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Debasis Kundu · Swagata Nandi

Statistical Signal Processing

Frequency Estimation

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Frequency Estimation

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To my parents

D. Kundu

To my parents

S. Nandi

Preface

We have worked on our Ph.D. theses on *Statistical Signal Processing* although in a gap of almost 15 years. The first author was introduced to the area by his Ph.D. supervisor Professor C. R. Rao, while the second author was introduced to this topic by the first author. It has been observed that frequency estimation plays an important role in dealing with different problems in the area of *Statistical Signal Processing*, and both the authors have spent a significant amount of their research career dealing with this problem for different models associated with Statistical Signal Processing.

Although an extensive amount of literature is available in the engineering literature dealing with the frequency estimation problem, not much attention has been paid to the statistical literature. The book by Quinn and Hannan [1] is the only book dealing with the problem of frequency estimation written for the statistical community. We were thinking of writing a review article on this topic for quite sometime. In this respect, the invitation from Springer to write a Springer Brief on this topic came as a pleasant surprise to us.

In this Springer Brief, we provide a review of the different methods available till date dealing with the problem of frequency estimation. We have not attempted an exhaustive survey of frequency estimation techniques. We believe that would require separate books on several topics themselves. Naturally, the choice of topics and examples are based, in favor of our own research interests. The list of references is also far from complete.

We have kept the mathematical level quite modest. Chapter 4 mainly deals with somewhat more demanding asymptotic theories, and this chapter can be avoided during the first reading without losing any continuity. Senior undergraduate level mathematics should be sufficient to understand the rest of the chapters. Our basic goal to write this Springer Brief is to introduce the challenges of the frequency estimation problem to the statistical community, which are present in different areas of science and technology. We believe that statisticians can play a major role in solving several problems associated with frequency estimation. In Chap. 8, we have provided several related models, where there are several open issues which need to be answered by the scientific community.

Every book is written with a specific audience in mind. This book definitely cannot be called a textbook. It has been written mainly for senior undergraduate and graduate students specializing in Mathematics or Statistics. We hope that this book will motivate students to pursue higher studies in the area of Statistical Signal Processing. This book will be helpful to young researchers who want to start their research career in the area of Statistical Signal Processing. We will consider our efforts to be worthy if the target audience finds this volume useful.

Kanpur, January 2012 Debasis Kundu
Delhi, January 2012 Swagata Nandi

Reference

1. Quinn, B.G., & Hannan, E.J. (2001). *The estimation and tracking of frequency*. New York: Cambridge University Press.

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Contents

Abbreviations

1-D	One-dimensional
2-D	Two-dimensional
3-D	Three-dimensional
AIC	Akaike's information criterion
ALSE(s)	Approximate least squares estimator(s)
AM	Amplitude modulated
AR	Autoregressive
ARIMA	Autoregressive integrated moving average
ARMA	Autoregressive moving average
BIC	Bayesian information criterion
CV	Cross-validation
ECG	Electrocardiograph
EDC	Efficient detection criterion
EM	Expectation maximization
ESPRIT	Estimation of signal parameters via rotational invariance technique
EVLP	Equivariance linear prediction method
i.i.d.	Independent and identically distributed
Im(z)	Imaginary part of a complex number z
ITC	Information theoretic criterion
IQML	Iterative quadratic maximum likelihood
KIC	Kundu's information criterion
LSE(s)	Least squares estimator(s)
MA	Moving average
MDL	Minimum description length
MFBLP	Modified forward backward linear prediction method
MLE(s)	Maximum likelihood estimator(s)
NSD	Noise space decomposition
PE(s)	Periodogram estimator(s)
QIC	Quinn's information criterion
QT	Quinn and Thomson

Re(z)	Real part of a complex number z
RGB	Red-green-blue
SIC	Sakai's information criterion
TLS-ESPRIT	Total least squares estimation of signal parameters via rotational invariance technique
WIC	Wang's information criterion
WLSE(s)	Weighted least squares estimator(s)

Symbols

$\arg(z)$	$\tan^{-1}\theta$ where $z = r\,e^{i\theta}$
$\mathcal{N}(a,\ b)^2$	Univariate normal distribution with mean a and variance b^2
$\mathcal{N}_p(\mathbf{0},\ \Sigma)$	p-variate normal distribution with mean vector $\mathbf{0}$ and dispersion matrix Σ
$X_n \xrightarrow{d} X$	X_n converges to X in distribution
$X_n \xrightarrow{p} X$	X_n converges to X in probability
a.s.	Almost surely
i.o.	Infinitely often
a.e.	Almost everywhere
$X_n \xrightarrow{a.s.} X$	X_n converges to X almost surely
$o(a_n)$	$(1/a_n)o(a_n) \to 0$ as $n \to \infty$
$O(a_n)$	$(1/a_n)O(a_n)$ is bounded as $n \to \infty$
$X_n = o_{\mathrm{p}}(a_n)$	$\lim_{n\to\infty} P(\lvert X_n/a_n\rvert \geq \epsilon) = 0,$ for every positive ϵ
$X_n = O_{\mathrm{p}}(a_n)$	X_n/a_n is bounded in probability as $n \to \infty$
$\lVert\mathbf{x}\rVert$	$\sqrt{x_1^2 + \ldots + x_n^2}$, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$
$\mathbb{R}$	Set of real numbers
$\mathbb{R}$	d-Dimensional Euclidean space
$\mathbb{C}$	Set of complex numbers
$\lvert\mathbf{A}\rvert$	Determinant of matrix $\mathbf{A}$
$X \stackrel{d}{=} Y$	X and Y are identically distributed

Chapter 1
Introduction

Signal processing may broadly be considered to involve the recovery of information from physical observations. The received signal is usually disturbed by thermal, electrical, atmospheric, or intentional interferences. Due to random nature of the signal, statistical techniques play important roles in analyzing the signal. Statistics is also used in the formulation of appropriate models to describe the behavior of the system, the development of an appropriate technique for the estimation of model parameters, and the assessment of the model performances. Statistical signal processing basically refers to the analysis of random signals using appropriate statistical techniques.

The main aim of this monograph is to introduce different signal processing models which have been used in analyzing periodic data, and different statistical and computational issues associated with them. We observe periodic phenomena everyday in our lives. The daily temperature of Delhi or the number of tourists visiting the famous Taj Mahal everyday or the ECG signal of a normal human being, clearly follow periodic nature. Sometimes, the observations/signals may not be exactly periodic on account of different reasons, but they may be nearly periodic. It should be clear from the following examples, where the observations are obtained from different disciplines, that they are nearly periodic. In Fig. 1.1, we provide the ECG signal of a healthy person. In Fig. 1.2, we present an astronomical data set which represents the daily brightness of a variable star on 600 successive midnights. Figure 1.3 represents a classical data set of the monthly international airline passengers from January 1953 to December 1960 and is collected from the Time Series Data Library http://www.robhyndman.info/TDSL.

The simplest periodic function is the sinusoidal function and it can be written in the following form:

$$y(t) = A\cos(\omega t) + B\sin(\omega t), \tag{1.1}$$

where $A^2 + B^2$ is known as the amplitude of $y(t)$ and ω is the frequency. In general, a *smooth* mean zero periodic function $y(t)$ can always be written in the form

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