Trends in Multiple Criteria Decision Analysis

International Series in Operations Research & Management Science

Volume 142

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Trends in Multiple Criteria Decision Analysis

Springer

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ISSN 0884-8289
ISBN 978-1-4419-5903-4 e-ISBN 978-1-4419-5904-1
DOI 10.1007/978-1-4419-5904-1
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2010932006

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

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Introduction

Matthias Ehrgott, José Rui Figueira, and Salvatore Greco

1 Introduction

When 5 years ago we edited the book "Multiple Criteria Decision Analysis: State of the Art Surveys" with 24 chapters written by 49 international leading experts, we believed that the book would cover the research field for several years. But over the last 5 years Multiple Criteria Decision Analysis (MCDA) has received an increasing interest and has experienced a development faster than we expected. Thus, what looked like a comprehensive collection of state-of-the-art surveys appears clearly partial and incomplete a few years later. New approaches and new methodologies have been developed which even contribute to change the paradigm of MCDA. A researcher who does not take into account the new contributed risks to be disconnected from the main trends of the discipline and to have a misleading conception of it. These thoughts convinced us to explore the map of the new trends in MCDA in order to recognize the most promising new contributions. This book comprises 13 chapters, once again written by leading international experts, that summarize trends in MCDA that were not covered in our previous book and that describe the development of rapidly evolving sub-fields of MCDA.

Po-Lung Yu and Yen-Chu Chen present the theory of dynamic multiple criteria decision analysis, habitual domains, and competence set analysis. In real life, most decisions are dynamic with multiple criteria. Even though most of the MCDA literature assumes that the parameters involved in decision problems – such as the set of alternatives, the set of criteria, the preference structures of the decision makers – are more or less fixed and steady, in reality – for most nontrivial decision problems – these parameters can change dynamically. In fact, satisfactory solutions are obtained only when those parameters are properly structured. To analyze the decision process in a dynamic context the concepts of habitual domain and competence set are of fundamental importance. A habitual domain is the set of ideas and concepts which we encode and store in our brain, gradually stabilized over a period of time. The competence set is a collection of ideas, knowledge, resources, skills, and effort for the effective solution of a decision problem. Competence set analysis and habitual domain theory suggest how to expand and enrich our competence set and how to maximize the value of our competence set. In this perspective, any decision problem can be dealt with by restructuring its elements and environmental facets in order to gain a broader and richer perception permitting to derive effective solutions.

Andrzej P. Wierzbicki discusses the need for and possible methods of objective ranking after observing that the classical approach in decision analysis and multiple criteria theory concentrates on subjective ranking. However, in many practical situations, the decision maker might not want to use personal preferences, but prefers to have some objective ranking. One reason for objectivity is that decisions of a given class might influence other people, e.g., some decision situations dominating in technology creation, such as constructing a safe bridge or a safe car. Thus, technologists stress objectivity but real managers also know well that there are many managerial situations where stressing objectivity is necessary. Therefore, even if it can be agreed that an absolute objectivity is not attainable, it is reasonable to treat the concept of objectivity as a useful ideal worth striving for, looking for objective ranking interpreted as an approach to ranking that is as objective as possible. Between many possible multiple criteria approaches, the reference point approach (already introduced in the literature to deal with interactive multiple criteria optimization) is mentioned as the best suited methodology for rational objective ranking, because reference levels needed in this approach can be established to some extent objectively – statistically from the given data set.

Jonathan Barzilai in his provocative chapter discusses preference function modelling, i.e., the mathematical foundations of decision theory. He formulates the conditions that must be satisfied for the mathematical operations of linear algebra and calculus to be applicable and claims that the mathematical foundations of decision theory and related theories depend on these conditions, which have not been correctly identified in the classical literature. He argues that Operations Research and Decision Analysis Societies should act to correct fundamental errors in the mathematical foundations of measurement theory, utility theory, game theory, mathematical economics, decision theory, mathematical psychology, and related disciplines. Consequences of this approach to some MCDA methodologies such as AHP or value theory are also discussed.

Hassene Aissi and Bernard Roy discuss robustness in MCDA. The term *robust* refers to a capacity for withstanding "vague approximations" and/or "zones of ignorance" in order to prevent undesirable impacts. Robustness concerns are related to the observation that an action is made, executed, and judged in a real-life context that may not correspond exactly to the model on which the decision analysis is based. The gap between formal representation and real-life context originates frailty points against which the robustness concern attempts to protect. Robustness concerns can be dealt with using approaches involving a single robustness criterion, completing a preference system that has been defined previously, or using several criteria. Robustness can be considered other than by using one or several criteria to compare the solutions in approaches that involve one or several properties designed to characterize the robust solution or to draw robust conclusions. The considerations developed in this chapter show that the use of multiple criteria for apprehending robustness in MCDA is a field of research open to future development, both theoretically and practically.

Bernard De Baets and János Fodor consider preferences expressed in a gradual way. The key concept is that the application of two-valued (yes or-no) preferences, regardless of their sound mathematical theory, is not satisfactory in everyday situations. Therefore, it is desirable to consider a degree of preference. There are two main frameworks in which gradual preferences can be modeled: fuzzy preferences, which are a generalization of Boolean (2-valued) preference structures, and reciprocal preferences, also known as probabilistic relations, which are generalization of the three-valued representation of complete Boolean preference relations. The authors consider both frameworks. Since the whole exposition makes extensive use of (logical) connectives, such as conjunctors, quasi-copulas and copulas, the authors provide an appropriate introduction on the topic.

Radko Mesiar and Lucia Vavríková present fuzzy set and fuzzy logic-based methods for MCDA. Alternatives are evaluated with respect to each criterion on a scale between 0 and 1, which can be seen as membership function of fuzzy sets. Therefore, alternatives can be seen as multidimensional fuzzy evaluations that have to be ordered according to the decision maker's preferences. This chapter considers several methodologies developed within fuzzy set theory to obtain this preference order. After discussion of integral-based utility functions, a transformation of vectors of fuzzy scores x into fuzzy quantity $U(x)$ is presented. Orderings on fuzzy quantities induce orderings on alternatives. Special attention is paid to defuzzification-based orderings, in particular, the mean of maxima method. Moreover, a fuzzy logic-based construction method to build complete preference structures over the set of alternatives is given.

Wassila Ouerdane, Nicolas Maudet, and Alexis Tsoukiàs discuss argumentation theory in MCDA. The main idea is that decision support can be seen as an activity aiming to construct arguments through which a decision maker will convince first herself and then other actors involved in a problem situation that "that action" is the best one. In this context the authors introduce argumentation theory (in an Artificial Intelligence oriented perspective) and review a number of approaches that indeed use argumentative techniques to support decision making, with a specific emphasis on their application to MCDA.

Valerie Belton and Theodor Stewart introduce problem structuring methods (PSM) in MCDA providing an overview of current thinking and practice with regard to PSM for MCDA. Much of the literature on MCDA focuses on methods of analysis that take a well-structured problem as a starting point with a well-defined set of alternatives from which a decision has to be made and a coherent set of criteria against which the alternatives are to be evaluated. It is an erroneous impression that arriving at this point is a relatively trivial task, while in reality this is not so simple even when the decision makers believe to have a clear understanding of the problem. Thus, PSM provides a rich representation of a problematic situation in order to enable effective multicriteria analysis or to conceptualize a decision, which is initially simplistically presented, in order for the multicriteria problem to be appropriately framed. The chapter outlines the key literature, which explores and offers suggestions on how this task might be approached in practice, reviewing several suggested approaches and presenting a selection of case studies.

Salvatore Greco, Roman Słowiński, José Rui Figueira, and Vincent Mousseau present robust ordinal regression. Within the disaggregation–aggregation approach, ordinal regression aims at inducing parameters of a preference model, for example, parameters of a value function, which represent some holistic preference comparisons of alternatives given by the decision maker. Usually, from among many sets of parameters of a preference model representing the preference information given by the DM, only one specific set is selected and used to work out a recommendation. For example, while there exist many value functions representing the holistic preference information given by the DM, only one value function is typically used to recommend the best choice, sorting, or ranking of alternatives. Since the selection of one from among many sets of parameters of the preference model compatible with the preference information given by the DM is rather arbitrary, robust ordinal regression proposes taking into account all the sets of parameters of the preference model compatible with the preference information, in order to give a recommendation in terms of necessary and possible consequences of applying all the compatible preference models on the considered set of alternatives. For example, the necessary weak preference relation holds for any two alternatives a and b if and only if all compatible value functions give to a a value greater than or equal to the value provided to b, and the possible weak preference relation holds for this pair if and only if at least one compatible value function gives to a a value greater than or equal to the value given to b. This approach can be applied to many multiple criteria decision models such as multiple attribute utility theory, fuzzy integral modeling interaction between criteria, and outranking models. Moreover, it can be applied to interactive multiple objective optimization and can be used within an evolutionary multiple objective optimization methodology to take into account preferences of the decision maker. Finally, robust ordinal regression is very useful in group decisions where it permits to detect zones of consensus for decision makers.

Risto Lahdelma and Pekka Salminen present Stochastic Multicriteria Acceptability Analysis (SMAA). SMAA is a family of methods for aiding multicriteria group decision making in problems with uncertain, imprecise, or partially missing information. SMAA is based on simulating different value combinations for uncertain parameters, and computing statistics about how the alternatives are evaluated. Depending on the problem setting, this can mean computing how often each alternative becomes most preferred, how often it receives a particular rank, or obtains a particular classification. Moreover, SMAA proposes inverse weight space analysis, using simulation with randomized weights in order to reveal what kind of weights make each alternative solution most preferred. After discussing several variants of SMAA the authors describe several real-life applications.

D. Marc Kilgour, Ye Chen, and Keith W. Hipel discuss multiple criteria approaches to Group Decision and Negotiation (GDN). After explaining group decision and negotiation, and the differences between them, the applicability of MCDA techniques to problems of group decision and negotiation is discussed. Application of MCDA to GDN is problematic because – as shown by the well-known Condorcet paradox and by Arrow's theorem on collective choices – collective preferences may not exist. While ideas and techniques from MCDA are directly applicable to GDN only rarely, it is clear that many successful systems for the support of negotiators, or the support of group decisions, have borrowed and adapted ideas and techniques from MCDA. The paper presents a review of systems for Group Decision Support and Negotiation Support, then highlights the contributions of MCDA techniques and some suggestions for worthwhile future contributions from MCDA are put forward.

Kalyanmoy Deb presents recent developments in Evolutionary Multi-objective Optimization (EMO). EMO deals with multiobjective optimization using algorithms inspired by natural evolution mechanisms using a population-based approach in which more than one solution participates in an iteration and evolves a new population of solutions at each iteration. This approach is a growing field of research with many applications in several fields. The author discusses the principles of EMO through an illustration of one specific algorithm (NSGA-II) and an application to an interesting real-world bi-objective optimization problem. Thereafter, he provides a list of recent research and application developments of EMO to paint a picture of some salient advancements in EMO research such as hybrids of EMO algorithms and mathematical optimization or multiple criterion decision-making procedures, handling of a large number of objectives, handling of uncertainties in decision variables and parameters, solution of different problem-solving tasks by converting them into multi-objective problems, runtime analysis of EMO algorithms, and others.

Jacek Malczewski introduces MCDA and Geographic Information Systems (GIS). Spatial decision problems typically involve sets of decision alternatives, of multiple, conflicting, and incommensurate evaluation criteria, and, very often, of individuals (decision makers, managers, stakeholders, interest groups). The critical aspect of spatial decision analysis is that it involves evaluation of the spatially defined decision alternative and the decision maker's preferences. This implies that the results of the analysis depend not only on the geographic pattern of decision alternatives, but also on the value judgments involved in the decision-making process. Accordingly, many spatial decision problems give rise to GIS-MCDA, being a process that combines and transforms geographic data (input maps) and the decision maker's preferences into a resultant decision (output map). The major advantage of incorporating MCDA into GIS is that a decision maker can introduce value judgments (i.e., preferences with respect to decision criteria and/or alternatives) into GIS-based decision making enhancing a decision maker's confidence in the likely outcomes of adopting a specific strategy relative to his/her values. Thus, GIS-MCDA helps decision makers to understand the results of GIS-based decision-making procedures, permitting the use of the results in a systematic and defensible way to develop policy recommendations.

The spectrum of arguments, topics, methodologies, and approaches presented in the chapters of this book is surely very large and quite heterogeneous. Indeed MCDA is developing in several directions that probably in the near future would need to be reorganized in a more systematic theoretical scheme. We know that not all new proposals currently discussed in the field are represented in the book and we are sure that new methodologies will appear in the next years. However, we believe that the book represents the main recent ideas in the field and that, together with the above quoted book “Multiple Criteria Decision Analysis – State of the Art Surveys,” it gives sufficient resources for an outline of the field of MCDA permitting to understand the most important and characterizing debates in the area being wholly aware of their origins and of their implications.

Acknowledgements A book like this involves many people and we would like to thank everyone who was involved in the publication of this book. Thanks go to Fred Hillier, the series editor, and Neil Levine from Springer for their encouragement and support. The authors of the chapters have to be thanked not only for writing their own chapters, but also for reviewing other chapters in order to ensure the high standard of the book. In addition, Michael Doumpos, Sébastien Damart, Tommi Tervonen, Yves De Smet, and Karim Lidouh acted as reviewers for some of the chapters and Tommi Tervonen as well as Augusto Eusébio assissted with the translation of some chapters from Word to Latex.

Chapter 1
Dynamic MCDM, Habitual Domains and Competence Set Analysis for Effective Decision Making in Changeable Spaces

Po-Lung Yu and Yen-Chu Chen

Abstract This chapter introduces the behavior mechanism that integrates the discoveries of neural science, psychology, system science, optimization theory and multiple criteria decision making. It shows how our brain and mind works and describes our behaviors and decision making as dynamic processes of multicriteria decision making in changeable spaces. Unless extraordinary events occur or special effort exerted, the dynamic processes will be stabilized in certain domains, known as habitual domains. Habitual domains and their expansion and enrichment, which play a vital role in upgrading the quality of our decision making and lives, will be explored. In addition, as important consequential derivatives, concepts of competence set analysis, innovation dynamics and effective decision making in changeable spaces will also be introduced.

Keywords Dynamic MCDM · Dynamics of human behavior · Habitual domains · Competence set analysis · Innovation dynamics · Decision making in changeable spaces

1.1 Introduction

Humans are making decisions all the time. In real life, most decisions are dynamic with multiple criteria. Take "dining" as an example. There are many things we, consciously or subconsciously, consider when we want to dine. Where shall we go?

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M. Ehrgott et al. (eds.), *Trends in Multiple Criteria Decision Analysis*,
International Series in Operations Research and Management Science 142,
DOI 10.1007/978-1-4419-5904-1_1,

Will we eat at home or dining out? What kind of meal shall we have? Location, price, service, etc. might be the factors that affect our decision of choosing the place to eat. Nutrition, flavor and the preference to food might influence our choices, too. Eating, an ordinary human behavior, is a typical multiple criteria decision problem that we all have to face in our daily life. Its decision changes dynamically as time and situation change. Dynamic multiple criteria decision making (MCDM) is, therefore, not unusual.

Indeed, human history is full of literature recording dynamic MCDM events. However, putting MCDM into mathematical analysis started in the nineteenth century by economists and applied mathematicians including Pareto, Edgeworth, Von Neumann, Morgenstern and many more.

Typically, the studies of MCDM are based on the following three patterns of logic. The first is "simple ordering" which states that a good decision should be such that there is no other alternative that can be better in some aspects and not worse in every aspect of consideration. This concept leads to the famous Pareto optimality and nondominated solutions [42] and quotes therein. The second one is based on human goal-setting and goal-seeking behavior, which leads to satisficing and compromise solution [42] and quotes therein. The third pattern is based on value maximization, which leads to the study of value function. The three types of logic lead to an abundant literature of MCDM [12, 37] and quotes therein. Most literature of MCDM assume that the parameters involved in decision problems such as the set of alternatives, the set of criteria, the outcome of each choice, the preference structures of the decision makers, and the players are, more or less, fixed and steady. In reality, for most nontrivial decision problems, these parameters could change dynamically. In fact, great solutions are located only when those parameters are properly restructured. This observation prompts us to study decision making in changeable spaces [38, 43, 48].

Note that the term "dynamic" could have diverse meanings. From the viewpoint of social and management science sense, it carries the implication of "changeable, unpredictable"; however, from the hard science and technological sense, it may also mean "changing according to inner laws of a dynamic process," which might, but not necessarily, imply unpredictability. Much works in MCDM were motivated by applying multiple criteria analysis to dynamic processes (in the second type of meaning), for example, see the concept of ideal point, nondominated decision, cone convexity and compromise solutions in dynamic problems of Yu and Leitmann [50, 51] and in technical control science of Salukvadze [31, 32]. In this article, we use "dynamic" to imply "changes with time and situation." The dimensions and structures of MCDM could dynamically change with time and situations, consistent with the changes of psychological states of the decision makers and new information.

As a living system, each human being has a set of goals or equilibrium points to seek and maintain. Multiple criteria decision problems are part of the problems that the living system must solve. To broaden our understanding of human decision making, it is very important for us to have a good grasp of human behavior. In order to facilitate our presentation, we first briefly describe three nontrivial decision problems which involve changeable parameters in Section 1.2. The examples will be used to illustrate the concepts introduced in the subsequent sections. In Section 1.3 we shall present a dynamic behavioral mechanism to capture how our brain and mind work. The mechanism is essentially a dynamic MCDM in changeable spaces. In Section 1.4, the concepts and expansion of habitual domains (HDs) and their great impact on decision making in changeable spaces will be explored. As important applications of habitual domains, concepts of competence set Analysis and innovation dynamics will be discussed in Section 1.5. Decision parameters for effective decision making in changeable spaces and decision traps will be described in Section 1.6. Finally in Section 1.7 conclusion and further researches will be provided.

1.2 Three Decision Makings in Changeable Spaces

In this section, three nontrivial decision problems in changeable spaces are briefly described in three examples. The examples illustrate how the challenge problems are solved by looking into the possible changes of the relevant parameters. The examples will lubricate our presentation of the concepts to be introduced in the subsequent sections.

Example 1.1. Alinsky's Strategy (Adapted from [1]) During the days of the Johnson-Goldwater campaign (in 1960s), commitments that were made by city authorities to the Woodlawn ghetto organization of Chicago were not being met. The organization was powerless. As the organization was already committed to support the Democratic administration, the president's campaign did not bring them any help. Alinsky, a great social movement leader, came up with a unique solvable situation. He would mobilize a large number of supporters to legally line up and occupy all the restroom facilities of the busy O'Hare Airport. Imagine the chaotic situation of disruption and frustration that occurred when thousands of passengers who were hydraulically loaded (very high level of charge or stress) rushed for restrooms but could not find the facility to relieve the charge or stress.

How embarrassing when the newspapers and media around the world (France, England, Germany, Japan, Soviet Union, Taiwan, China, etc.) headlined and dramatized the situation. The supporters were extremely enthusiastic about the project, sensing the sweetness of revenge against the City. The threat of this tactic was leaked to the administration, and within 48 hours the Woodlawn Organization was meeting with the city authorities, and the problem was, of course, solved graciously with each player releasing a charge and claiming a victory.

Example 1.2. The 1984 Olympics in LA

The 1984 Summer Olympics, officially known as the Games of the XXIII Olympiad, were held in 1984 in Los Angeles, CA, United States of America. Following the news of the massive financial losses of the 1976 Summer Olympics in Montreal, Canada, and that of 1980s Games in Moscow, USSR, few cities wished to host the Olympics. Los Angeles was selected as the host city without voting because it was the only city to bid to host the 1984 Summer Olympics.

Due to the huge financial losses of the Montreal and that of the Moscow Olympics, the Los Angeles government refused to offer any financial support to the 1984 Games. It was then the first Olympic Games that was fully financed by the private sector in the history. The organizers of the Los Angeles Olympics, Chief Executive Officer Peter Ueberroth and Chief Operating Officer Harry Usher, decided to operate the Games like a commercial product. They raised fund from corporations and a great diversity of activities (such as the torch relay) and products (for example, "Sam the Eagle," the symbol and mascot of the Games), and cut operating cost by utilizing volunteers. In the end, the 1984 Olympic Games produced a profit of over $ 220 million.

Peter Ueberroth, who was originally from the area of business, created the chances to let ordinary people (not just the athletes) and corporations to take part in the Olympic Games, and alter people's impression of hosting Olympic Games.

Example 1.3. Chairman Ingenuity (adapted from [43])

A retiring corporate chairman invited to his ranch two finalists (A and B) from whom he would select his replacement using a horse race. A and B, equally skillful in horseback riding, were given a black and white horse, respectively. The chairman laid out the course for the horse race and said, "Starting at the same time now, whoever's horse is slower in completing the course will be selected as the next Chairman!" After a puzzling period, A jumped on B's horse and rode as fast as he could to the finish line while leaving his horse behind. When B realized what was going on, it was too late! Naturally, A was the new Chairman.

In the first two examples, new players, such as the passengers and the media in Example 1.1 and all the potential customers to the Olympic Games besides the athletes in Example 1.2, were introduced into the decision problem. In the third example, new rule/criteria were introduced, too. These examples show us that in reality, the players, criteria and alternatives (part of decision parameters) are not fixed; instead, they are dynamically changed. The dynamic changes of the relevant parameters play an important role in nontrivial decision problems. To help us understand the dynamic changes, let us introduce first the dynamics of human behavior, which basically is a dynamic MCDM in changeable spaces.

1.3 Dynamics of Human Behavior

Multicriteria decision making is only a part of human behaviors. It is a dynamic process because human behaviors are undoubtedly dynamic, evolving, interactive and adaptive processes. The complex processes of human behaviors have a common denominator resulting from a common behavior mechanism. The mechanism depicts the dynamics of human behavior.

In this section, we shall try to capture the behavior mechanism through eight basic hypotheses based on the findings and observations of psychology and neuron science. Each hypothesis is a summary statement of an integral part of a dynamic system describing human behavior. Together they form a fundamental basis for understanding human behavior. This section is a summary sketch of Yu [40–43, 48].

1.3.1 A Sketch of the Behavior Mechanism

Based on the literature of psychology, neural physiology, dynamic optimization theory, and system science, Yu described a dynamic mechanism of human behavior as presented in Fig. 1.1.

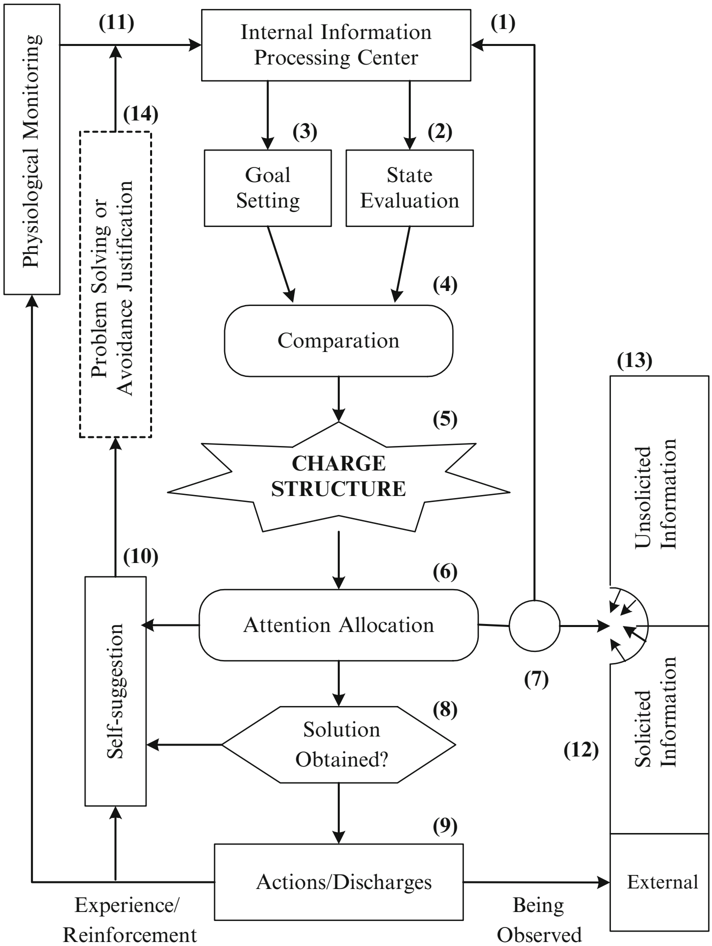

Fig. 1.1 The behavior mechanism

Although Fig. 1.1 is self-explanatory, we briefly explain it as follows:

1. Box (1) is our brain and its extended nerve systems. Its functions may be described by the first four hypotheses (H1–H4) shortly.
2. Boxes (2)–(3) describe a basic function of our mind. We use H5 to explain it.
3. Boxes (4)–(6) describe how we allocate our attention to various events. It will be described by H6.
4. Boxes (8)–(9), (10) and (14) describe a *least resistance principle* that humans use to release their charges. We use H7 to describe it.
5. Boxes (7), (12)–(13) and (11) describe the *information input* to our information processing center (Box (1)). Box (11) is internal information inputs. Boxes (7) and (12)–(13) are for external information inputs, which we use H8 to explain.

The functions described in Fig. 1.1 are interconnected, meaning that through time they can be rapidly interrelated. The outcome of one function can quickly become an input for other functions, from which the outcomes can quickly become an input for the original function.

Observe that the four hypotheses related to Box (1) which describe the information processing functions of the brain are four basic abstractions obtained from the findings of neuron science and psychology. The other Boxes (2)–(14) and hypotheses describe the input, output and dynamics of charges, attention allocation and discharge. They form a complex, dynamic multicriteria optimization system which describes a general framework of our mind. These eight hypotheses will be described in the following subsection.

1.3.2 Eight Hypotheses of Brain and Mind Operation

While the exact mechanism of how the brain works to encode, store and process information is still largely unknown, many neural scientists are still working on the problem with great dedication. We shall summarize what is known into four hypotheses to capture the basic workings of the brain. They are *Circuit Pattern Hypothesis (H1), Unlimited Capacity Hypothesis (H2), Efficient Restructuring Hypothesis (H3) and Analogy/Association Hypothesis (H4).*

The existence of life goals and their mechanism of ideal setting and evaluation lead to dynamic charge structures which not only dictate our attention allocation of time, but also command the action to be taken. This part of the behavior mechanism is related to how our mind works. We shall use another four hypotheses to summarize the main idea: *Goal Setting and State Evaluation Hypothesis (H5), Charge Structure and Attention Allocation Hypothesis (H6), Discharge Hypothesis (H7)* and *Information Inputs Hypothesis (H8).*

1. *Circuit Pattern Hypothesis (H1): Thoughts, concepts or ideas are represented by circuit patterns of the brain. The circuit patterns will be reinforced when the corresponding thoughts or ideas are repeated. Furthermore, the stronger the circuit*

patterns, the more easily the corresponding thoughts or ideas are retrieved in our thinking and decision making processes.

Each thought, concept or message is represented as a circuit pattern or a sequence of circuit patterns. Encoding is accomplished when attention is paid. When thoughts, concepts or messages are repeated, the corresponding circuit patterns will be reinforced and strengthened. The stronger the circuit patterns and the greater the pattern redundancy (or the greater the number of the circuit patterns), the easier the corresponding thoughts, concepts or messages may be retrieved and applied in the thinking and interpretation process.

2. *Unlimited Capacity Hypothesis (H2): Practically, every normal brain has the capacity to encode and store all thoughts, concepts and messages that one intends to.*

   In normal human brains, there are about 100 billion neurons that are interconnected by trillions of synapses. Each neuron has the potential capacity to activate other neurons to form a pattern. To simplify the situation for the moment and to ease computations, let us neglect the number of possible synapses between neurons and simply concentrate on only activated neurons. Since each neuron can be selected or not selected for a particular subset, mathematically the number of possible patterns that can be formed by 100 billion neurons is 2^{10^9}. To appreciate the size of that number, consider the fact that 2^{100} is equal to 1,267,650,600,228,329,401,496,703,205,376 (or 100 neurons). It suggests that the brain has almost infinite capacity, or for practical purposes, all the capacity that will ever be needed to store all that we will ever intend to store. According to neural scientists (see [2, 3, 27, 30]), certain special messages or information may be registered or stored in special sections of the brain, and only a small part of human brain (about percent) is activated and working for us at any moment in time. Therefore, the analogy described above is not a totally accurate representation of how the brain works. However, it does show that even a small section of the brain, which may contain a few hundred to a few million neurons, can create an astronomical number of circuit patterns which can represent an astronomical number of thoughts and ideas. In this sense, our brain still has a practically unlimited capacity for recording and storing information.

3. *Efficient Restructuring Hypothesis (H3): The encoded thoughts, concepts and messages (H1) are organized and stored systematically as data bases for efficient retrieving. Furthermore, according to the dictation of attention they are continuously restructured so that relevant ones can be efficiently retrieved to release charges.*

   Our brain puts all concepts, thoughts and messages into an organizational structure represented by the circuit patterns discussed earlier as H1. Because of charge structure, a concept to be discussed later, the organizational structure within our brain can be reorganized rapidly to accommodate changes in activities and events which can arise rapidly. This hypothesis implies that such restructuring is accomplished almost instantaneously so that all relevant information can be retrieved efficiently to effectively relieve the charge.

4. *Analogy/Association Hypothesis (H4): The perception of new events, subjects or ideas can be learned primarily by analogy and/or association with what is already known. When faced with a new event, subject or idea, the brain first investigates its features and attributes in order to establish a relationship with what is already known by analogy and/or association. Once the right relationship has been established, the whole of the past knowledge (preexisting memory structure) is automatically brought to bear on the interpretation and understanding of the new event, subject or idea.*
   Analogy/Association is a very powerful cognitive ability which enables the brain to process complex information. Note that there is a preexisting code or memory structure which can potentially alter or aid in the interpretation of an arriving symbol. For example, in language use, if we do not have a preexisting code for a word, we have no understanding. A relationship between the arriving symbol and the preexisting code must be established before the preexisting code can play its role in interpreting the arriving symbol.
5. *Goal Setting and State Evaluation (H5): Each one of us has a set of goal functions and for each goal function we have an ideal state or equilibrium point to reach and maintain (goal setting). We continuously monitor, consciously or subconsciously, where we are relative to the ideal state or equilibrium point (state evaluation). Goal setting and state evaluation are dynamic, interactive, and are subject to physiological forces, self-suggestion, external information forces, current data bank (memory) and information processing capacity.*
   There exist a set of goal functions in the internal information processing center which are used to measure the many dimensional aspects of life. Basically our mind works with dynamic multicriteria. A probable set is given in Table 1.1. Goal functions can be mutually associated, interdependent and interrelated.

Table 1.1 A structure of goal functions

1	*Survival and Security*: physiological health (correct blood pressure, body temperature and balance of biochemical states); right level and quality of air, water, food, heat, clothes, shelter and mobility; safety; acquisition of money and other economic goods
2	*Perpetuation of the Species*: sexual activities; giving birth to the next generation; family love; health and welfare
3	*Feelings of Self-Importance*: self-respect and self-esteem; esteem and respect from others; power and dominance; recognition and prestige; achievement; creativity; superiority; accumulation of money and wealth; giving and accepting sympathy and protectiveness
4	*Social Approval*: esteem and respect from others; friendship; affiliation with (desired) groups; conformity with group ideology, beliefs, attitudes and behaviors; giving and accepting sympathy and protectiveness
5	*Sensuous Gratification*: sexual; visual; auditory; smell; taste; tactile
6	*Cognitive Consistency and Curiosity*: consistency in thinking and opinions; exploring and acquiring knowledge, truth, beauty and religion
7	*Self-Actualization*: ability to accept and depend on the self, to cease from identifying with others, to rely on one's own standard, to aspire to the ego-ideal and to detach oneself from social demands and customs when desirable

6. *Charge Structures and Attention Allocation Hypothesis (H6): Each event is related to a set of goal functions. When there is an unfavorable deviation of the perceived value from the ideal, each goal function will produce various levels of charge. The totality of the charges by all goal functions is called the charge structure and it can change dynamically. At any point in time, our attention will be paid to the event which has the most influence on our charge structure.*
   The collection of the charges on all goal functions created by all current events at one point in time is the charge structure at that moment in time. The charge structure is dynamic and changes (perhaps rapidly) over time. Each event can involve many goal functions. Its significance on the charge structure is measured in terms of the extent of which its removal will reduce the levels of charges. Given a fixed set of events, the priority of attention to events at a moment in time depends on the relative significance of the events on the charge structure at that moment in time. The more intense the remaining charge after an event has been removed, the less its relative significance and the lower its relative priority. Thus attention allocation is a dynamic multicriteria optimization problem.
7. *Discharge Hypothesis (H7): To release charges, we tend to select the action which yields the lowest remaining charge (the remaining charge is the resistance to the total discharge) and this is called the least resistance principle.*
   Given the charge structure and the set of alternatives at time t, the selected alternative for discharge will be the one which can reduce the residual charge to the lowest level. This is the least resistance principle which basically is a concept of dynamic multicriteria optimization. When the decision problem involves high stakes and/or uncertainty, active problem solving or avoidance justification can be activated depending on whether or not the decision maker has adequate confidence in finding a satisfactory solution in due time. Either activity can restructure the charge structure and may delay the decision temporarily.
8. *Information Input Hypothesis (H8): Humans have innate needs to gather external information. Unless attention is paid, external information inputs may not be processed.*
   In order to achieve life goals, humans need to continually gather information. Information inputs, either actively sought or arriving without our initiation, will not enter the internal information processing center unless our attention is allotted to them. Allocation of attention to a message depends on the relevancy of the message to the charge structures. Messages which are closely related to long lasting events which have high significance in the charge structures can command a long duration of attention, and can, in turn, impact our charge structures and decision/behavior. Thus information inputs play an important role in dynamic MCDM.

1.3.3 Paradoxical Behavior

The following are some observations of human paradoxical behavior described in [43, 48]. They also appear in the decision making process regularly. We can verify them in terms of H1–H8 and specify under what conditions these statements may or may not hold.

- *Each one of us owns a number of wonderful and fine machines – our brain and body organs. Because they work so well, most of the time we may be unaware of their existence. When we are aware of their existence, it is very likely we are already ill. Similarly, when we are aware of the importance of clean air and water, they most likely have already been polluted.*
  This is mainly because when they work well, the charge structures are low and they will not cause our attention (H6). Once we are "aware" of the problems, the charge structure must be high enough so that we will pay attention to it. The reader may try to explore the charge structures and attention allocation of those people involved in Examples 1.1–1.3. The high levels of charges and dissolution make the examples interesting to us because they go beyond our habitual ways of thinking.
- *Dr. H. Simon, a Nobel Prize laureate, states that people have a bounded rationality. They do not like information overload. They seek satisfying solutions, and not the solution which maximizes the expected utility (see [35, 36]).*
  People gather information from different sources and channels, these messages may have significance in the charge structures and impact our decision making behavior (H8). They do not like information overload because it will create charges (H6). To release charges, people tend to follow the least resistance principle (H7) and seek for satisfying solutions instead of the solution which maximizes the expected utility (because the latter may create high charge structure!) Solutions that make people satisfied are those ones that meet people's goal setting and state evaluations (H5), they might not be the best answers but they are fair enough to solve the problems and make people happy. Again, here we clearly see the impact of the charge structure and attention allocation hypothesis (H6). Note that the challenging problems of Examples 1.1–1.3 were solved by jumping out of our habitual ways of thinking, no utility or expected utility were used.
- *Uncertainty and unknown are no fun until we know how to manage them. If people know how to manage uncertainty and unknown, they do not need probability theory and decision theory.*
  Uncertainty or unknown comes from messages that we are not able to judge or respond by our previous experiences or knowledge (H8). These experiences and knowledge form old circuit patterns (H1) in our brain/mind. When facing decision problems, we do not like the uncertainty and unknown which are new to us and we are unable to find matching circuit patterns to deal with them. This might create charges and makes us feel uncomfortable (H6). However, our brain has unlimited capacity (H2), by restructuring the circuit patterns (H1, H3) and the ability of analogy/association (H4), we can always learn new things and expand our knowledge and competence sets (the concept will be discussed in Section 1.5) to manage uncertainty/unknown. Examples 1.1–1.3 illustrate that much uncertainty and unknown are solved by expanding our competence for generating effective concepts and ideas, rather by using probability or utility theory.

- *Illusions and common beliefs perpetuate themselves through repetition and word of mouth. Once deeply rooted they are difficult to change (see [15]).*
  Illusions and common beliefs are part of the information inputs people receive everyday (H8) and they will form the circuit patterns in our brain. Through repetition and word of mouth, these circuit patterns will be reinforced and strengthened because the corresponding ideas are repeated (H1). Also, the stronger the circuit patterns, the more easily the corresponding thoughts are retrieved in our thinking and decision making processes, this explains why illusions, common beliefs or rumors can usually be accepted easier than the truth. In history, many famous wars were won by creating effective illusions and beliefs. Such creation in fact is an important part of war games.
- *When facing major challenges, people are charged, cautious, exploring, and avoiding making quick conclusions. After major events, people are less charged and tend to take what has happened for granted without careful study and exploration.*
  This is a common flaw when we are making decisions. Major challenges or serious problems are information that have high significance in the charge structures and can command our attention, so we will be cautious and avoiding making rough diagnostic (H6, H8). After major events, the decision maker's stake is low so that his/her attention will be paid to other problems that cause higher charge structures. As we read Examples 1.1–1.3, we are relaxed and enjoying. Those people involved in the examples might, most likely, be fully charged, nervous and exploring all possible alternatives for solving their problems.

For more paradoxical behaviors, please refer to [43, 48].

1.4 Habitual Domains

Our behavior and thinking are dynamic as described in the previous section. This dynamic change of charge makes it difficult to predict human behavior. Fortunately, these dynamic changes will gradually stabilize within certain domains. Formerly, the set of ideas and concepts which we encode and store in our brain can over a period of time gradually stabilize in certain domain, known as *habitual domains*, and unless there is an occurrence of extraordinary events, our thinking processes will reach some steady state or may even become fixed. This phenomenon can be proved mathematically [4, 42] as a natural consequence of the basic behavior mechanism (H1–H8) described in Section 1.3. As a consequence of this stabilization, we can observe that every individual has his or her own set of habitual ways of thinking, judging and responding to different problems, events and issues. Understanding the habitual ways of making decisions by ourselves and others is certainly important for us to make better decisions or avoid expensive mistakes. Habitual domains was first suggested in 1977 [38] and further developed [4, 40–44, 48] and quotes therein by Yu and his associates.

In this section, we shall discuss the stability and concepts of habitual domains and introduce the tool boxes for their expansion and enrichment so that we can make good use of our habitual domains to improve the quality of decision making and upgrade our lives. In fact, the concept of habitual domains is the underlying concept of competence set analysis to be introduced subsequently.

1.4.1 Definition and Stability of Habitual Domains

By the *habitual domain at time t*, denoted by HD_t, we mean the collection of ideas and actions that can be activated at time t. In view of Fig. 1.1, we see that habitual domains involve self-suggestion, external information, physiological monitoring, goal setting, state evaluation, charge structures, attention allocation and discharges. They also concern encoding, storing, retrieving and interpretation mechanisms (H1–H4). When a particular aspect or function is emphasized, it will be designated as "habitual domain on that function." Thus, habitual domain on self-suggestion, habitual domains on charge structures, habitual domain on attention, habitual domain on making a particular decision, etc. all make sense. When the responses to a particular event are of interest, we can designate it as "habitual domains on the responses to that event," etc. Note that conceptually habitual domains are dynamic sets which evolve with time.

Recall from H1 that each idea (thought, concept, and perception) is represented by a circuit pattern or a sequence of circuit patterns; otherwise, it is not encoded and not available for retrieving. From H2, we see that the brain has an infinite capacity for storing encoded ideas. Thus, $|HD_t|$, the number of elements in the habitual domain at time t, is a monotonic nondecreasing function of time t.

From H4 (analogy and association), new ideas are perceived and generated from existing ideas. The larger the number of existing ideas, the larger the probability that a new arriving idea is one of them; therefore, the smaller the probability that a new idea can be acquired. Thus, $|HD_t|$, although increasing, is increasing at a decreasing rate. If we eliminate the rare case that $|HD_t|$ can forever increase at a rate above a positive constant, we see that $|HD_t|$ will eventually level off and reach its steady state. Once $|HD_t|$ reaches its steady state, unless extraordinary events occur, habitual ways of thinking and responses to stimuli can be expected.

Theoretically our mind is capable of almost unlimited expansion (H2) and with sufficient effort one can learn almost anything new over a period of time. However, the amount of knowledge or ideas that exist in one's mind may increase with time, but the rate of increment tends to decrease as time goes by. This may be due to the fact that the probability of learning new ideas or concepts becomes lower as a number of ideas or actions in the habitual domain are larger. These observations enable us to show that the number of ideas in one's HD_t converges when suitable conditions are met. The followings are mathematically precise models which describe conditions for stability on the number of elements in habitual domains.

Let us introduce the following notation:

1. Let a_t be the number of additional new ideas or concepts acquired during the period $(t-1,\ t]$. Note that the timescale can be in seconds, minutes, hours, or days, etc. Assume that $a_t \geq 0$, and that once an idea is registered or learned, it will not be erased from the memory, no matter whether it can be retrieved easily or not. When a particular event is emphasized, a_t designates the additional ideas or concepts acquired during $(t-1,\ t]$ concerning that event.
2. For convenience, denote the sequence of a_t throughout a period of time by a_t. Note that due to the biophysical and environmental conditions of the individuals, a_t is not necessarily monotonic. It can be up or down and subject to certain fluctuation. For instance, people may function better and more effectively in the morning than at night. Consequently, the a_t in the morning will be larger than that at night. Also observe that a_t may display a pattern of periodicity (day/night for instance) which is unique for each individual. The periodicity can be a result of biophysical rhythms or rhythms of the environment.

The following can readily be proved by applying the ratio test of power series. The interested readers please refer to [4, 42] for further proof.

Theorem 1.1. *Suppose there exists T such that whenever* $t > T$, $\frac{a_t+1}{a_t} \leq r < 1$. *Then as* $t \to \infty$, $\sum_{t=0}^{\infty} a_t$ *converges.*

Theorem 1.2. *Assume that (i) there exists a time index s, periodicity constant* $m > 0$, *and constants D and M, such that* $\sum_{n=0}^{\infty} a_{s+nm} \leq D$, *where* a_{s+nm} *is a subsequence of* a_t *with periodicity m, and (ii) for any period n,* $\left(\sum_{i=1}^{m} a_{s+nm+i}\right)/ma_{s+nm} \leq M$. *Then* $\sum_{t=0}^{\infty} a_t$ *converges.*

Note that for habitual domain to converge, Theorem 1.2 does not require a_t to be monotonically decreasing as required in Theorem 1.1. As long as there exists a convergent subsequence, and the sum of a_t within a time period of length m is bounded, then a_t can fluctuate up and down without affecting the convergence of HD_t. Thus the assumptions in Theorem 1.2 are a step closer to reality than those in Theorem 1.1.

Another aspect of the stability of habitual domains is the "strength" of the elements in HD_t to be activated, which is called *activation probability*.

Define $x_i(t)$, $i \in HD_t$, to be the *activation probability* of element i at time t. For simplicity let $HD_t = 1, 2, \ldots, n$ and $x = (x_1, \ldots, x_n)$. Note that n, the number of elements in HD_t, can be very large. As $x_i(t)$ is a measurement of the force for idea i to be activated, we can assume that $x_i(t) \geq 0$. Also $x_i(t) = 0$ means that idea i cannot be activated at time t, by assigning $x_i(t) = 0$ we may assume that HD_t contains all possible ideas of interest that may be acquired now and in the future.

Similar to charge structure, $x_i(t)$ may be a measurement of charge or force for idea i to occupy the "attention" at time t. Note that $x_i(t)/\sum_i x_i(t)$ will be a measurement of relative strength for idea i to be activated. If all $x_i(t)$ become stable after some time, the relative strength of each i to be activated will also be stable. For stability of $x_i(t)$, the interested reader may refer to [4, 42] for mathematical derivation and further discussion.

1.4.2 Elements of Habitual Domains

There are two kinds of thoughts or memory stored in our brain or mind: (1) the ideas that can be activated in thinking processes; and (2) the operators which transform the activated ideas into other ideas. The operators are related to thinking processes or judging methods. In a broad sense, operators are also ideas. But because of their ability to transform or generate (new) ideas, we call them operators. For instance, let us consider the numerical system. The integers $0, 1, 2, \ldots$ are ideas, but the operation concepts of $+, -, \times, \div$, are operators, because they transform numbers into other numbers.

Habitual domains at time t, HD_t, have the following four subconcepts:

1. *Potential domain*, designated by PD_t, which is the collection of all ideas and operators that can be potentially activated with respect to specific events or problems by one person or by one organization at time t. In general, the larger the PD_t, the more likely that a larger set of ideas and operators will be activated, holding all other things equal.
2. *Actual domain*, designated by AD_t, which is the collection of ideas and operators which actually occur at time t. Note that not all the ideas and operators in the potential domain can actually occur. Also note that the actual domain is a subset of the potential domain. That is $AD_t \subset PD_t$.
3. *Activation probability*, designated by AP_t, which is defined for each subset of PD_t and is the probability that a subset of PD_t is actually activated or is in AD_t. For example, people who emphasize profit may have a greater frequency to activate the idea of money. Similarly, people who study mathematics may have a greater frequency to generate equations.
4. *Reachable domain*, designated by $R(I_t, O_t)$, which is the collection of ideas and operators that can be generated from the initial idea set (I_t) and the initial operator set (O_t). In general, the larger the idea set and/or operator set, the larger the reachable domain.

At any point in time, without specification, habitual domains (HD_t) will mean the collection of the above four subsets. That is $HD_t = \{PD_t, AD_t, AP_t, R(I_t, O_t)\}$. In general, the actual domain is only a small portion of the reachable domain, while the reachable domain is only a small portion of the potential domain, and only a small portion of the actual domain is observable. This makes it very difficult for us to observe other people's habitual domains and/or even our own habitual domains. A lot of work and attention is therefore needed in order to accomplish that. For further discussion, see [42, 43, 48].

As a mental exercise, it might be of interest for the reader to answer: "With respect to the players or rules of games, how the PD_t, AD_t and RD_t evolve over time in Examples 1.1–1.3?" Note that it is the expansion of the relevant HD_t that get the challenge problems solved. We will further discuss this later.