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Eddy Covariance

A Practical Guide to Measurement and Data Analysis

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A Practical Guide to Measurement and Data Analysis

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Preface

As soon as the first eddy covariance networks developed, in the mid-1990s, the need for standardisation became clear. Standardisation concerned not only material but also data treatment, corrections, computation. In order to harmonise these procedures in the frame of the EUROFLUX network, some software intercomparison exercises were developed: "golden files" were circulated between teams and treated with different software packages with the aim to compare the computation results. It rapidly appeared that beyond some bugs that appeared in new software and were corrected immediately, important differences remained between different computations that were due to the use of different hypotheses. The necessity to clarify these choices, and to propose a standardised (even if perfectible) eddy covariance flux computation procedure, led us to publish a first methodological paper (Aubinet et al. 2000). Eleven years later, this paper remains an often cited reference in the field.

However, as the theory and measurement techniques progressed since, and since the eddy covariance techniques is becoming also a monitoring exercise and not more only a purely scientific activity, the necessity of an update of this paper and of creating something that could help to install an eddy covariance site and manage it correctly grew. In December 2008, during a meeting at the Hyytiälä Forestry Field Station (Finland) celebrating the tenth anniversary of the EUROFLUX network constitution, the idea was launched (originally by Samuli Launiainen) to produce such an update. However, it appeared rapidly that if we wanted to produce a self-standing document, useful to eddy flux practisers, we could not limit its size to those of a paper.

We thus decided to tackle the edition of a book with the general objective to give to eddy flux practisers the theoretical and practical information necessary in order to develop eddy covariance measurements, from site installation to data treatment. After preparing a book plan, structured in 17 chapters, we chose different first authors, known for their skills in the field and asked them to constitute a team of co-authors and prepare their chapters. The present book is the result of the two and half year long work that followed.

After a first chapter recalling the theoretical bases on which eddy covariance method relies, Chap. 2 describes technical requirements of the eddy covariance set-up: tower positioning and dimensioning (height, position, system positioning on the tower), sonic and gas analysers, dimensioning, calibration and maintenance.

Chapter 3 describes the general procedure used in order to get "uncorrected" fluxes and to discuss the pros and cons of different computation alternatives. This implied especially a description of the data acquisition set-ups, and a detailed discussion on flux computation (fluctuation computation, first quality control on raw data, time lagging, rotation and flux computation).

Chapter 4 concentrates on the different corrections procedures necessary in order to get good quality fluxes and on the quality tests on these fluxes.

Chapter 5 focuses on the problem of night flux underestimation, its causes and its impact on flux measurements. It described different screening or correction procedures and discussed their pros and cons.

Chapter 6 specifies the conditions when data gap filling is necessary and which precautions should be taken when performing data gap filling. It presented and compared the different data gap filling procedures and their (dis)advantages.

Chapter 7 identifies and quantifies the different causes of uncertainty in flux measurements and analyses how they combine during scaling up.

Chapter 8 describes the main footprint models and the way they could be combined with vegetation cover maps (in order to identify the sources/sinks of flux) or with quality tests (in order to evaluate the general quality of data).

Chapter 9 presents the different possibilities to partition eddy flux into ecosystem respiration and gross ecosystem photosynthesis. Different approaches based on night-time or on day-time data were described.

Chapter 10 focuses on disjuncted eddy covariance technique, which is especially adapted to capture tracer gas.

Chapters 11–16 describe the specific requirements for flux measurements in specific ecosystems like forests, grasslands, croplands, wetlands, lakes or urban environment.

Finally, Chap. 17 describes the objectives of a data base, the way it should be maintained and managed. In addition, it proposes some policies for data use, exchange and publication.

The editors would like to thank the co-authors of the chapters for their enthusiasm and their involvement in this long (but, hopefully, useful) work that we hope can contribute to reinforce the links between the different eddy covariance networks in the world.

Dario Papale and Timo Vesala, although editors of this book, would like to thank M. Aubinet for taking care of the lion's share of the editing job.

The book idea and preparation has been also supported by the IMECC EU project and the ABBA Cost Action.

This book is dedicated to all the field (often anonymous) technicians whose continuous system care, maintenance and follow up constitute an inestimable contribution to ecosystem studies and to the Ph.D. students that decide to base their work on these unique measurements.

Folks, mark already in your calendars “the 20th Anniversary of EUROFLUX” to be held around 10 December in 2018, once again in Hyytiälä. We do not know yet what will be the main product of the meeting then.

Marc Aubinet
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Chapter 1
The Eddy Covariance Method

Thomas Foken, Marc Aubinet, and Ray Leuning

1.1 History

The eddy covariance method for measuring exchanges of heat, mass, and momentum between a flat, horizontally homogeneous surface and the overlying atmosphere was proposed by Montgomery (1948), Swinbank (1951), and Obukhov (1951). Under these conditions, net transport between the surface and atmosphere is one-dimensional and the vertical flux density can be calculated by the covariance between turbulent fluctuations of the vertical wind and the quantity of interest.

Instrumentation limitations hampered early implementation of this approach. In 1949, Konstantinonov (Obukhov 1951) developed a wind vane with two hot wire anemometers to measure the shear stress but the full potential of the eddy covariance method only emerged after the development of sonic anemometers, for which the basic equations were given by Schotland (1955). After the development of the first sonic thermometer (Barrett and Suomi 1949), a vertical sonic anemometer with a 1 m path length (Suomi 1957) was used during the O'Neill experiment in 1953 (Lettau and Davidson 1957). The design of today's anemometers was developed by Bovscheverov and Voronov (1960) and later by Kaimal and Businger (1963) and

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Mitsuta (1966). These phase shift anemometers have now been replaced by running time anemometers with delay time measurements (Hanafusa et al. 1982; Coppin and Taylor 1983).

Early micrometeorological experiments from the 1950s to 1970s were designed to study fundamental aspects of atmospheric turbulence over homogeneous surfaces, whereas studies in the 1980s investigated the turbulent fluxes of momentum, sensible, and latent heat over heterogeneous surfaces. Similar experiments were conducted in the United States (FIFE, Sellers et al. 1988), in France (HAPEX, André et al. 1990), and in Russia (KUREX, Tsvang et al. 1991). These experiments were to become the basis of many further micrometeorological experiments (Foken 2008) that needed researchers who were highly experienced in micrometeorology and sensor handling.

The possibility of continuous eddy flux measurements arose in the 1990s with the development of a new generation of sonic anemometers (see reviews by Zhang et al. 1986; Foken and Oncley 1995) and infrared gas analyzers for water vapor and carbon dioxide, together with the first comprehensive software packages for the eddy covariance method (McMillen 1988). In the early 1990s, the eddy covariance method became more and more widely used by the ecological community for the measurement of the carbon dioxide and water exchange between an ecosystem and the atmosphere. The first measuring towers of what later became the international FLUXNET network (Baldocchi et al. 2001) were installed, and introductions into techniques new for nonmicrometeorologists were written (Aubinet et al. 2000; Moncrieff et al. 1997a, b). In parallel, the development of new analyzer types allowed an extension of the investigated trace gas spectrum. In particular, Tunable Diode Laser and Quantum Cascade Laser spectrometers were used for the measurement of methane and nitrous oxide (Smith et al. 1994; Laville et al. 1999; Hargreaves et al. 2001; Kroon et al. 2010), Proton Transfer Reaction Mass Spectrometers for volatile organic compounds (Karl et al. 2002; Spirig et al. 2005), and Chemiluminescent sensors for Ozone (Güsten and Heinrich 1996; Gerosa et al. 2003; Lamaud et al. 1994, a.o.).

Some milestones in the development of the eddy covariance method are given in Table 1.1 with the reference to the Chapters of this book.

1.2 Preliminaries

1.2.1 Context of Eddy Covariance Measurements

Eddy covariance measurements are typically made in the surface boundary layer, which is approximately 20–50 m high in the case of unstable stratification and a few tens of meters in stable stratification (see Stull 1988; Garratt 1992; Foken 2008; for complete definitions of layers in the atmosphere). Fluxes are approximately constant with height in the surface layer; hence measurements taken in this layer

Table 1.1 History of the development of the eddy covariance method

Historical milestone	References	See chapter/ section
Theoretical basis of the eddy covariance method	Montgomery (1948), Swinbank (1951), Obukhov (1951)	Section 1.2
Three-dimensional sonic anemometer	Bovscheverov and Voronov (1960), Kaimal and Businger (1963), Mitsuta (1966)	Chapter 2
Instrumental requirements	McBean (1972)	Chapter 2
Gas analyzer for water vapor (UV)	Buck (1973), Kretschmer and Karpovitsch (1973), Martini et al. (1973)	
Gas analyzer for water vapor (IR)	Elagina (1962), Hyson and Hicks (1975), Raupach (1978)	Chapter 2
Correction of the effect of the air density	Webb et al. (1980)	Section 4.1
Gas analyzer for carbon dioxide (IR)	Ohtaki and Matsui (1982), Elagina and Lazarev (1984)	Chapter 2
Transformation of buoyancy flux into sensible heat flux	Schotanus et al. (1983)	Section 4.1
System of transfer functions for spectral correction	Moore (1986)	Section 4.1
Fetch conditions	Gash (1986)	Chapter 8
Real-time data processing software	McMillen (1988)	Chapter 3
Source regions for fluxes (footprint), based on Gash (1986)	Schmid and Oke (1990), Schuepp et al. (1990)	Chapter 8
Relaxed eddy accumulation method, based on Desjardins (1977)	Businger and Oncley (1990)	
Influence of tubing of closed path sensors	Leuning and Moncrieff (1990)	Section 4.1.3 Chapter 3
Theoretical basis for flux footprints and sampling strategies	Horst and Weil (1994), Lenschow et al. (1994)	Chapter 8
Addressing the problem of the unclosed energy balance at the surface	Foken and Oncley (1995)	Section 4.2
Quality tests for eddy covariance data	Foken and Wichura (1996), Vickers and Mahrt (1997)	Section 4.3
Addressing the problem of vertical advection	Lee (1998) and many others	Section 1.3, Chapter 5
Methodology for FLUXNET network (EuroFlux)	Aubinet et al. (2000)	All chapters
Gap filling in the FLUXNET network	Falge et al. (2001a, b)	Chapter 6
Organization of an international network (FLUXNET)	Baldocchi et al. (2001)	All chapters

Foken et al. (1995), Foken (2008), Moncrieff (2004), modified

are representative of the fluxes from the underlying surfaces which are desired to be known. Here atmospheric turbulence is the dominant transport mechanism, justifying the use of the eddy covariance approach to measure the fluxes.

Some preliminary definitions are necessary before discussing the eddy covariance approach in detail.

1.2.2 Reynolds Decomposition

The description of turbulent motions in the following theory sections requires the decomposition of the time-series of each variable ζ into a time-mean part, $\bar{\zeta}$, and a fluctuating part, ζ', the so-called Reynolds decomposition (Fig. 1.1). This can be written as:

$$\zeta = \bar{\zeta} + \zeta' \tag{1.1a}$$

where:

$$\bar{\zeta} = \frac{1}{T} \int_{t}^{t+T} \zeta(t)dt \tag{1.1b}$$

The application of Reynolds decomposition requires some averaging rules for the turbulent valu*e* ζ' which are termed Reynolds postulates:

$$\begin{aligned} &\text{I} \quad \overline{\zeta'} = 0 \\ &II \quad \overline{\zeta\xi} = \bar{\zeta}\,\bar{\xi} + \overline{\zeta'\xi'} \\ &III \quad \overline{\bar{\zeta}\xi} = \bar{\zeta}\,\bar{\xi} \\ &IV \quad \overline{a\zeta} = a\bar{\zeta} \\ &V \quad \overline{\zeta + \xi} = \bar{\zeta} + \bar{\xi} \end{aligned} \tag{1.2}$$

where a is a constant.

Stricto sensu, these relations are valid only when averages are by "ensemble" averaging (i.e., averaging over many realizations under identical conditions, Kaimal and Finnigan 1994). However, this is never possible in atmospheric measurements, so averages are most often computed on the basis of time series of statistical quantities by making use of the ergodic hypothesis which states that time averages are equivalent to ensemble averages (Brutsaert 1982; Kaimal and Finnigan 1994). To fulfil this assumption, the fluctuations have to be statistically stationary during the averaging time chosen (see Chap. 4).

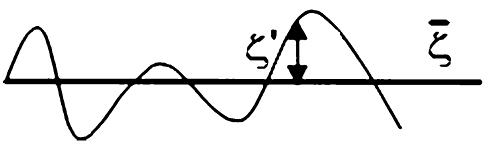

Fig. 1.1 Schematic presentation of Reynolds decomposition of the value ζ (Foken 2008)

1.2.3 Scalar Definition

The following variables are commonly used in the literature (and throughout this book) to define the scalar intensity of an atmospheric constituent *s*: *density* (ρ_s, kg m^{-3}) and *molar concentration* (c_s mol m^{-3}) represent the mass and the number of moles of *s* per volume of air, respectively. The *mole fraction* (mole mole^{-1}) is the ratio of the moles of *s* divided by the total number in the mixture (also equal to the ratio of the constituent partial pressure to the total pressure), the *molar mixing ratio* ($\chi_{s,m}$, mole mole^{-1}) is the ratio of the constituent mole number to those of dry air, and the *mass mixing ratio* (χ_s, kg kg^{-1}) is the ratio of the mass of the constituent to the mass of dry air. These variables are related by the perfect gas and the Dalton laws.

However, among these variables, only the molar and mass mixing ratios are conserved quantities in the presence of changes in temperature, pressure, and water vapor content (see Kowalski and Serrano-Ortiz (2007) for a more complete discussion). Unfortunately, the variables that are directly measured in the field by infrared gas analyzers are rather density and molar concentration, quantities that are not conserved during heat conduction, air compression/expansion or evaporation, and water vapor diffusion. Therefore, variations in these quantities may appear even in the absence of production, absorption, or transport of the component. The corrections that are necessary to take these effects into account were extensively discussed by Webb et al. (1980) and reexamined by Leuning (2003, 2007). They will be presented in Sect. 4.1.4.

The conservation equations developed in the section below are written using the mass mixing ratio but, for convenience, the other variables will also appear in this book. Conversion factors of one variable into another are given in Table 1.2.

Table 1.2 Conversion factors between different variables characterizing scalar intensity

Conversion factor	Molar mixing Ratio, $\chi_s=$	Mass mixing Ratio, $\chi_{sm}=$	Molar concentration, $c_s=$	Density, $\rho_s=$
Molar mixing ratio, χ_s X	1	$\frac{m_s}{m_d}$	$\frac{p_d}{R\overline{\theta}}$	$\frac{m_s p_d}{R\overline{\theta}}$
Mass mixing Ratio, χ_{sm} X	$\frac{m_d}{m_s}$	1	$\frac{m_d p_d}{m_s R\overline{\theta}}$	$\frac{m_d p_d}{R\overline{\theta}}$
Molar concentration, c_s X	$\frac{R\overline{\theta}}{p_d}$	$\frac{m_s R\overline{\theta}}{m_d p_d}$	1	m_s
Density, ρ_sX	$\frac{R\overline{\theta}}{m_s p_d}$	$\frac{R\overline{\theta}}{m_d p_d}$	$\frac{1}{m_s}$	1

Note that p_d corresponds to the dry air pressure (namely $p - p_v$). As a result, the exact conversion of mass or molar mixing ratio into concentration or density needs the knowledge of water vapor pressure (for details see list of symbols)

1.3 One Point Conservation Equations

The equation describing the conservation of any scalar or vector quantity ζ in the atmosphere may be written as

$$\underbrace{\frac{\partial \rho_d \zeta}{\partial t}}_{I} + \underbrace{\vec{\nabla}(\vec{u}\rho_\mathrm{d}\zeta)}_{II} + \underbrace{K_\zeta \Delta(\rho_d \zeta)}_{III} = \underbrace{S_\zeta}_{IV} \tag{1.3}$$

where $\overrightarrow{u}$ is the wind velocity vector, $\overrightarrow{\nabla}$ and Δ represent the divergence $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ and Laplacian $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ operators, ρ_d is the dry air density, K_ζ is the molecular diffusivity of the quantity ζ, and S_ζ represents its source/sink strength. This equation is instantaneous and applies to an infinitesimal volume of air. It states that the *rate of change of the quantity* (I) can be due to its *atmospheric transport* (II) to *molecular diffusion* (III) or to its *production by a source/absorption by a sink* into the infinitesimal volume (IV). It can be applied to any scalar or vector quantity provided source terms are defined accordingly. In particular, if ζ is 1, Eq. 1.3 is the continuity equation, if ζ is air enthalpy, it is the enthalpy conservation equation, and if ζ is the mixing ratio of an atmospheric component (water vapor, carbon dioxide, etc.), it is the scalar conservation equation. If the quantity is a component of the velocity vector in one given direction, Eq. 1.3 expresses the conservation of the momentum component in this direction. The three equations describing the momentum conservation in the three directions constitute the Navier Stokes equations.

Application of these equations to the surface boundary layer requires application of the Reynolds decomposition rules: the variables ζ, $\rho_\mathrm{d}, \vec{u}$, and S_ζ should each be decomposed into a mean and a fluctuating part according to Eq. 1.1, followed by application of the averaging operator, and appropriate rearrangement and simplification. This procedure will be applied to each equation below.

1.3.1 Dry Air Mass Conservation (Continuity) Equation

By replacing ζ by 1 in Eq. 1.3, one obtains

$$\frac{\partial \rho_\mathrm{d}}{\partial t} + \vec{\nabla}(\vec{u}\rho_\mathrm{d}) = 0 \tag{1.4}$$

as there is neither a source nor sink of dry air in the atmosphere. Application of the time- averaging operator gives immediately:

$$\overline{\frac{\partial \rho_\mathrm{d}}{\partial t}} + \vec{\nabla}(\overline{\vec{u}\rho_\mathrm{d}}) = 0 \tag{1.5}$$

1.3.2 Momentum Conservation Equation

By replacing ζ in Eq. 1.3 with the component of wind velocity in one given direction, u_i, one obtains the momentum conservation equation in this direction:

$$\frac{\partial \rho_d u_i}{\partial t} + \vec{\nabla}(\vec{u}\rho_d\, u_i) = S_i \tag{1.6}$$

In Eq. 1.6, the source/sink terms correspond to momentum source/sink, namely to forces. Forces that can act on air parcels in the atmospheric boundary layer are drag, pressure gradient, Coriolis forces, viscous forces, or buoyancy. The first three forces are considered negligible for a flat, horizontally homogeneous surface boundary layer above the roughness elements (i.e. not including vegetation) (Businger 1982; Foken 2008; Stull 1988). Buoyancy appears only in the equation for vertical momentum. The horizontal component of momentum parallel to the mean wind is dominant in the surface boundary layer and thus the buoyancy term is not considered. In a Cartesian coordinate system (*x, y, z*) where *x* corresponds to the horizontal, parallel to the average wind velocity, *y* to the horizontal, perpendicular to the average velocity, and *z* to the vertical; *u, v, w* are the *x, y*, and *z* components of velocity, respectively, and this equation is written as

$$\frac{\partial \rho_d u}{\partial t} + \frac{\partial \rho_d u^2}{\partial x} + \frac{\partial \rho_d v u}{\partial y} + \frac{\partial \rho_d w u}{\partial z} = 0 \tag{1.7}$$

Application of the Reynolds decomposition to Eq. 1.7 and use of the following simplifications (Businger 1982; Stull 1988):

$$\begin{aligned} &I\ \ \left|p'/\bar{p}\right| \ll \left|\rho'_d/\overline{\rho_d}\right| \\ &II\ \ \left|p'/\bar{p}\right| \ll \left|\theta'/\bar{\theta}\right|, \\ &III\ \ \left|\rho'_d/\overline{\rho_d}\right| \ll 1 \\ &IV\ \ \left|\theta'/\bar{\theta}\right| \ll 1 \end{aligned} \tag{1.8}$$

where p is the pressure and θ the air temperature, leads to

$$\frac{\overline{\partial u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} + \frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{v'u'}}{\partial y} + \frac{\partial \overline{w'u'}}{\partial z} = 0 \tag{1.9}$$

Equation 1.8, III corresponds to the *Boussinesq-approximation* (Boussinesq 1877), which neglects density fluctuations except in the buoyancy (gravitation) term, because the acceleration of gravity is relatively large in comparison with the other accelerations in the momentum equation. By choosing a coordinate system such that