
$f_{ij}^{\mathrm{I}}, f_{ij}^{\mathrm{II}}, f_{ij}^{\mathrm{III}}$	Mode I, II and II angular functions
f_k^{pu}	Set of PU functions
$F_l(\mathbf{x}), F_\alpha(r, \theta)$	Crack tip enrichment functions
$F(\sigma, \alpha)$	Delamination function
F_{ij}	Deformation gradient
$g(\theta)$	Angular function for a crack-tip kink problem
$g_j(\theta), \bar{g}_k(\theta)$	Orthotropic crack-tip enrichment functions
G	Shear modulus
$G, G(\theta)$	Fracture energy release rate
G_c	Critical fracture energy release rate
G_1, G_2	Mode I and II fracture energy release rates
h	Intrinsic shear band thickness
h_t	Characteristic thickness of the bonding layer
H	Slope of linear softening curve
$\bar{H}$	Intrinsic hardening coefficient
$H(\xi), H(\mathbf{x})$	Heaviside function
i	Complex number $\mathrm{i}^2 = -1$
$\mathbf{I}$	2^{nd} order identity tensor
I	4^{th} order symmetric identity tensor
$I(t)$	Corresponding creep compliance
$\mathbf{J}$	Jacobian matrix
J, J^s	J integral
J^{act}	Actual J integral
J^{aux}	Auxiliary J integral
J_1, J_2	Components of the J vector
J_k^{d}	Dynamic J integral

k_0	Dimensionless constant for the power hardening law
K	Bulk modulus
$\mathbf{K}$	Stiffness matrix
$\mathbf{K}_{ij}^{\mathrm{rs}}$	Components of stiffness matrix
K	Stress intensity factor
$K, \bar{K}$	Complex stress intensity factor
K_{c}	Critical stress intensity factor
K_0	Reference stress intensity factor
$K_{\mathrm{I}}, K_{\mathrm{II}}, K_{\mathrm{III}}$	Mode I, II and III stress intensity factors
$\bar{K}_{\mathrm{I}}, \bar{K}_{\mathrm{II}}$	Normalized mode I and mode II stress intensity factors
$K_{\mathrm{I}}^{\mathrm{aux}}, K_{\mathrm{II}}^{\mathrm{aux}}$	Auxiliary mode I and mode II stress intensity factors
$K_{\mathrm{Ic}}, K_{\mathrm{IIc}}$	Critical mode I and mode II stress intensity factors
$K_{\mathrm{Ic}}^1, K_{\mathrm{Ic}}^2$	Fracture toughnesses along the principal planes of elastic symmetry
K_{Ic}^{θ}	Fracture toughness at propagation
$K_{\mathrm{Ic}}^{\mathrm{d}}$	Dynamic crack initiation toughness
$K_{\mathrm{Ic}}^{\mathrm{D}}$	Dynamic crack growth (propagation) toughness
$K_{\theta\theta}, K_{tt}$	Hoop stress intensity factor
l_{ij}	Coefficient
L	Length of the singular element
$L(v_c)$	Dynamic matrix for orthotropic materials
m	Number of enrichment functions
$m\mathrm{t}$	Number of nodes to be enriched by crack-tip enrichment functions
$m\mathrm{h}$	Number of nodes to be enriched by Heaviside enrichment functions
$m\mathrm{f}$	Number of crack-tip enrichment functions
$m\mathrm{m}$	Number of weak discontinuity enrichment functions
$m\mathrm{st}$	Number of transition enrichment functions 1
$m\ \mathrm{sh}$	Number of transition enrichment functions 2
m_k	Roots of characteristic equation $m_k = m_{kx} + im_{ky}$
M	Concentrated bending moment
M	Interaction integral
$M^{(1)}, M^{(2)}$	Interaction integral associated with two modes I and II
M^{d}	Dynamic interaction integral
$\mathbf{M}$	Mass matrix
$\mathbf{M}_{ij}$	Components of mass matrix
n	Power number for the HKK plastic model
n	Number of nodes for each finite element
ng^A	Number of gauss points inside contour area A
ng^{Γ}	Number of gauss points on contour Γ
$n\mathrm{p}$	Number of independent domains of partition of unity

n_{n}, n_{nodes}	Number of nodes in a finite element
n_{e}, $n_{\text{elem.}}$	Number of finite elements
n_{cp}	Number of control points
n_{cells}	Number of background cells of EFG
n_{DOFs}	Number of degrees of freedom
$\mathbf{n}$	Normal vector
$\mathbf{N}_j$	Matrix of shape functions
N_j	Shape function
N_{elements}	Number of finite elements
N_{nodes}	Number of nodes
$N_{\text{enrich.}}$	Number of enrichment functions
N_{DOFs}	Number of degrees of freedom
$\bar{N}_i$	Hierarchical shape functions for the transition domain
$\bar{N}_j$	New set of GFEM shape functions
$p(\mathbf{x})$	Basis function
p	A point on curvilinear coordinate system $p = \alpha + i\beta$
p, p_k, $\bar{p}_k$	Orthotropic parameters
$\boldsymbol{p}$	Basis function
$\boldsymbol{p}^{\text{enr}}$	Enrichment basis function
$\boldsymbol{p}^{\text{lin}}$	Linear basis function
p_k	k-th basis function
$p^l(\mathbf{x})$	l-order polynomial function
P	Concentrated force
$\boldsymbol{P}$	External load vector
P_{cr}	Critical load
q	Arbitrary smoothing function
q, q_k, $\bar{q}_k$	Orthotropic parameters
q_i	Nodal values of the arbitrary smoothing function
(r, θ)	Local crack tip polar coordinates
r_J	Radius of J integral contour
r_{d}, r_{s}	Dilatational and shear distance functions
r_{l}, r_{s}	Orthotropic distance functions
r_{p}, r_{p1}, r_{p2}	Crack tip plastic zone
R	Ramp function
R_K	Ratio of dynamic stress intensity factors
R_δ	Ratio of opening to sliding displacements
$R_i^p(\xi)$	NURBS function of order p
s	Roots of characteristic equation $s = s_1 + \text{i}s_2$
s_k	Roots of characteristic equation $s_k = s_{kx} + \text{i}s_{ky}$

$\bar{s}_k$	Roots of characteristic equation $\bar{s}_k = \bar{s}_{kx} + \mathrm{i}\bar{s}_{ky}$
s_m	Roots of characteristic equation $s_m = s_{m1} + \mathrm{i}s_{m2}$
$\bar{S}_\mathrm{n}$	Slope of softening curve
S_{ij}	Material constants
$\mathbf{S}(\xi_1, \xi_2)$	NURBS surface
t	Time
$\mathbf{t}$	Traction
$\mathbf{t}_h$	Unit vector for tangential direction
t_{ij}	Material function
t_0	Time for the wave to reach the crack tip
t_p	FRP thickness
$T_i(t)$	Time shape functions
T_j	Enriched time interval
T_j	Transformation matrix
$\mathbf{T}_\mathrm{i}$	Control points
$\mathbf{u}$	Displacement vector
$\dot{\mathbf{u}}$	Velocity vector
$\bar{\mathbf{u}}$	Prescribed displacement
$\bar{\dot{\mathbf{u}}}$	Prescribed velocity
$\ddot{\mathbf{u}}$	Acceleration vector
$\mathbf{u}_i$	Displacement field component
$\dot{\mathbf{u}}_i$	Velocity field component
$\ddot{\mathbf{u}}_i$	Acceleration field component
$\mathbf{u}_i^{\mathrm{aux}}$	Auxiliary displacement field component
$\dot{\mathbf{u}}_i^{\mathrm{aux}}$	Auxiliary velocity field component
$\mathbf{u}^{\mathrm{enr}}$	Enriched displacement field
$\mathbf{u}^{\mathrm{FE}}$	Classical finite element displacement field
$\mathbf{u}^{\mathrm{XFEM}}$	XFEM displacement field
$\mathbf{u}^{\mathrm{tra}}$	Transition enrichment part of the displacement field
$\mathbf{u}^{\mathrm{H}}$	Heaviside enrichment part of the displacement field
$\mathbf{u}^{\mathrm{tip}}$	Crack-tip enrichment part of the displacement field
$\mathbf{u}^{\mathrm{mat}}$	Weak discontinuity enrichment part of the displacement field
$\mathbf{u}_{\mathrm{tip}}^{\mathrm{Enr}}(\mathbf{x}, t)$	Crack-tip part of the approximation
u_n^h	Displacement at time n
$\dot{u}_n^h$	Velocity at time n
$\ddot{u}_n^h$	Acceleration at time n
$\mathbf{u}^h$, $\mathbf{u}^h(\mathbf{x})$	Approximated displacement field
$\dot{\mathbf{u}}^h$	Approximated velocity field
$\ddot{\mathbf{u}}^h$	Approximated acceleration field

$\bar{\mathbf{u}}_j$	Nodal displacement vector
$\bar{\mathrm{u}}_{ij}$	Displacement angular functions
u_x, u_y, u_z	*x, y* and *z* displacement components
$u_{x'}, u_{y'}$	Local displacements of the nodes along the crack in the singular element
U^I, U^{II}	Symmetric and antisymmetric crack tip displacements
U_s	Strain energy
$U_\mathrm{s}^\mathrm{e}, U_\mathrm{s}^\mathrm{p}$	Elastic and plastic strain energies
U_Γ	Surface energy
v_c	Crack-tip velocity
$\mathbf{v}$	Velocity vector
$\mathbf{v}^\mathrm{c}$	Classical velocity DOFs
$\mathbf{v}^e$	Additional velocity DOFs
$\mathbf{V}(t)$	Vector of approximated velocity degrees of freedom
V	Volume
W	External work
w^aux	Auxiliary work
W^ext	Virtual work of the external loading
W_g	Gauss weight factor
W_i	Weights associated with each control point i
W_g^Γ	Gauss weighting factor for contour Γ
W_g^A	Gauss weighting factor for area inside contour integral J
W^int	Internal virtual work
w^M	Interaction work
w_d	Kinetic energy density
w_s	Strain energy density
$W_t(t)$	Time weight function
x, y, z	Cartesian coordinates
(X, Y)	Global coordinate system
(X_1, X_2)	Global coordinate system
$\mathbf{x}$	Position vector
$\mathbf{x}_\mathrm{c}$	Position of crack or discontinuity
$\mathbf{x}_\mathrm{tip}$	Position of the crack tip
$\mathbf{x}_\Gamma$	Position of the projection point on an interface
(x_1, x_2)	2D coordinate system
(x_1, x_2)	Material axes
(x', y')	Local crack tip coordinate axes
z	Complex variable $z = x + \mathrm{i}y$
$\bar{z}$	Conjugate complex variable $\bar{z} = x - \mathrm{i}y$
z_i	Complex parameters

$\frac{\mathrm{d}}{\mathrm{d}t}$	Time derivative
$\frac{\mathrm{D}}{\mathrm{D}t}$	Material time derivative
$\dot{f}$, $\ddot{f}$	The first and second temporal derivatives of a function
f', f''	The first and second spatial derivatives of a function
$\bar{f}$, $\bar{\bar{f}}$	The first and second integrals of a function
$\nabla = {}^{\partial}\!/_{\partial x}$	Nabla operator
$\langle\rangle$	Jump operator across an interface
$:$	Inner product of two second order tensors
$\otimes$	Tensor product of two vectors
Re	Real part of a complex number
Im	Imaginary part of a complex number
$\propto$	Proportional
BEM	Boundary element method
CAD	Computer aided design
CNT	Composite nanotube
COD	Crack opening displacement
CTOD	Crack-tip opening displacement
DCT	Displacement correlation technique
DEM	Discrete element method
DOF	Degree of freedom
EDI	Equivalent domain integral
EFG	Element free Galerkin
ELM	Equilibrium on line
EPFM	Elastic plastic fracture mechanics
FDM	Finite difference method
FE	Finite element
FEM	Finite element method
FGM	Functionally graded materials
FMM	Fast marching method
FPM	Finite point method
FRP	Fibre reinforced polymer
GFEM	Generalized finite element method
GNpj	Generalized Newmark approximation of degree p for equations of order j
HRR	Hutchinson-Rice-Rosengren
IGA	Isogeometric analysis
LEFM	Linear elastic fracture mechanics
LSM	Level set method
MCC	Modified crack closure
MLPG	Meshless local Petrov Galerkin

MLS	Moving least squares
NURBS	Non-uniform rational B-splines
OUM	Ordered upwind method
PU	Partition of unity
PUFEM	Partition of unity finite element Method
RKPM	Reproducing kernel particle method
SAR	Statically admissible stress recovery
SIF	Stress intensity factor
SPH	Smoothed particle hydrodynamics
SSpj	Single step approximation of degree p for equations of order j
STXFEM	Space time extended finite element method
TXFEM	Time extended finite element method
WLS	Weighted least squares
XFEM, X-FEM	Extended finite element method
XIGA	Extended isogeometric analysis