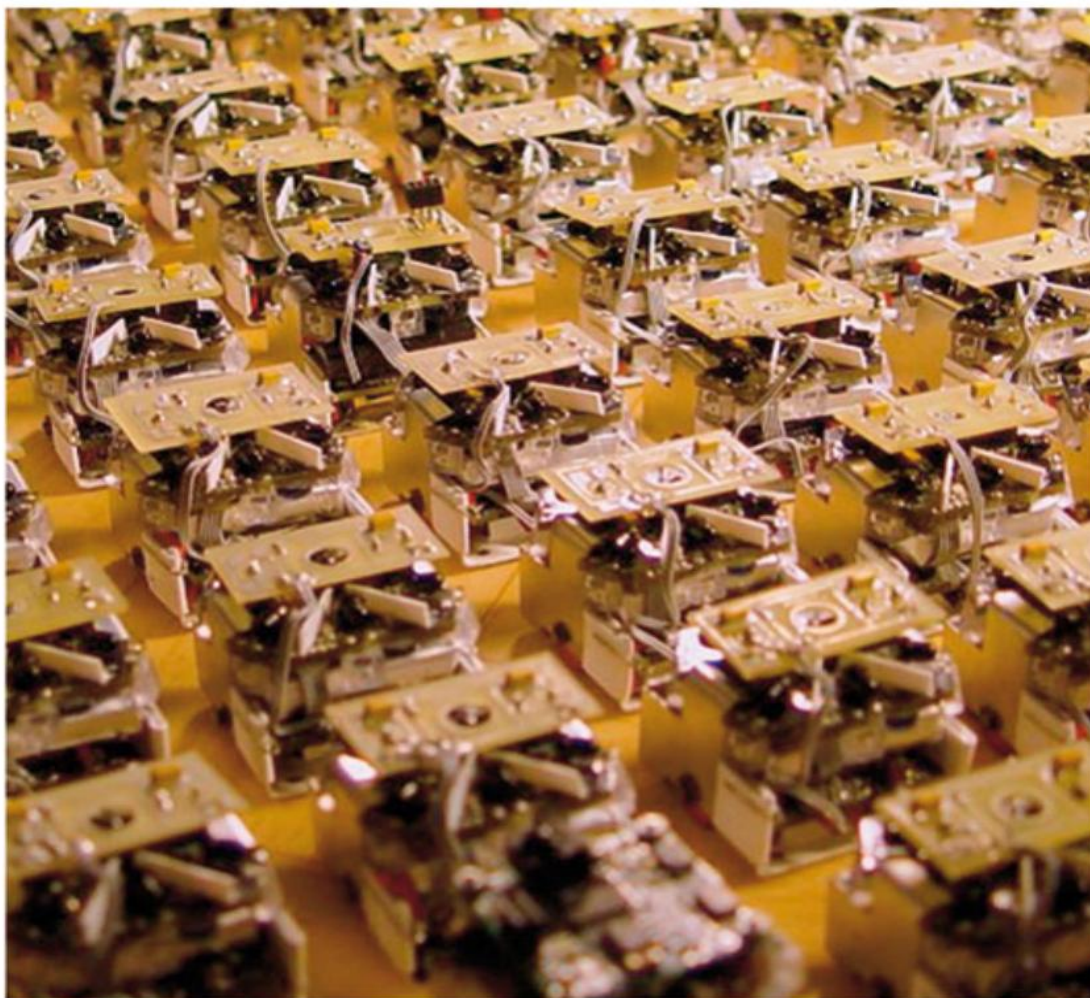


Hermann Haken, Paul Levi

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Synergetic Agents

From Multi-Robot Systems to Molecular Robotics



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Preface

This book presents entirely new vistas in the following two disciplines:

- 1.** For the first time, it applies basic principles of synergetics - the science of cooperation - to multirobot systems.
- 2.** It applies a modern method developed for active, nonequilibrium quantum systems to *molecular* robots - a rapidly developing, fascinating field within nanoscience and technology.

In both cases (1) and (2), this book deals with *active units*, that is, robots or molecules, capable of forming spatiotemporal structures or collective action based on *cooperation*. In other words, it deals with *synergetic agents*.

In order to reach a broad audience, it is written in a pedagogical style that will allow even nonspecialists to acquaint themselves with our approach. (A few more technical sections are marked by asterisk.)

In fact, both fields, that is, multirobot systems and molecular robots have become highly interdisciplinary endeavors that comprise disciplines such as robotics, mechanical and electrical engineering, physics, informatics, chemistry, biology, medicine, mathematics, and other fields. Our book applies to graduate students, professors, and scientists. Though occasionally we refer to experiments, our emphasis is laid on theoretical approaches. Among our numerous results are

- the Haken-Levi theorem in its classical and quantum mechanical formulation relating robot motion to probability distribution;
- a whole chapter presenting our quantum theory of muscle contraction based on actin-myosin interaction;

- a detailed quantum theoretical model of the motion of molecular robots.

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Stuttgart, April 2012

Hermann Haken
Paul Levi

Prologue I: Synergetic Agents: Classical

Self-Organization in Collective Systems

Collective systems in technique and biology move more and more into the focus of basic research in the fields of natural science such as physics, biology, chemistry, and engineering science such as mechanical engineering, computer science, cybernetics, and robotics. In biology, swarms of bees or ants, flocks of fish or birds, and networks of natural neural networks such as those realized in different kinds of brains demonstrate very impressively the power and the abilities of such collective systems. These advantages are also expected in technical collective systems like cooperative production systems, distributed traffic managements, all facets of internet, and last but not least swarms of mobile robots (on land, in water, and in air). What are the dominant features of natural and artificial collective systems that are so fascinating for all of us?

The two basic scientifically most relevant features of all these aforementioned collaborative systems that immediately catch our eye are the ability to create distributed intelligence (meaning the emergence of intelligence; the whole is more than the sum of all parts) and the competence of self-organization (Levi and Kernbach, 2010). In addition to the dominant property of an “intelligent” collectivity generated by self-organization is the increase of adaptation, of reliability, of flexibility, of self-development, and of self-healing.

In standard artificial collective systems, for example, in traffic management, the intelligence is brought into the system by engineers, but the interplay that creates the self-organization and all the other complementary features like reliability is still very important for inanimate or artificial systems. A very prominent example of a self-organized technical system that is constructed by physicists and engineers is the laser. It demonstrates very clearly the phase transition from noncoherent light of a lamp to coherent light of a laser by self-organization.

The basic concept to define and to implement self-organization is given by the methods of synergetics (Haken, 2004). It is the theory of the cooperation of parts of a system that generate by themselves an “order parameter field” that in turn exerts a strong feedback to its many originators (circular causality). In this book, the parts of a collective system are mainly (but not exclusively) inanimate units.

All features of an intelligent and self-organized system are the result of the cooperative interplay between the global structure (organization principle) of the system, the behavior of the individual units, and the different functionalities “generated” by individual members. As an example of this assertion, consider a swarm of bees. Relevant questions are here: how they organize their foraging, how they perform their navigation and exploration tasks, how they do the foraging, how they distribute the collected nutrition, how they regulate the homeostatic mechanisms, and so on?

The transfer of these features and behaviors to inanimate, artificial swarms of robots, which was mentioned before, is primarily motivated by the approach to get answers to these basic, biological questions by picking up these questions of animated systems and trying to get relevant responses by technical systems. In view of classical artificial

intelligence (AI) and its more philosophically based connection to cognitive science that are both characterized by a top-down approach we will present in this book, the new bottom-up approach of collective robotics (Pfeifer and Scheier, 1999) starts from the microscopic parts (e.g., robots) and studies the emergence of intelligence, self-organization, and cognition, for example, in a swarm or even in an organism that is generated out of such a swarm (Flozano and Mattiussi, 2008; Levi, 2009; Siciliano and Khatib, 2008).

Such a distinction between a swarm mode (phase) and an organism mode offers the possibility to analyze the essential features of a part that are inevitable to generate an intelligent swarm (e.g., swarms of house flies never show swarm intelligence). What is different if a swarm is going together in order to build an organism (may be considered a morphogenetic phase transition)? What features of a swarm member are changed if it “mutates” to a “cell” of an organism? How do these new “cells” differentiate themselves to different organs or parts of an organism? Such questions are considered in the so-called symbiotic collective robotics (Levi and Kernbach, 2010). Swarm behavior is also very beneficial in soccer games, for example, in RoboCup, where the robots are no longer small robot cells (about 5 cm^3) but have a bigger volume of about 20 cm^3 (Rajaie *et al.*, 2011).

Besides these basic questions of swarm mode and organism mode, the bottom-up approach in robotics is characterized by the so-called “embodiment,” meaning that there can be no intelligence and cognition if there is no body (matter) available; intelligence and cognition require a body.

This statement is augmented by the concept of “situatedness,” denoting that each part of such a system can acquire information about the current situation under given environmental conditions, perform an individual

interpretation of the existing situation (e.g., by pattern recognition), and finally it makes an individual decision concerning its next activities. The bodies and the situations can be simple or complex. According to the bottom-up approach that is accomplished by our approach, we consider as the first step simple bodies and nonsophisticated situations. In order to complete the two strongly interwoven concepts of “embodiment” and “situatedness,” we include in our approach the additional concept of an agent. This is an active part of a whole system (the so-called multiagent system, MAS) that is afflicted with a corpus, is autonomous, and is aware of situations (Weiss, 1999). An agent realizes internally the two concepts of “embodiment” and “situatedness,” and it is able to learn. An agent represents a basic concept of robotics and artificial intelligence.

In this book, we create the concept of a synergetic agent. This is an agent that uses internally the methods of synergetics to calculate the sensor-based acquired information and comes to appropriate decisions and actions in response to the calculated information (situation description). The correct information handling is the engine of progress of the interplay between the theory of synergetics - here we mean especially the circular causality of self-organization, the emergence of new qualities by nonequilibrium phase transitions in open systems, and reduction of complexity - and the theory of the emergence of cognition and intelligence of an agent that finally is condensed in intelligent decisions.

The construction and design principles for synergetic agents are based on those of informatics-based agents, but they must be dominantly extended by a paradigm change in physical descriptions of synergetic processes and by a new principle of information.

The paradigm change can very clearly be explained if we compare the classical laser paradigm (a nearly inexhaustible

source of inspiration) and a multirobot system (MAS), be it in swarm or in organism mode. The most dominant commonality is, for example, the “circular causality”: the participating parts generate one or more “order parameter fields” that operate recurrently and therefore “enslave” the originating parts. Another important commonality is the supplement of the principal coupling to the environment. Here, the following effects have to be considered: damping, fluctuations, and dissipation processes of open systems.

The essential difference is that all atoms that generate the coherent electromagnetic field (order parameter field) are passive and are neither intelligent nor situated, nor able to learn. Robots also obey equations of motion, but their real movement must be generated by controllers for steering (Shen *et al.*, 2004), where an internal force that mimics an external force constrains them to move on a prescribed trajectory. Such controllers have to consider details of the transaction type (type of drive system) and details of the properties of the underground (e.g., land or water) and unforeseen situations (like obstacles or holes). As a result, the “cognitive” decision making of a robot (realized as an agent) generates the appropriate response to unforeseen situations (Levi, 2010). A good example for this latter statement is the kind of response of a soccer robot if it is attacked by one or more robots of the opponent team. This decision is highly influenced by the learned team strategy (how to play the game and if possible to win).

An important supplement for every kind of motion is the coupling to environment. In a classical physical approach, these are the effects of damping, fluctuations (noise), and dissipation. But for mobile robots, we have also to consider new and different types of uncertainties. These are failures in sensor data, aged sensor data, and incorrect steering statements (more generally spoken: degraded information). The correct handling of such degraded information (also

including trustworthiness in information source) demands implementation of cognitive processes. In human decision making as part of a cognitive process, the anchoring bias is an example of a dominant focus on a trait of information that is degraded (Kahneman and Tversky, 1996).

This kind of cognitive response has clearly to be distinguished from the elementary stimulus-response cycle that occurs very often on lower levels in biological systems (e.g., consider the Braitenberg vehicle (Braitenberg, 1984)).

In view of information theory, the reaction on unexpected situations is dictated by a minimum of individual information. As bigger the surprise concerning an event the smaller is the probability for this event or for other features like anchoring.

The close connection between the acquired information of an agent and the reaction to this information is formulated by the Haken-Levi information principle.

Each individual synergetic agent minimizes its local information.

$$(I.1) \quad i_s(\text{agent}) = -\ln p_s(\xi_s, \xi_u),$$

where p_s is the joint probability of the value ξ_s of the variable of agent "s" and of the value ξ_u of the order parameter of the whole system.

This implies that each individual synergetic agent disposes of information that regulates already the feedback caused by the circular causality already mentioned.

This relationship is most clearly expressed when we use the relation

$$(I.2) \quad p_s(\xi_s, \xi_u) = p_u(\xi_u) p_s(\xi_s | \xi_u),$$

where p_u is the probability distribution of the order parameter that is *collectively* generated by all the agents of the system, and $p_s(\xi_s | \xi_u)$ the *conditional* probability that the enslaved variable ξ_s acquires that value provided the value ξ_u

is given. By using (I.1) and (I.2), we define the conditional information $i_{s,c}$ of agent s :

$$(I.3) \quad i_{s,c} = -\ln p_s(\xi_s | \xi_u).$$

If we consider the whole system (e.g., let it be an organism assembled by robot cells), the total system information is maximized. This means the expectation value of all individuals, where information is maximized

$$(I.4) \quad \text{Information } i(\text{system}) = i_u + \sum_s \langle i_s \rangle,$$

where

$$(I.5) \quad i_u = -\sum_{\xi_u} p_u \ln p_u$$

is the information of the order parameter and

$$(I.6) \quad \langle i_s \rangle = \sum_{\xi_u, \xi_s} p_s(\xi_s, \xi_u) i_{s,c}$$

the expectation value of $i_{s,c}$ (I.3). (Haken, 2006).

By means of the local information (I.1), we may express the equation of motion (or more generally the behavior) of an agent (a robot vehicle):

$$(I.7) \quad m \ddot{\xi}_s + \gamma \dot{\xi}_s = -(Q/\gamma) \nabla i_s + F_s(t).$$

(This is a special case of the H-L principle). In (I.7), ξ_s is - in general - the three-dimensional position vector, m the mass of the robot, γ a damping constant, Q the strength of the random force $F_s(t)$ acting on the robot, and ∇ the nabla operator. (For details, cf. Chapter 2.)

There are three main effects attributable to agents. First, we can formulate the circular causality of self-organization by the combined application of the H-L principle to each individual robot and to the system of all robots. Second, we can formulate the equations of motion for one robot or for all robots by the calculation of the gradient of information. Third, we can store the individual and total information gain by the calculation of the Kullback measure. If we perform this calculation by iteration and store each information gain

or information loss, then we have implemented a dedicated method to learn.

At first sight, [Equation \(1.7\)](#) might look like a simple rewriting of the equation of motion of a robot agent (as part of a multirobot system), namely, instead of using a potential function $v(\xi_s)$ directly, we write it in a somewhat disguised form. In other words, [\(1.7\)](#) seems to rest on some tautology. In mathematics, tautologies are surely not a crime; rather the individual steps used there are just a sequence of tautologies! In the present case, the situation is different, however. First of all, the concepts of the equations of (mechanical) motion and of information in its scientific, mathematical form stemming from information theory originate from two conceptually quite different scientific disciplines. Thus, [\(1.7\)](#) provides us with a qualitatively new insight. As a consequence, we may interpret and use information i under entirely new aspects. Namely, in practice, a robot must *acquire* the appropriate information by its own activities and rather limited preprogramming. Since it does not “know” the positions of the other robots and objects (e.g., obstacles) beforehand, it must measure their relative directions and distances. It then has to attribute to these quantities appropriate artificial potentials. To this end, it has to distinguish between other robots, obstacles, and attractive objects (e.g., energy sources). In specific situations, for example, soccer games, it must distinguish between friend and foe. All these cases require specific preprogrammed potentials (leaving aside aspects of robot learning and evolution).

As we shall see in detail, for instance, when we study docking maneuvers, the robot information may switch from one kind of information to another, depending on the situation. To mention a simple example, the information may switch from the use of one potential function to another one.

Clearly, higher order programs may also be installed in the expression for the information. We will discuss some examples in our book, for example, the self-organized formation of letters by suitable configurations of robots.

Let us discuss how the robot uses the instructions enfolded in the information $i_s(\xi_s, \xi_w)$, (1.1), or, in other words, how it unfolds its information. In principle, it may solve its equation of motion according to (1.7) and use a control mechanism to secure the realization of the wanted motion. In practice, the situation is quite different, at least in general in a multirobot system. First of all, to calculate its future path, the robot must be informed on the future paths of all other robots and vice versa. This requires the action of a “master” computer of very high capacity outside the multirobot system.

In a swarm situation, such a procedure is not possible at all and contradicts the principle of *self-organization*. The practical procedure must be quite different. Based on its measurements of distances and directions to all other objects (including the other robots), the robot under consideration acts on its actuators from moment to moment in such a way that for a *given, measured* value of the r.h.s. of (1.7) the robot accelerates or decelerates, including damping. Because the robot relies on the measured r.h.s., it can even act if slip is present. It may follow its path, though with some *time delay*. In this way, the artificial potential appears as an evaluation function of the quality of reaching the robot's goal.

As we know from the theory of swarms, an essential ingredient of their collective behavior is the requirement that each individual keeps a mean distance to all its nearest neighbors.

There is yet another aspect to our approach: The whole system altogether acts as a *parallel computer* (in contrast to a sequential computer). All its components (the agents!) collect their information in parallel and act in parallel. This

information acquisition may be active (e.g., measuring distances to objects) or both active and passive (e.g., communication among agents in collective pattern recognition; see Chapter 3). We believe that our information-based approach opens new vistas to dealing with multirobot or, more generally, multiagent systems. For readers who wish to learn more about the scientific concept of information, we include the following section.

The Tricky Concept of Information (Shannon)

“Information” in ordinary sense is a message (e.g., birth of a child, accident, winning of an election, etc.), an instruction, a set of data, and so on. In more technical terminology, information is essentially a measure of probabilistic uncertainty (not of principal uncertainty, for example, in quantum mechanics (Genz, 1996)). In terms of the discipline of stochastics, the appropriate methodological terms are stochastic events (e.g., unexpected obstacle during an exploration tour of a mobile robot), stochastic variables (e.g., set of sensor data), and stochastic “functions” (e.g., instructions and algorithms). In our book, “information” is a *terminus technicus* that allows a quantitative treatment in terms of the three aforementioned basic definitions. However, the meaning of information is often not very clearly defined, and we will try in the first step to elucidate this meaning before we present it as a useful concept in robotics.

Let us start with the first step by explaining Shannon information ((Shannon, C.E., 1948), (Shannon, C.E., Weaver, W., 1949)) (originally conceived as a measure of the capacity of data transmission channels). We begin our “explanation route” by a set of discrete events labeled by

an index λ , where N is a fixed number. Typical examples of such events are tossing of a coin that yields the two events head or number, rolling a die with six outcomes (i.e., events are $\lambda = 1, \dots, 6$). A more sophisticated example that is for our wanted robot applications more illustrative is the exploration tour of mobile robots in an unknown environment as a task that is a typical part of probabilistic robotics (Thrun, Burgard, and Fox, 2005). This new methodology imposes weaker restrictions on the accuracy (greater uncertainty) of sensor data than the classical deterministic interpretation of measurement data. Typical events during such an exploration tour are the emergence of unexpected obstacles, possibilities of several navigation paths with different lengths (stochastic variable), and the stability (robustness) of the internal power supply (stochastic function in the sense of a homeostasis).

We consider now a very frequent repetition of trials. The probability (frequency) of outcome of event λ is defined by

$$p_\lambda = \frac{\text{number of positive outcomes}}{\text{number of all possible trials}},$$

where we require the normalization of the distribution function $p = (p_1, p_2, \dots, p_N)$ by

$$\sum_\lambda p_\lambda = 1.$$

The information that a positive occurrence of an individual event delivers is called information of event λ and is defined by

$$(1.8) \quad i_\lambda = -\ln p_\lambda.$$

We can also use \log_2 instead \ln because $\log_2 = c \ln$, where $c = \log_2 e$, and both logarithmic expressions differ only by a constant factor c .

Shannon defined information as the expectation value of all individual information

$$(1.9) \quad i_s(p) = -\sum_\lambda p_\lambda \ln p_\lambda = \sum_\lambda p_\lambda i_\lambda.$$

Formula [\(1.9\)](#) calculates information as a measure of stochastic uncertainty. This term is also called information entropy. The reason for this other naming is the fact that [\(1.9\)](#) is the same mathematical expression as it is used in thermodynamics for entropy. Therefore, von Neumann suggested to Shannon not to use two different names for the identical formula. Today, this argument is no longer fully accepted since the relationship between the information as a measure of (probabilistic) uncertainty and the physical meaning of entropy as the number of microstates, for example, in gases, is clearly distinguished (Penrose, 2006). If we want to point out the equality of the same expression for two different approaches and meanings, [\(1.9\)](#) will be called information entropy in order to accentuate the nonphysical aspect.

Furthermore, in this book we focus on nonequilibrium phase transitions that are characteristics of many dissipative, open systems that not only include living beings but also, for example, robots as artificial “ingredients” of inanimate nature. In open systems, the information can even be increased if a nonequilibrium phase transition occurs and a final system is generated after a bifurcation that has an increased order (Haken, 2006). An example for this declaration is the transition from a lamp (below the bifurcation threshold) to a laser (well above the bifurcation threshold). In closed systems, the opposite effect occurs. In an equilibrium state (constant energy) phase transition, the information entropy decreases if (after the bifurcation) a more ordered system state is achieved.

Maximum and Minimum Principles of Information

We get a feeling on the significance of individual information [\(1.1\)](#) and the total system information of Shannon [\(1.2\)](#) if we treat the following two examples:

1. All but one p_λ are zero: $p_\lambda = 0$; $p_K = 1$, $\ln 1 = 0$, then the individual event information and the total system information are both zero:

$$i_s = i_\lambda = 0.$$

This means that there is no uncertainty, no surprise with respect to the outcome of a trial. Or, in other words, there is complete certainty as long as we use the information available to us.

2. All p_λ are equal. The considered probability distribution is given by the uniform distribution $p = (p_1 = (1/N), \dots, p_N = (1/N))$, $p_\lambda = 1/N$. In this case, the system information i_s maximal:

$$i_s(p) = \ln N.$$

This is the case if there are no additional constraints besides the standard restriction of normalization of the probability distribution. The uncertainty is maximal since all outcomes are equally likely. Laplace called it the “principle of insufficient reason,” which states that all outcomes are equally likely if there is no reason to the contrary (Kapur and Kesavan, 1992). In physics, this result corresponds to the equipartition theorem.

The Kullback measure $K(p, q)$ (Kullback, 1951) calculates the difference between two probability distributions

$p = (p_1, \dots, p_N)$ and $q = (q_1, q_2, \dots, q_N)$:

$$K(p, q) = \sum_{\lambda} p_{\lambda} \ln \frac{p_{\lambda}}{q_{\lambda}}, (p, q) \geq 0,$$

where each probability distribution (density) is separately normalized to 1, and $K(p, q)$ is nonnegative and vanishes if and only if $p = q$. Usually, the Kullback measure can also be called “information gain” since q is a fixed *a priori* distribution and p is a probability distribution that is searched with the aid of K in order to maximize the divergence of p from q . But this expression can also be used

to minimize the difference (“information adjustment”). Closer the distance from p to q , the more the probabilities of the different observed events confirm the *a priori* experiences (knowledge of the experimenter). The application of this method is then directed to find a distribution p that is closest to q and fulfills the same restrictions as q .

However, despite the existing conceptual differences of Shannon measure and Kullback measure there is a central relation between both approaches. There is a tight connection between the maximization of $i_s(p)$ and the minimization of $K(p, q)$ if we assume that q is given by the uniform probability distribution u :

$$(I.10) \quad K(p, u) = \sum_{\lambda} p_{\lambda} \ln \frac{p_{\lambda}}{1/N} = \ln N - i_s(p).$$

Maximizing $i_s(p)$ is identical to minimizing $K(p, q)$ if the *a priori* probability distribution is uniform ($q = u$). By maximizing the uncertainty, we minimize the probabilistic distance to a given distribution.

We close this short excursion to two often used expressions for information calculation by the remark that the Shannon approach is not invariant under coordinate transformations, whereas the Kullback approach is invariant under coordinate transformations.

After these remarks, we address ourselves again to the further investigation of the Shannon information in the light of the “maximum information principle” of Jaynes (Jaynes, 1957). This theorem postulates that we are looking for probability distribution p that guarantees $i_s(p) = \text{maximum}$ is fulfilled under all given constraints that also include normalization. In more details, this means that a system tries to realize all “allowed” configurations, that is, configurations that obey the constraints.

For more illustration, we treat another example. Let $N = 2$, $\lambda = 1, 2$ be and $p = (p_1, p_2) = (x, 1-x)$, $0 \leq x \leq 1$. We introduce again the

information of event λ : $i_\lambda = -\ln p_\lambda$.

We discuss two questions that are basic for understanding Jaynes' principle by analyzing the results of [Figures I.1-I.3](#):

1. For which x does $i_s(p)$ become a maximum?

According to [Figure I.3](#), we find $x = 1/2$, $p_1 = p_2$; $i_s(p) = \ln 2$

2. For which x does p_1 or (p_2) get a maximum?

According to [Figures I.1](#) and [I.2](#), we find $x=1$, $p_1 = 1$, $i_1 = 0$. The probability p_1 is maximum if the information for event 1, i_1 , is a minimum.

These two resulting answers lead us to the formulation of two principles that will be important in our book. We start with the general case:

(I.11)
$$i_s(p) = -\sum_{\lambda} p_{\lambda} \ln p_{\lambda}, \quad \sum_{\lambda} p_{\lambda} = 1$$

and additional constraints that all together will be taken care of by the use of Lagrange multipliers to maximize $i_s(p)$ (for more details, consult Section 1.4). We consider [Figure I.4](#) and reformulate the two questions put above into one similar, combined question.

Figure I.1 p_λ versus x ; l.h.s.: $\lambda = 1$, r.h.s.: $\lambda = 2$.

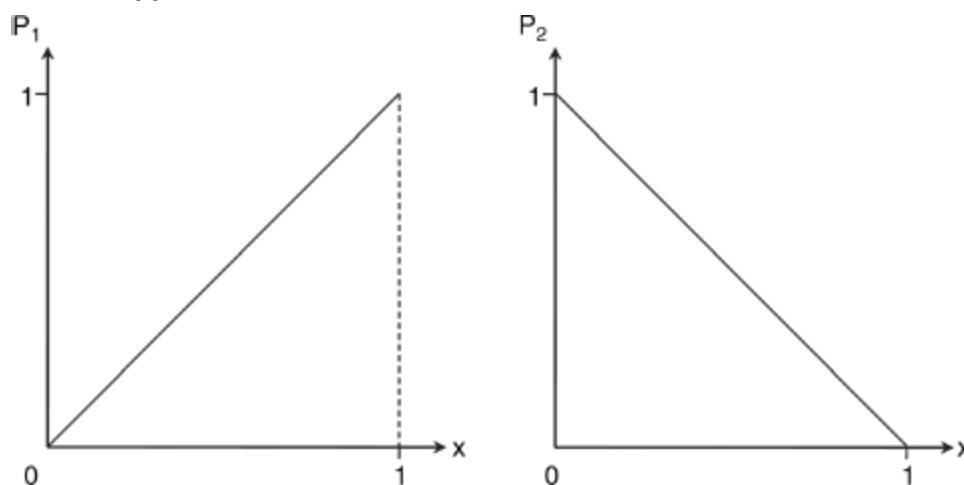


Figure I.2 $-\ln p_\lambda$ versus x ; l.h.s.: $\lambda = 1$, r.h.s.: $\lambda = 2$.

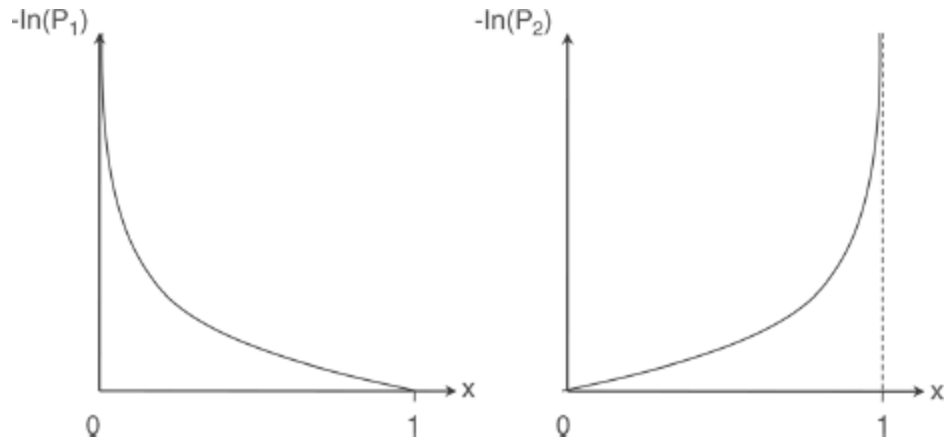


Figure I.3 i versus x .

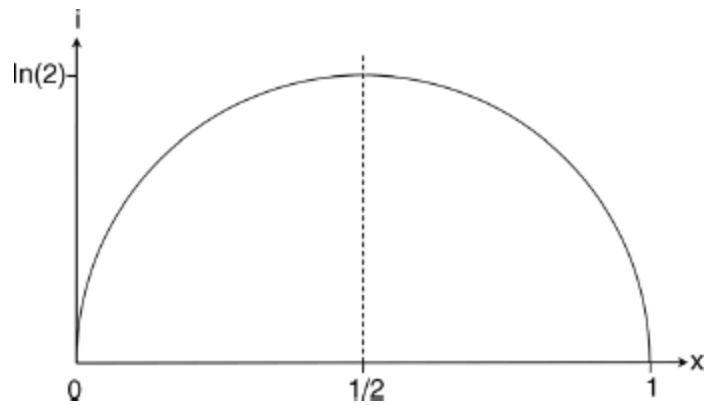
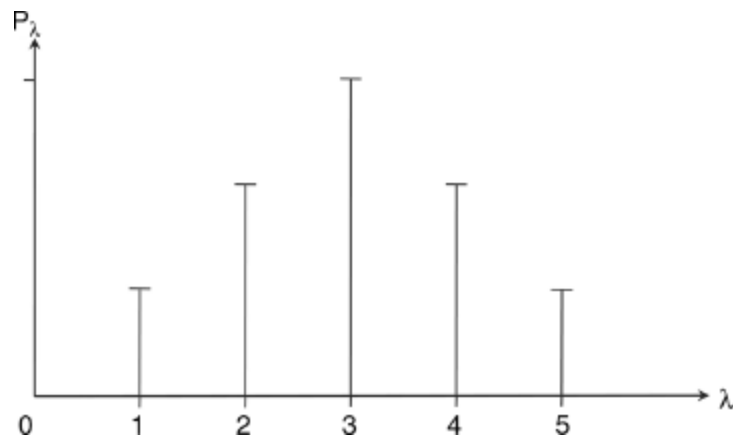


Figure I.4 Discrete probability distribution p_λ as function of λ .



Which p_λ is(are) maximum or which $i_\lambda = -\ln p_\lambda$ is minimum? The answer is: a maximal p_λ minimizes the individual event information i_λ . Or, to turn the argument around, a small p_λ means that the event λ is rare or unexpected, which implies that i_λ must be large. This is the reason why some scientists