Nerida Ellerton M.A. (Ken) Clements

Rewriting the History of School Mathematics in North America 1607–1861

The Central Role of Cyphering Books



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Foreword by Jeremy Kilpatrick



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Foreword

In an encyclopedia by the German monk Gregor Reisch first published in 1503, a well-known woodcut *Typus Arithmeticae* portrays Dame Arithmetic presiding over what is apparently a competition between Boethius, who is writing calculations in Hindu-Arabic numerals using a pen, and Pythagoras, who is using a counting board. Arithmetic gazes approvingly toward the confident algorist Boethius, who seems to have finished his calculation, while the abacist Pythagoras looks stumped. Although Reisch has made a curious choice of competitors, he has correctly anticipated the outcome of the contest. The replacement of the abacus by written algorithms for computing in Western Europe took several centuries and was not completed until the late sixteenth century. Once writing had been established, however, as the way arithmetic—not to mention the rest of mathematics—was to be done, it enjoyed a long reign and spread quickly to the colonies.

North American historians of education have not always attended to the prominent role that writing played in teaching pupils mathematics. Instead, they have focused attention on the heavy reliance that American mathematics teachers have had on textbooks. Certainly that reliance has been profound. After the U.S. Civil War, the rise of the age-graded school coupled with the rapid Western expansion of education and remarkable growth of public high schools led to a shortage of welleducated teachers, especially teachers of mathematics. Commercially published mathematics textbooks, along with so-called ancillary materials, were written and marketed to help semi-skilled teachers cope with mathematical ideas that they might not understand very well. Although it is impossible to determine whether, over the past century and a half, mathematics teachers in North America have relied more heavily on textbooks than teachers in other countries, their textbooks have certainly been bulkier, more elaborate, and more heavily scripted than textbooks elsewhere. Moreover, textbooks have been the principal conduits for attempts to change the mathematics curriculum, as shown by federally funded projects to create new textbooks during the roughly two decades of the new math era and the more recent so-called standards era.

A preoccupation with textbooks has led historians to ignore or devalue the way cyphering books were used to learn mathematics in North America during the two and a half centuries preceding 1861. They have tended to assume that in a time before textbooks became commonplace in mathematics classrooms, pupils simply copied from each other or from whatever book their teacher had without thinking too much about what they were doing. The pedagogical value of writing as a means of formulating and consolidating one's mathematical thinking is easily overlooked once printed materials become readily available. But for centuries, the principal way that young scholars learned and remembered mathematics was by writing it carefully in their own treasured cyphering books.

As Ellerton and Clements show, the cyphering approach to learning mathematics has both an extensive history and a complex rationale. In the approach, the learner was to become an independent problem solver by learning to identify problems of various types, learning the rules for their solution, solving such problems, having each solution checked by a tutor or teacher, and only when the solution was correct, copying that solution into a cyphering book using exemplary calligraphy. The handwriting process was not meant to be only copying; it was to be an accompaniment to thinking. The learner was not simply inscribing the solution on to paper; he or she was inscribing it into memory.

Cyphering books are a much neglected resource for understanding and appreciating the early history of North American mathematics education. The present book marks a major advance in putting that resource to effective use as well as in raising some challenging questions for subsequent research.

Athens, Georgia

Jeremy Kilpatrick

Abstract

An analysis is presented of the historical, societal, and mathematics education practices that defined what we call the "cyphering tradition" which prevailed in the North American (mainly British) colonies between 1607 and 1776, and then in the United States of America for most of the period between 1776 and 1861. The main data set comprises 270 handwritten cyphering books, of which 212 originated in North America between 1701 and 1861.

The following five research questions provided the foci for the study:

- 1. Where and when were the cyphering books prepared, and by whom?
- 2. Were the manuscripts consistent with *abbaco* traditions, especially in relation to mathematical content and sequencing, genre, and "writing as also arithmeticke"?
- 3. What theoretical base, if any, can be identified to encapsulate the educational purposes of the cyphering books?
- 4. Why, between 1840 and 1861, was there such a sharp decline in the use of the cyphering approach in the Unites States?
- 5. What were the main educational advantages and disadvantages of the cyphering approach?

It was found that most of the North American manuscripts were prepared between 1780 and 1850 in the North-Eastern regions of what is now the United States of America. About 83 percent of them were prepared by males, usually during winter months. After 1840, the extent of the use of cyphering books declined sharply. Three reasons are identified for the decline: first, cyphering books were no longer regarded as important in relation to evaluating the quality of an instructor's teaching or of a student's learning; second, key state education leaders favoured pedagogical reform by which mathematics would be taught to graded classes, rather than to individuals; and third, young teachers graduating from state normal schools discouraged the use of the cyphering approach, especially in the new high schools that were being established.

Preface

The term "rewriting" in the title of this book merits comment. In this work we argue that published histories dealing with the teaching and learning of school mathematics in the North American colonies, and during the first 85 years of the United States of America, have not recognized the fundamental importance of what we call the "cyphering tradition," and have over-emphasized the role of textbooks. We provide evidence from numerous primary sources that printed texts were less used and, from a mathematics teaching and learning perspective, were less important, than handwritten manuscripts prepared by learners. These handwritten documents were referred to as "cyphering books" by the students who prepared them, and by their instructors. We argue that a cyphering tradition, imported from European nations to the North American colonies, provided both the practical and the theoretical base for early North American mathematics education.

After reviewing the European background to the cyphering tradition, we show how the cyphering tradition was translated into the North American colonies, where it remained the dominant influence on mathematics teaching and learning until well into the 19th century.

Historians of mathematics education in North America have often written more from the college than from the school perspective. Furthermore, because it has been relatively easy to access a wide range of mathematics textbooks, and descriptions of mathematics courses used in various colleges in the 17th, 18th centuries and early 19th centuries, histories have often been more concerned with elaborating the *intended* rather than *implemented* or *attained* curricula.

The principal data set, on which much of the analyses summarized in this book were based, is the largest-known single collection of North American cyphering books. We have linked our analyses of these data with analyses of other published and unpublished materials, including textbooks. Most of the 212 cyphering books in the principal data set were prepared by school students and not college students, and hence we have been able to look closely at what went on in the name of mathematics for students who did not necessarily intend to proceed to college.

We wish to thank librarians and archivists in the Phillips Library (Peabody Essex Museum, Salem, Massachusetts), the Butler Library (Columbia University, New York), the Clements Library (University of Michigan), the Houghton Library

(Harvard University), the American University (Washington, DC), the Wilson Library (University of North Carolina at Chapel Hill), the Special Collections Research Center in the Swem Library at the College of William and Mary, and the Rockefeller Library (both in Williamsburg, Virginia), the New York Public Library, Guildhall Library (London), the London Metropolitan Archives, and the Milner Library (Illinois State University) for locating relevant manuscripts, artifacts, and books for us.

We would particularly like to thank Dr George Seelinger, the Head of the Mathematics Department at Illinois State University (in which we both work) for encouraging us in our research endeavors. We also thank Dr Roger Eggleton for insightful comments which helped us finalize the title of the book, and improve the wording of parts of this preface.

Normal, Illinois

Nerida Ellerton M.A. (Ken) Clements

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Chapter 1 The Historical Challenge

Abstract Between 1607 and 1840 school mathematics in North America was very different from school mathematics today. Throughout that period, the cyphering tradition dictated the mathematics curriculum and the ways mathematics was taught and learned. Early North American teachers did not stand at the front of the room and teach, and most students, even those studying mathematics, did not own a mathematics textbook. Written examinations of any kind were not used. Most teachers of any branch of mathematics did not have formal qualifications in mathematics. Those studying mathematics copied, into their own cyphering books, rules, cases and examples from existing handwritten cyphering books. Definitions of key terms (e.g., "cyphering book") are given.

Mathematics teachers are often accused of being full of inertia, in the sense that they are reluctant to change curricula or teaching approaches (Taylor, 2002). The standard explanation for this perceived state of affairs is that because teachers' daily professional lives are prescribed by regulations, timetables, entitlements and expectations, they become comfortable working in ways which, they believe, work for them (Lortie, 1975). Furthermore, evaluators tend to assess curricula and teaching approaches in terms of what has occurred in the past, rather than what might be possible if new opportunities (arising out of, say, technological advances) were grasped (Papert, 1980).

Thus, so the argument goes, no matter how much society might think large-scale changes are needed in mathematics teaching and learning, these are very difficult to achieve. From this line of argument, it is not difficult to reach the conclusion that present mathematics teaching methods are likely to be similar to those used 100 or even 200 years ago.

But, in fact, such a conclusion would be well wide of the mark, for mathematics education practices in the United States of America are very different now from what they were in the 18th and early 19th centuries in the colonies and states. In the early 1800s many school-age children in the United States rarely attended school. Of those who did attend, many (especially boys in the New England colonies or states) attended during winter months only, and did not study any mathematics beyond elementary arithmetic. As late as the 1870s, the hero of Mark Twain's *Adventures of Huckleberry Finn* went to school only "three or four months" during which time he learnt the rudiments of reading, writing, spelling, and "the multiplication table up to six times seven is thirty-five" (Clemens, 1885, p. 18).

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Two centuries ago, American teachers did not stand at the front of the room and teach, and most students, even those studying mathematics, did not own a mathematics textbook. Written examinations of any kind were not used. Most teachers of any branch of mathematics did not have formal qualifications in mathematics.

In this book we review changes to the principal method of teaching and learning mathematics in North America in the 17th and 18th centuries. Our analysis for the period 1840–1860 reveals that in just two decades a cyphering tradition, that had spread to Europe and America after originating in India and in Arabic nations, and had endured for over 600 years, was done away with in North American schools. We shall offer an explanation for how and why this occurred.

In providing details of the early traditions relating to the teaching and learning of mathematics in the North American colonies and, between 1776 and 1861, within the United States of America, we shall present an account of developments that differs from those offered in other histories. For example, unlike other analyses of school mathematics in colonial times, we will not emphasize the role of mathematics textbooks, arguing that until about 1800 such textbooks were little used in the North American colonies, except in colleges like Harvard and Yale. Our thesis will be that the main teaching and learning aid for mathematics education was the cyphering book, and that instruction and learning were consistent with what we have called the "cyphering tradition." We shall also draw attention to ethnomathematical contexts in which children and adults learned mathematics outside of formal education institutions.

Much of the focus of this book will therefore be on the cyphering tradition as it operated in North America before the 1860s. We shall begin by defining the terms "cyphering book," "*abbacus* school," and "mathematical activity," and then move on to a description of what constituted the cyphering tradition. We shall argue that this tradition was transplanted from Europe to the North American colonies, and had a pronounced influence on mathematics education practices during the first 250 years of European settlement in North America.

Following a historical overview of factors influencing the cyphering tradition, the set of questions that guided the research will be presented, and the principal data set will be summarized. Subsequent analyses will form the basis of our reconstruction of the early history of North American school mathematics.

Clarifying Concepts, and Associated Nomenclature

Schools

The concept of "school" in this book will embrace a wide range of educational settings. Included will be "dame schools," "common schools," "public schools," "subscription schools," "private schools," "evening schools," "writing schools," "high schools," and "apprenticeship schools." Although, sometimes, words like "academy" will be used instead of "school," whenever we use the word "school"

(or "academy," etc.) we will be referring to education environments, both formal and informal, in which at least one "teacher" regularly met with a gathering of "students," at an agreed place, for the purpose of helping those students to learn the elementary facts, concepts, and skills associated with at least one particular area of knowledge. Our use of "school" embraces far more than just "private" and "public schools."

Writing in 1921, Robert Seybolt pointed out that evening schools in urban centers had played important roles in the history of education in 17th-, 18th- and early 19th-century North America. Seybolt (1921) wrote of evening schools in colonial New York:

The available records indicate that the earliest ... offered instruction only in the rudiments—reading, writing and cyphering. It is probable that these were attended exclusively by apprentices. In some instances, adults may have received such evening instruction, but on this matter the records are silent. (p. 630)

Right through the colonial period, schools were an important part of society. Thus, for example, although the population of the "city" of New York around 1700 was only about 5000 (Oswald, 1917), there were several evening schools within the city at that time (Seybolt, 1921).

Note that in this book higher-level colleges—such as Columbia, Harvard, William and Mary, and Yale—will not be regarded as "schools," for in the 17th and 18th centuries, such higher-level institutions were usually sharply distinguished from schools.

Cyphering Books

In this book we argue that before 1840, students in North America who attempted to learn any branch of mathematics usually did so through a cyphering approach (Meriwether, 1907). That statement raises at least two questions: (a) What were the main elements of the cyphering approach? And, (b) Why was this approach used so extensively in the United States until about 1840? We shall answer both of those questions.

We begin by defining a *cyphering book* as a handwritten manuscript with the following four properties:

- 1. Either the contents were written by a student who, through the act of preparing it, was expected to learn and be able to apply whatever content was under consideration; or, the book was prepared by a teacher who wished to use it as a model that could be followed by students preparing their own cyphering books.
- 2. All entries in the book appeared in ink—either as handwriting or as illustrations. Headings and sub-headings were presented in a decorative, calligraphic style.
- 3. The book was dedicated to setting out rules and cases associated with a sequence of mathematical topics, with each topic being followed by problems linked with that topic. The problems were in arithmetic, especially business arithmetic, or

in algebra, or geometry, or trigonometry, or were applications of mathematics in the fields of navigation, surveying, military strategy, etc.

4. The topics covered were sequenced so that they became progressively more difficult. The content also reflected the expectation that, normally, no child less than 10 years of age would be assigned the task of preparing a cyphering book.

Most of the cyphering books in the principal data set (hereafter "PDS") for this study dealt with just one branch of mathematics (e.g., usually arithmetic, but sometimes algebra or geometry, etc.). Occasionally, a cyphering book had entries from several areas of mathematics. In the 18th and 19th centuries in North America, cyphering books were usually made up of unlined, rectangular folio-sized paper sheets (in the 18th century, "rag" paper was often used) with dimensions about 12.5" by 8". The pages, grouped as sections, were routinely sewn together to form books (in the form of handwritten manuscripts). Sometimes, protective covers were added. Typically, the first page of a book was beautifully decorated, and indicated the name of the owner as well as the year(s) and location in which the book was prepared.

Most cyphering books dealt with a number of topics. Although, usually, the treatment of any one topic occurred on successive pages, it was not uncommon for topics to be revisited at various times in the same cyphering book. Cyphering books were intended to serve as reference books for future use by those who prepared them.

In addition to our collection of cyphering books we own other handwritten manuscripts that, although intended for school mathematics purposes, do not fit our definition of a cyphering book. Thus, for example, we have an original handwritten manuscript written by an 18th-century author who had prepared it for possible publication as a textbook on geometry, trigonometry, and navigation. We also have a manuscript, prepared by a school administrator during the period 1828–1852, that set out how arithmetic was to be taught in a Canadian school that adopted the Lancastrian monitorial approach. In addition, we have a collection of handwritten documents in which writers concentrated solely on stating and solving mathematics problems—in these manuscripts, introductory summaries, and rules and cases were not stated, and therefore we have not regarded them as cyphering books.

Abbacus School

According to Jens Høyrup (2008), *abbacus* schools, which operated in Italy from the 13th century, were

... primarily frequented by merchant and artisan youth for ... two years (around the age of 11), who were taught the mathematics needed for commercial life: calculation with the Hindu-Arabic numerals; the rule of three; how to deal with the complicated metrological and monetary systems; alloying; partnership; simple and composite discount; the use of "single false position"; and area computation. Smaller towns might employ a master; in towns like Florence and Venice private *abbacus* schools could flourish. In both situations *abbacus* masters had to compete, either for communal positions or for the enrolment of students. (pp. 4–5)

Mathematical Activity and Mathematics Education

The term "mathematical activity" will be identified with making numerical calculations or measurements of quantity, or reasoning about relationships between physical or abstract quantities. Thus, for example, arithmetic, algebra, geometry, trigonometry, mensuration, calculus, and their direct applications (for example, in surveying or navigation), will be considered to be forms of pure or applied mathematics.

Mathematics education encompasses all matters concerned with the teaching or learning of mathematics, or with curricular issues arising from considerations of what mathematics might profitably be taught and learned by well-defined groups of people.

Nomenclature

In this book we shall use the term "cyphering book" rather than any of "ciphering book," "cypher book," "cipher book," "copybook," "copy book," "notebook," or "exercise book." In the 18th and early 19th centuries, in Great Britain and in North America, the three terms "cyphering book," "ciphering book," and "copybook" were often used to describe the same thing-sometimes the same person used the different terms interchangeably. We shall use "cyphering" in preference to "ciphering" because an "advanced Google search" revealed that during the period 1607–1820 "cyphering" (with a "y") was more commonly used in North America and in England than was "ciphering" (with an "i"). However, in 1788, Isaiah Thomas published an American version of William Perry's (1788) Royal Standard English Dictionary, and in this "cipher" was spelt with an "i." This continued in subsequent American editions of the Royal Standard Dictionary. In 1828, Noah Webster released his influential American Dictionary of the English Language, and in that book Webster gave "ciphering" priority over "cyphering." An "advanced Google search" revealed that after 1820 "ciphering" began to be more commonly used in published works than "cyphering." After about 1850, "cyphering" was rarely used.

The term "copybook" (or "copy book") suggests that everything was *copied* from another text. That was not always the case for manuscripts in our principal data set (PDS) described later in this book. Furthermore, the term copybook was also commonly used to describe handwritten manuscripts that dealt solely, or mainly, with non-mathematical content. The word "cyphering" implies the *writing of numerals*, which was the case in every manuscript in our PDS—including those mainly concerned with algebra, geometry, trigonometry, navigation or surveying.

Some writers (e.g., Williamson, 1928; Yeldham, 1936) have preferred the term "exercise book" to "cyphering book" or "ciphering book"—indeed, Yeldham, in her history of the teaching of arithmetic in Great Britain between 1535 and 1935, devoted a whole chapter to "the exercise book." However, we distinguish between cyphering books and exercise books for four reasons. First, in the 18th and 19th centuries "exercise book" was less used in North America than in Great Britain in

relation to mathematical writing of students. Second, the term has subsequently been applied to manuscripts for any subject (and not just to mathematics manuscripts). Third, North American "cyphering books" were often larger in format than "exercise books"; also, their pages were usually unlined, whereas those in exercise books were usually lined. Fourth, exercise books used for mathematics were often manuscripts in which students wrote little more than solutions to problems set by the teacher. Since it was intended that cyphering books would become personal reference manuals, they contained definitions and statements of rules and cases, as well as problem statements and solutions.

We also distinguish between *abacus* boards and *abbacus* books. The former, which were physical devices on which reckoning was carried out (Howson, 1982), will not be the subject of much attention in this book. Although reckoning masters and students often used *abbacus* books and *abacus* boards in the same transactions, there was no connection, other than etymological, between the two (Franci, 2009; Van Egmond, 1980).

Chapter 2 Development of a Tradition

Abstract The cyphering tradition which controlled North American school mathematics between 1607 and 1840 was imported from Western European nations. The curriculum within the tradition had a strong mercantile emphasis, and was most suited to boys who wished to gain employment as clerks or reckoners, or as navigators or surveyors. This chapter traces how the cyphering tradition, which was originally developed in India and in Arab nations, was introduced into Italian city states around 1200. Gradually, it was adopted in Western Europe, including France, the Germanic states, Spain, Portugal, the Netherlands, and Great Britain.

Origins of the Cyphering Approach to Mathematics Education in Western Europe

In order to be in a position to understand the origins of the cyphering approach in mathematics education, one needs to appreciate how commercial, political, military and socio-cultural factors affected the development of numeracy in Western European nations and city-states or republics between 1200 and 1600. During that 400-year period, merchants increasingly exported and imported goods to and from distant, and hitherto unreachable, markets (Burke & Burke, 1758; Kretschmer, 1909; Long, McGee, & Stahl, 2009). Despite the ever-present danger of piracy and the difficulty of ensuring accurate navigation (Falola & Warnock, 2007), the increasing availability of faster and safer ships encouraged entrepreneurs to risk overseas venture capital (Howson, Keitel, & Kilpatrick, 1981; Swetz, 2005). The need for improved navigation practices spawned a group of "practical mathematicians" (Taylor, 1954), who invented magnetic compasses and other aids that helped speed up international commercial transactions (Collinder, 1954; Mahan, 1898).

The 17th century was the "golden age" in Dutch history, a time when the Netherlands was the world's greatest sea power. The Dutch East India Company (*Vereenigde Oostindishche Compagnie*, or VOC, in Dutch) was the first company in the world to issue stock, and by 1669 it had become the richest private company the world had ever seen. Not only did it have 150 merchant ships, 40 warships, 50,000 employees, and a private army of 10,000 soldiers, but, with respect to commercial activities it increasingly assumed many of the functions of the national government (Ames, 2008). Within that context, mathematics education became

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a vitally important matter, not only to the Netherlands, but also to competing commercial powers such as England.

Because the Dutch were instrumental in the early development of New York, Dutch coinage joined the Spanish, Portuguese, and British coins used for daily transactions in the American colonies. The arithmetic associated with currency exchange, and commercial investment, and the geometry associated with fast and accurate navigation, were matters of great importance for both Europe and America, especially for government authorities and merchants who had their eyes on local and national economic advantage (Kretschmer, 1909).

Merchants, keen to monitor and maximize profits, introduced new accounting and business procedures involving double entry bookkeeping, letters of credit, bills of exchange, and marine insurance (Long et al., 2009; Swetz, 1987, 2005). They increasingly looked for partnerships that would last for years rather than for a single transaction. Merchants began to specialize in what they did and in the materials they handled. Furthermore, the exploitation of cheap labor in Africa, the Caribbean Islands, and the Americas, and the use of indentured servants (often brought from England), promised to usher in an era of economic prosperity for the colonizers (Archer, 1988; Burke & Burke, 1758; Franci, 1992, 2009; Franci & Rigatelli, 1982, 1989; Long et al., 2009; Radford, 2003; Taylor, 1956; Van Egmond, 1980).

Eventually, prosperous merchant traders could be found in major West European cities—such as Florence, Genoa, Milan, Pisa, Venice, Geneva, Marseilles, Lisbon, London, Nuremberg, Leiden, and Amsterdam. Companies which needed to ship their wares to other cities and other nations developed central headquarters with networks of representatives along the principal trade routes. In the 13th century the principal trading city-states linked with Germany went so far as to create a semi-formal alliance of trading cities and guilds known as the Hanseatic League. This was established for the specific purpose of maintaining and extending a trade monopoly, and it remained in operation for several hundred years. Its primary purpose was to improve profits of member organizations by protecting trade routes, and coordinating commerce. The complex web of captains of business, accountants, shipping agents, other shipping personnel, distributors, labor-controllers, and manufacturers rapidly changed the face of international commerce (Kretschmer, 1909; Long, 2009).

Business leaders increasingly recognized that being able to calculate, predict, and control profit was important, and hence they sought out skilled "reckoners" who understood the new sophisticated arithmetic that had developed in relation to Hindu-Arabic numeration (Franci, 2009). These reckoners were expected to take advantage of place-value notation within the Hindu-Arabic numeration system and, in particular, to apply the four operations on whole numbers and fractions with currency exchange, weights, and other measures, discount, simple and compound interest, ratio and proportion tasks, and partnership calculations. The aim was to maximize the profits of those merchants who employed them.

As mathematics developed, the reckoners were expected to be responsible for harnessing its powers within areas of commerce involving *abbacus* concepts such as alligation, false position, fellowship, gauging, the rules of three, and tare and tret