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Spatial Analysis along Networks

Statistical and Computational Methods

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Spatial Analysis along Networks

Statistical and Computational Methods

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Preface

As its title indicates, this book is devoted to spatial analysis along networks, referred to as *network spatial analysis*, or more explicitly, statistical and computational methods for analyzing events occurring on and alongside networks. Network spatial analysis is of practical use for analyzing, among other things, the occurrence of traffic accidents on highways, the incidence of crime on streets, the location of stores alongside roads, and the contamination of rivers (Chapter 1 introduces many applications). This usefulness is the main reason we focus on network spatial analysis in this volume. However, there is also a more general and somewhat more ambitious justification for this work. That is, when viewed from a broader perspective, we expect that network spatial analysis will prove to be a first step toward next-generation spatial analysis.

Having reviewed the extant literature on spatial analysis, we note that most empirical studies incorporate spatially aggregated data across subareas, such as administrative districts, census tracts, and postal zones. We refer to this type of spatial analysis as *subarea-based spatial analysis* or *meso-scale spatial analysis*. One of the earliest and most notable examples of this type of spatial analysis is included in a compilation titled *The City* (Park, Burgess, and McKenzie, 1925), written by sociologists at the Chicago School (sometimes described as the Ecological School). More specifically, Burgess (1925) surveyed land use of subareas in Chicago and formulated the concentric-zone model, subsequently followed by Hoyt's (1939) sector model and the Harris–Ullman multiple nuclei model (Harris and Ullman, 1945).

Since then, subarea-based spatial analysis has become one of the most important approaches to empirical spatial analysis. Even today, we frequently employ subarea-based spatial analysis for empirical studies because subarea data, including population and other census-related data, are widely available and because it is generally straightforward to apply ordinary statistical techniques, including regression analysis, to the attribute values of subareas. Unlike the empirical literature, we find that the development of most theoretical work on spatial analysis has assumed an 'ideal space', that is, real space is represented by unbounded homogeneous space with Euclidean distance. This ideal space is convenient for developing pure theories of spatial analysis or spatial stochastic processes; indeed, the derivations of many useful theorems employ this assumption (see, e.g., Illian *et al.*, (2008)). However, ideal space is far from the real world.

In the late twentieth century, the availability of detailed spatial data increased dramatically thanks to rapid progress in data acquisition technologies, such as the global positioning system (GPS) and many kinds of geosensors. Better data availability potentially enables us to analyze spatial events in detail by representing individual entities in the real world in terms of geometric objects in two- or three-dimensional Cartesian space instead of aggregating them into subareas (see Chapter 2 for this representation). We describe this possible form of spatial analysis as *object-based spatial analysis* or *micro-scale spatial analysis*, in contrast to the well-established subarea-based spatial analysis or meso-scale spatial analysis. At present, however, the methods for micro-scale spatial analysis are at an early stage. We believe that one clue to micro-scale spatial analysis would be to represent real space by networks embedded in two- or three-dimensional Cartesian space. This is because many kinds of events or activities in the real world are constrained by networks, such as streets, railways, water and gas pipe lines, rivers, electric wires, and communication networks. A first step toward micro-spatial analysis would thus appear to be network-constrained spatial analysis, which is the main concern of this volume.

In network spatial analysis, we measure the shortest-path distance. Unfortunately, its computation is much more difficult than that of Euclidean distance because it requires the management of network topology. Therefore, network spatial analysis becomes practical only when efficient computational methods are available. Dijkstra (1959) developed a key algorithm for this purpose in the middle of last century. Since then, there has been extensive study of location problems on networks by a variety of researchers, mainly in operations research (Handler and Mirchandani, 1979; Daskin, 1995; for a review, see Labbe, Peeters, and Thisse (1995)). We should note that the focus in these studies has been locational optimization or the computing of network characteristics (e.g., Kansky, 1963; Haggett and Chorley, 1969), with rather less attention paid to the statistical analysis of events on networks.

To fill this gap in the literature, we develop statistical and computational methods for network spatial analysis by introducing computational methods originally developed for operations research and computational geometry (Preparata and Shamos, 1985; Chapter 3 in this volume presents some basic computational methods). In this sense, the network spatial analysis presented in this volume is a first step toward micro-scale spatial analysis. However, we cannot present real world space by either network or Euclidean space alone as it is a complex hybrid system with elements of both. The next step, then, would be object-based spatial analysis in a hybrid space consisting of a discrete network space with shortest-path distance and a continuous space with Euclidean, or more generally, geodesic distance. An initial attempt is Cressie *et al.* (2006).

We are now in the midst of an ongoing revolution brought about by information and communication technologies. In the future, microcomputer tips, tags, and geosensors will be embedded in almost every entity (including moving objects) in our environment, and the integration of these devices with communication systems (e.g., the Internet) will establish an intellectual system joining the virtual world of computers and the global real world. This system will then realize a society we refer to as the *ubiquitous computing society*, in which at any time and in any place, people can receive the most appropriate personalized information for action given their particular circumstances in time and space (Sakamura and Koshizuka, 2005). To construct this system, micro-scale spatial analysis is expected to extend to *real-time spatial analysis*, that is, spatial analysis in which the circumstances of an acting body (including a person, a group of persons, a company, or possibly a robot) are analyzed and appropriate personalized information for action is derived almost instantaneously (Okabe, 2009a, 2009b). We intend that this volume, in presenting state-of-the-art methodology for network spatial analysis, will contribute a first step toward micro-scale spatial analysis and encourage our readers to further develop micro-scale spatial analysis and, from there, tackle the challenge of real-time spatial analysis.

Atsuyuki Okabe
Kokichi Sugihara
March 2012

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1

Introduction

This book presents statistical and computational methods for analyzing events that occur on or alongside networks. To this end, the first three chapters are concerned with preparations. This chapter shows the scope of this book, Chapter 2 fixes a general framework for spatial analysis, and Chapter 3 describes computational methods commonly used throughout the subsequent chapters. In this introductory chapter, we first describe the events under consideration, i.e., events that occur on and alongside networks, termed *network events*. Second, we show that if traditional spatial analysis assuming a plane with Euclidean distances, referred to as *planar spatial analysis*, is applied to network events, then it is likely to lead to false conclusions. Third, to overcome this shortcoming, we propose a new type of spatial analysis, namely *network spatial analysis*, which assumes a network with shortest-path distances. Fourth, we review studies on network events in the related literature and show how to apply network spatial analysis to those studies. Last, we describe the structure of the twelve chapters of the book and suggest how to read them according to the reader's interests. Note that network spatial analysis viewed from a board perspective is described in the preface of this volume.

1.1 What is network spatial analysis?

To introduce this new type of spatial analysis, we first define a key concept, *network events*, and next consider typical questions about network events to be solved by network spatial analysis. We then describe the salient features of network spatial analysis in contrast to the traditional planar spatial analysis.

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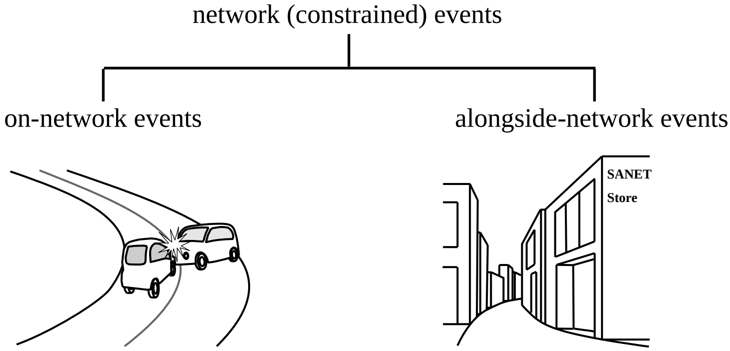


Figure 1.1 Network (constrained) events consisting of on-network events and alongside-network events.

1.1.1 Network events: events on and alongside networks

In the real world, there are numerous and various events that are strongly constrained by networks, such as car crashes on roads and fast-food shops located alongside streets. We call them *network-constrained events* (Yamada and Thill, 2007) or *network events* for short. Network events can be classified into two classes: events that occur directly on a network (e.g., car crashes on a road), and events that occur alongside a network rather than directly on it (e.g., fast-food shops located alongside a street). We refer to the former as *on-network events* and the latter as *alongside-network events*. Consequently, network events consist of on-network events and alongside-network events (Figure 1.1). Note that we sometimes use ‘along’ for both ‘on’ and ‘alongside.’

Figure 1.2 illustrates an actual example of on-network events, where each dot represents a traffic accident around Chiba station, Japan. As with this example, many types of network event have been reported in the related literature, including pedestrian and motor vehicle street accidents, roadkills of animals on forest roads, street crime sites, tree spacing along the roadside, seabirds located along a coastline, beaver lodges in watercourses, levee crevasse distribution on river banks, leakages in gas and oil pipelines, breaks in a wiring network, disconnections on the Internet, and blood clots in a vascular network (studies on network events including these examples will be reviewed in Section 1.2).

Figure 1.3 depicts an actual example of alongside-network events, where the black dots indicate advertisement agency sites alongside streets in Shibuya ward, one of the subcentral districts in Tokyo. There are many facilities that are located alongside street networks within densely inhabited areas. In fact, the entrances to almost all facilities in a city are adjacent to streets and users access amenities through these (Figure 1.1). Consequently, the locations of almost all facilities within an urbanized area can be regarded as alongside-network events.

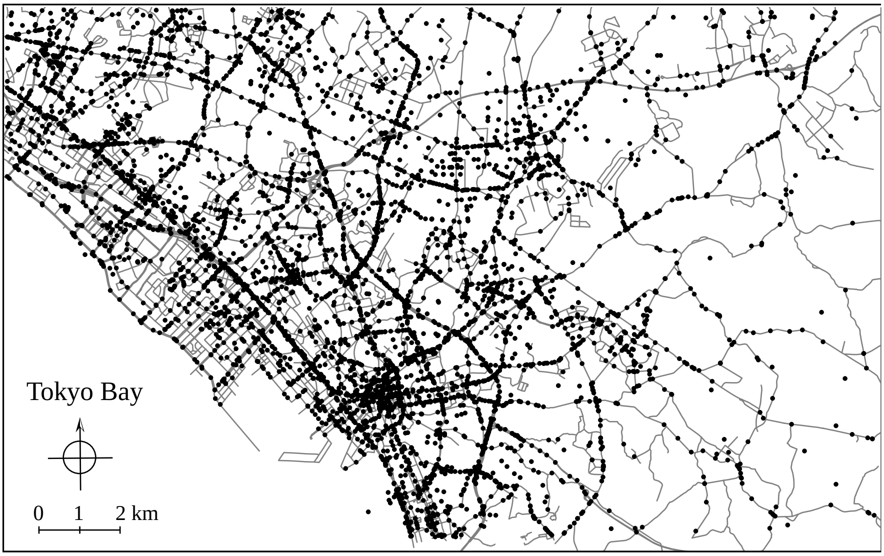


Figure 1.2 Sites of traffic accidents around Chiba station, Japan (private roads are not shown).

Figure 1.3 The distribution of advertisement agency sites (the black points) alongside streets (the gray line segments) in Shibuya ward, one of the subcentral districts in Tokyo.

On- and alongside-network events such as those in the above examples are the major concern of this book. More specifically, this book primarily focuses on spatial distributions and relationships of such events on and alongside networks. Typical questions to be discussed in this volume are as follows:

Q1: How can we obtain the catchment areas of parking lots in a downtown area including one-way streets, assuming that drivers access their nearest parking lots?

Q2: Do boutiques tend to stand side-by-side alongside streets in a downtown area?

Q3: Do street burglaries tend to take place near railway stations?

Q4: Is the roadside land price of a street segment similar to those of the adjacent street segments?

Q5: How can we locate clusters of fashionable boutiques alongside downtown streets?

Q6: How can we estimate the density of traffic accidents and street crimes incidence, and how can we identify locations where the densities of those occurrence are high, referred to as *black spots* and *hot spots*?

Q7: How can we spatially interpolate an unknown NO_x (nitrogen oxides) density at an arbitrary point on a road using known NO_x densities at observation points in a high-rise building district, such as Midtown Manhattan?

Q8: How can we estimate the probability of a consumer choosing a specific fast-food shop among alternative shops located alongside streets in a downtown area?

1.1.2 Planar spatial analysis and its limitations

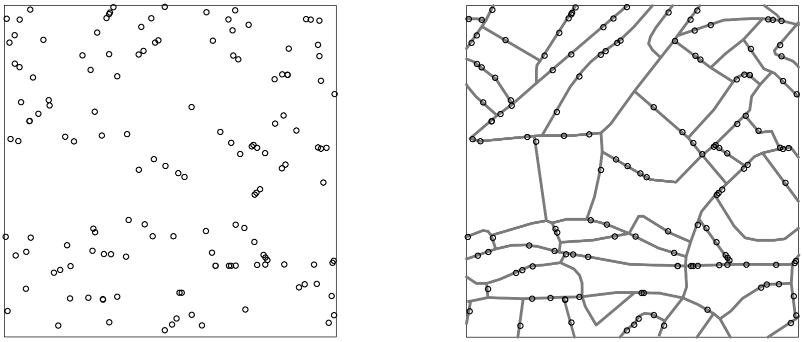
To answer the above types of question, we might conventionally use spatial methods that assume:

AP1: Events occur on a continuous (unbounded) plane.

AP2: If a method for analyzing the events includes distance variables, the distances are measured by Euclidean distance.

These types of spatial approach are referred to as *planar spatial methods*, and analyses made in this way are termed *planar spatial analyses*. Originally, planar spatial methods were designed for analyzing events on a plane, but in practice, as a matter of convenience, planar spatial methods are often applied to network events. However, this use is likely to lead to false conclusions, which are clearly demonstrated in Figure 1.4.

Having assessed the distribution of points in Figure 1.4a, nobody would consider that the points are randomly distributed. This view is true if the points are considered as being distributed on a plane; however, this becomes false when the points are seen to be located on a network indicated by the line segments in



(a) (b)

Figure 1.4 Point distributions: (a) nonrandomly distributed points on a bounded plane, (b) randomly distributed points on a network (note that the point distributions in (a) and (b) are the same).

Figure 1.4b. In fact, the points in this figure are randomly generated according to the uniform distribution across the network (for details, see Section 2.4.2 in Chapter 2). Figure 1.4 provides the following warning: analysis of network events using a planar spatial method is likely to lead to false conclusions. We shall show examples in subsequent chapters.

The second assumption AP2, i.e., the Euclidean distance assumption, is also arguable. The reasons for making this assumption are:

- it is much easier to compute Euclidean distance on a plane than the shortest-path distance on a network; and
- it is believed that the shortest-path distance is approximated by Euclidean distance.

The first reason remains true, although the difficulty is nowadays reduced because the use of geographical information systems (GIS) makes it easy to manage network data and to calculate shortest-path distances (a concise introduction to GIS is provided by Okabe (2004, 2005, Chapter 1)). The second reason may be true over a large region, but the validity of this concept is questionable across a small area or within a city. For example, Maki and Okabe (2005) report that in Kokuryo, a suburb of Tokyo, the difference between shortest-path distances and their corresponding Euclidean distances is significant if the Euclidean measurement is less than 400 m (see Figure 1.5). In addition, as shown in Table 1.1, the average radii of the service areas of many types of downtown store, exemplified by Shibuya ward in Tokyo, are less than 400 m. Planar spatial methods may be inappropriate therefore for analyzing alongside-network location events affected by trip behavior (for a further discussion, see Section 6.3 in Chapter 6).

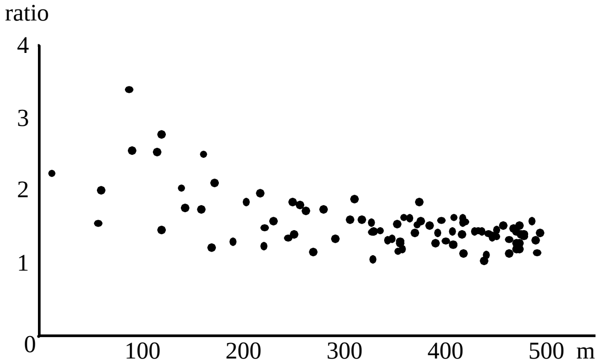


Figure 1.5 Ratio of the shortest-path distance to its corresponding Euclidean distance for the street network in Kokuryo, a suburb in Tokyo (data source: Maki and Okabe (2005)).

1.1.3 Network spatial analysis and its salient features

To overcome the above limitations of planar spatial methods, we now introduce a new type of spatial analysis that assumes:

AN1: Events occur on and alongside a network.

AN2: If a method for analyzing the events includes distance variables, the distances are shortest-path distances.

Corresponding to the planar spatial methods mentioned above (AN1 and AN2 correspond to AP1 and AP2, respectively), we call these methods *network spatial*

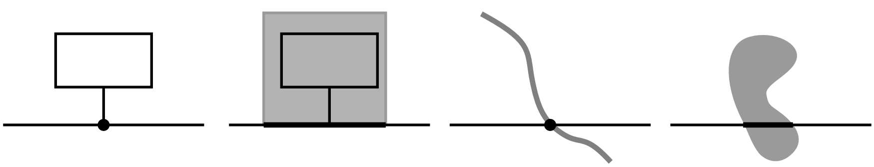
Table 1.1 Average radii of service areas in Shibuya ward, Tokyo.

Store type	Average radius (m)
Aromatherapy shop	282
Bag shop	271
Interior design shop	249
Daily necessities store	217
Preparatory school	216
Apartment estate agent	175
Printing store	167
Cafe	130
Japanese-style restaurant	106
Clothing store	85
Beauty shop	73

methods, and analyses that use network spatial methods, we call *network spatial analyses*. It should be noted that network spatial analysis does not imply the analysis of a network itself, such as geographical network analysis (Haggett and Chorley, 1969), communication network analysis (Kesidis, 2007), and circuit network analysis (Stanley, 2003). To avoid this confusion, we could use the terms *on-* or *alongside-network spatial analysis*, *network-constrained spatial analysis* (Yamada and Thill, 2004), *network-based spatial analysis* (Downs and Horner, 2007a, 2007b; Shiode, 2008) or more strictly, *spatial analysis on and alongside networks*. In this text, we use *network spatial analysis* for short.

We make a few remarks on the above two assumptions, AN1 and AN2. The first assumption AN1 describes places where events occur. The *on-network relation* is obvious. Events occur exactly on a network, such as traffic accidents. The *alongside-network relation* includes fairly broad spatial relations. It implies that the physical unit of an event (e.g., a store located at a site) has an access point on a network (the entrance of the store indicated by the black circle in Figure 1.6a) or the physical unit (the lot of the store) shares a common boundary line segment with a network (the bold line segment in Figure 1.6b). In addition, the alongside-network relation includes relations in which the physical unit of an event may intersect a network, for instance, a river intersects a road (Figure 1.6c) or a network goes through a forest area (Figure 1.6d). Computational treatments of these alongside-network relations are developed in Chapter 3 in detail.

The second assumption, AN2, specifies distance variables included in spatial methods. Consider, for instance, the analysis of boutique clusters in a downtown area using cluster analysis (for details, see Chapter 8). Because boutiques in clusters in a downtown area are located side-by-side alongside streets and customers access boutiques from entrances facing streets, it is natural to measure the closeness in terms of the shortest-path distance along streets. If a river separates two boutiques, it is not natural to assume that the boutiques belong to the same cluster even if the Euclidean distance between them is short. Underlying activities that result in boutique clusters are trips through streets, for example, window-shopping on sidewalks. In addition, many kinds of activities in a city are achieved through a street network, and so the configuration of activities may be influenced by trip



(a) (b) (c) (d)

Figure 1.6 Alongside-network relations: (a) an access point (the black circle) of a polygon to a network (the horizontal line segment), (b) a boundary line segment of a polygon shared with a network (the bold line segment) (c) an intersection point of two networks (the black circles), (d) a network intersecting an area (the bold line segment).

behavior constrained by a street network. Consequently, network events may be best analyzed in terms of the shortest-path distance.

It should be noted, however, that there may be cases in which the shortest-path distance is not appropriate even if events occur on a network. For instance, consider the service area of a cell phone antenna. Although cell phone antennas stand on the edge of a street, their service areas are determined by Euclidean distance, because electric waves go straight through the air. The reader who wants to use a network spatial method should confirm whether or not the network spatial method is appropriate even when events are network events.

We notice from the above definition of network spatial analysis that it has salient features distinct from those of planar spatial analysis. First, by definition, network spatial analysis can properly analyze events occurring on and alongside a network. As a result, we can avoid the misleading conclusion illustrated by Figure 1.4. It is apparent from that figure that the selected points inevitably form clusters on a plane, because the points can exist only on a network. In fact, Yamada and Thill (2004) claimed that a planar spatial method (the *K* function method) overestimates clusters of traffic accidents in Buffalo (for details, see Chapter 6). Lu and Chen (2007) gave similar warning when analyzing urban crime distributed along streets. Such an overestimation is likely to happen not only for on-network events but also for alongside-network events. Therefore, clusters of stores in a city examined by planar spatial methods should be reexamined by network spatial methods.

Second, network spatial analysis can easily take account of directions, such as directions of current in a river and traffic flow regulation on a street network. In cities, particularly in downtown areas, many streets are one-way. In fact, about one third of streets in the downtown area of Kyoto are one-way (Okabe *et al.*, 2008). This implies that we cannot precisely estimate the delivery service areas of retail stores (e.g., pizza delivery stores) with Euclidean distances. Alternatively, we estimate the service areas in terms of the shortest-path distance on a directed network, and this estimation is investigated in detail in Chapter 4 (deterministic service areas) and Chapter 11 (probabilistic service areas).

Third, network spatial analysis can treat detailed networks using a common data structure. In a simple case, we represent a street by a line segment, but the street may consist of several components. For example, a street consists of vehicular roads (with two-way lanes), sidewalks, and crossings (Figure 1.7a). We can represent these details by a set of networks, as shown in Figure 1.7b, that share the same data structure (see Chapters 2 and 3).

Fourth, network spatial analysis can easily treat networks in three-dimensional space, such as underpaths and crossover bridges. This easy treatment is powerful when we analyze, for example, the incidence of pickpockets in a department store, egg-laying sites in an ant nest or blood clots in a vascular network. Figure 1.8 illustrates walkways, stairs, up/down escalators, and elevators in a department store, which are represented by a directed network.

Fifth, as will be shown in Section 2.3, network spatial analysis can treat nonuniform activities on a network more easily than planar spatial analysis can. While traditional spatial analysis methods are mostly designed to test the null

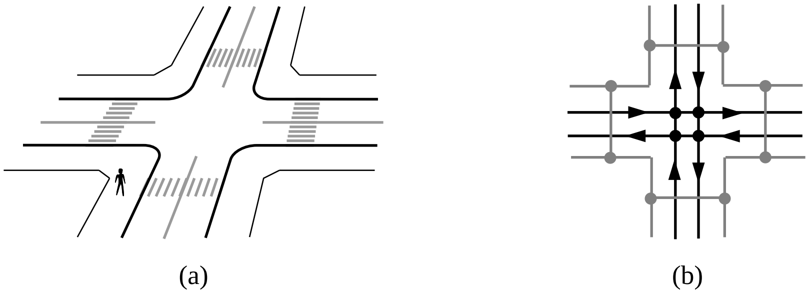


Figure 1.7 Entities represented by networks: (a) sidewalks, vehicular roads and crossing (entities), (b) the networks representing those entities.

hypothesis that events are uniformly distributed over a plane or network, this assumption is often violated in real-world phenomena. Consider, for example, traffic accidents on a road network. Obviously, traffic accidents do not occur uniformly across the network. Traffic accidents result from many factors, one of which is traffic volume (see Section 1.2.2). It is likely that the density of traffic accidents is proportional to traffic volume which naturally varies over a road network. Therefore, we cannot directly apply the traditional methods that assume uniform traffic volume to the distribution of traffic accidents resulting from nonuniform traffic volume. It is difficult to incorporate such nonuniformity in planar spatial analysis. Fortunately, however, we have good 'magic' that transforms a nonuniform density of an activity to a uniform density of the activity (to be shown in Section 2.4 in Chapter 2), to which we can apply traditional spatial methods assuming a uniform density. Through this transformation, we can easily analyze nonuniform activities on networks.

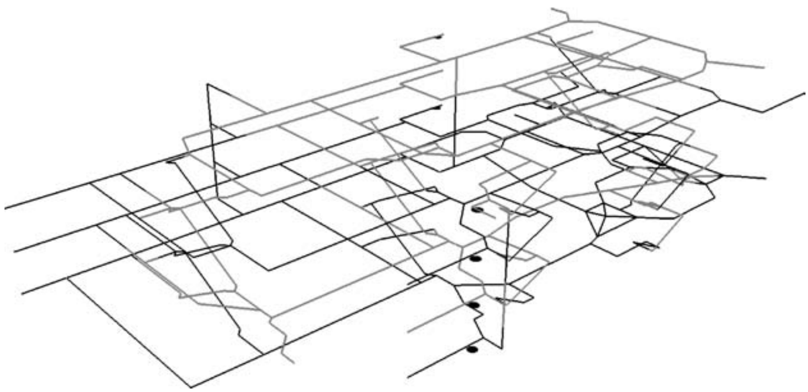


Figure 1.8 Walkways, stairs, up/down escalators, and elevators in a department store in Tokyo, where the circles indicate toilets (provided by T. Satoh). The subnetworks in different gray colors indicate the Voronoi cells of the three-dimensional network Voronoi diagram generated by the toilets (for definition, see Chapter 4).

Sixth, network spatial analysis gains analytical tractability because a network consists of one-dimensional line segments. Mathematical derivations on a one-dimensional space are more tractable than those on a two-dimensional space. For instance, to derive indexes or statistics, we often do integral computation, and single-integral computation is easier than double-integral computation. Therefore, we may obtain exact statistics for a network that could not be obtained for a plane.

Last, we should note the shortcomings of network spatial methods. On a plane, once the coordinates of points are given, we can easily compute the Euclidean distance between them. On a network, however, computation of the shortest path is not so simple and requires several steps. First, we must construct a database for managing a network. In practice, point data and network data are obtained from different sources and points that are supposed to be on a network are likely to be off the network rather than exactly on the network. Therefore, second, we must assign the points to the network. Third, we must use an algorithm for computing the shortest path on a network. In addition, we must perform many kinds of geometrical computation inherent in network spatial analysis. As a result, it is not straightforward in practice to extend statistical methods for planar spatial analysis to those for network spatial analysis. Network spatial analysis becomes practical only when its computation is possible. That is why the subtitle of this book is *Statistical and Computational Methods*. The computational methods in each chapter show how to solve difficult geometric computations encountered in network spatial methods in practice.

1.2 Review of studies of network events

As the above salient features indicate, network spatial analysis provides a suitable and powerful approach to the analysis of events occurring on and alongside networks. In fact, we can find many empirical studies of network events in various fields, although they do not always call their analyses network spatial analysis. In this subsection, we review these studies, but note that our review is not exhaustive and that our intent is merely to provide illustrative examples to be discussed in the following chapters.

1.2.1 Snow's study of cholera around Broad Street

Primitive qualitative network spatial analysis might date back many centuries ago when, for instance, a Roman ruler considered the location of colony settlements along Roman roads (Hodder and Orton, 1976). As far as we know, scientific quantitative network spatial analysis originated from John Snow's study in the mid-nineteenth century (Snow, 1855, 1936). John Snow's cholera map (Figure 1.9), which he called a diagram of the *topography of the outbreak* (Snow, 1855), illustrated one of the worst outbreaks of cholera that occurred around Broad Street and Golden Square in London in the mid-nineteenth century. The black bars along streets in Figure 1.9 indicate the number of victims. To find the source of the