

SPRINGER BRIEFS IN STATISTICS

Debasis Kundu · Swagata Nandi

Statistical Signal Processing Frequency Estimation



Springer

SpringerBriefs in Statistics

For further volumes:
<http://www.springer.com/series/8921>

Debasis Kundu · Swagata Nandi

Statistical Signal Processing

Frequency Estimation

Debasis Kundu
Department of Mathematics and Statistics
Indian Institute of Technology
Kanpur, Uttar Pradesh 208016
India

Swagata Nandi
Theoretical Statistics and Mathematics Unit
Indian Statistical Institute
7, S.J.S. Sansanwal Marg
New Delhi 110016
India

ISSN 2191-544X
ISBN 978-81-322-0627-9
DOI 10.1007/978-81-322-0628-6
Springer New Delhi Heidelberg New York Dordrecht London

ISSN 2191-5458 (electronic)
ISBN 978-81-322-0628-6 (eBook)

Library of Congress Control Number: 2012938023

© The Author(s) 2012

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To my parents

D. Kundu

To my parents

S. Nandi

Preface

We have worked on our Ph.D. theses on *Statistical Signal Processing* although in a gap of almost 15 years. The first author was introduced to the area by his Ph.D. supervisor Professor C. R. Rao, while the second author was introduced to this topic by the first author. It has been observed that frequency estimation plays an important role in dealing with different problems in the area of *Statistical Signal Processing*, and both the authors have spent a significant amount of their research career dealing with this problem for different models associated with Statistical Signal Processing.

Although an extensive amount of literature is available in the engineering literature dealing with the frequency estimation problem, not much attention has been paid to the statistical literature. The book by Quinn and Hannan [1] is the only book dealing with the problem of frequency estimation written for the statistical community. We were thinking of writing a review article on this topic for quite sometime. In this respect, the invitation from Springer to write a Springer Brief on this topic came as a pleasant surprise to us.

In this Springer Brief, we provide a review of the different methods available till date dealing with the problem of frequency estimation. We have not attempted an exhaustive survey of frequency estimation techniques. We believe that would require separate books on several topics themselves. Naturally, the choice of topics and examples are based, in favor of our own research interests. The list of references is also far from complete.

We have kept the mathematical level quite modest. [Chapter 4](#) mainly deals with somewhat more demanding asymptotic theories, and this chapter can be avoided during the first reading without losing any continuity. Senior undergraduate level mathematics should be sufficient to understand the rest of the chapters. Our basic goal to write this Springer Brief is to introduce the challenges of the frequency estimation problem to the statistical community, which are present in different areas of science and technology. We believe that statisticians can play a major role in solving several problems associated with frequency estimation. In [Chap. 8](#), we have provided several related models, where there are several open issues which need to be answered by the scientific community.

Every book is written with a specific audience in mind. This book definitely cannot be called a textbook. It has been written mainly for senior undergraduate and graduate students specializing in Mathematics or Statistics. We hope that this book will motivate students to pursue higher studies in the area of Statistical Signal Processing. This book will be helpful to young researchers who want to start their research career in the area of Statistical Signal Processing. We will consider our efforts to be worthy if the target audience finds this volume useful.

Kanpur, January 2012
Delhi, January 2012

Debasis Kundu
Swagata Nandi

Reference

1. Quinn, B.G., & Hannan, E.J. (2001). *The estimation and tracking of frequency*. New York: Cambridge University Press.

Acknowledgments

The authors would like to thank their respective Institutes and to all who contributed directly and indirectly to the production of this monograph. The first author would like to thank his wife Ranjana for her continued support and encouragement.

Contents

1	Introduction	1
	References	6
2	Notations and Preliminaries	7
2.1	Prony's Equation	8
2.2	Undamped Exponential Model	10
2.3	Sum of Sinusoidal Model	11
2.4	Linear Prediction	11
2.5	Matrix Pencil	12
2.6	Stable Distribution: Results	13
	References	15
3	Estimation of Frequencies	17
3.1	ALSEs and PEs	18
3.2	EVLP	19
3.3	MFBLP	20
3.4	NSD	22
3.5	ESPRIT	24
3.6	TLS-ESPRIT	25
3.7	Quinn's Method	25
3.8	IQML	26
3.9	Modified Prony Algorithm	27
3.10	Constrained Maximum Likelihood Method	28
3.11	Expectation Maximization Algorithm	29
3.12	Sequential Estimators	33
3.13	Quinn and Fernandes Method	33
3.14	Amplified Harmonics Method	37
3.15	Weighted Least Squares Estimators	38
3.16	Nandi and Kundu Algorithm	39
3.17	Super Efficient Estimator	40

3.18 Conclusions. 41

References 42

4 Asymptotic Properties 45

4.1 Introduction. 45

4.2 Sinusoidal Model with One Component 46

4.3 Strong Consistency of LSE and ALSE of θ 47

4.3.1 Proof of the Strong Consistency of $\hat{\theta}$, the LSE of θ 48

4.3.2 Proof of Strong Consistency of θ , the ALSE of θ 49

4.4 Asymptotic Distribution of LSE and ALSE of θ 51

4.4.1 Asymptotic Distribution of $\hat{\theta}$ Under Assumption 3.2 51

4.4.2 Asymptotic Distribution of $\hat{\theta}$ Under Assumption 4.2 53

4.4.3 Asymptotic Equivalence of LSE $\hat{\theta}$ and ALSE θ 55

4.5 Superefficient Frequency Estimator 58

4.6 Multiple Sinusoidal Model 66

4.7 Weighted Least Squares Estimators 67

4.8 Conclusions. 69

References 77

5 Estimating the Number of Components 79

5.1 Introduction. 79

5.2 Likelihood Ratio Approach 81

5.3 Cross Validation Method 83

5.4 Information Theoretic Criteria 84

5.4.1 Rao’s Method 85

5.4.2 Sakai’s Method 86

5.4.3 Quinn’s Method. 86

5.4.4 Wang’s Method. 87

5.4.5 Kundu’s Method 88

5.5 Conclusions. 89

References 90

6 Real Data Example 91

6.1 Introduction. 91

6.2 ECG Data 91

6.3 Variable Star Data 92

6.4 “uuu” Data 94

6.5 Airline Passenger Data 95

6.6 Conclusions. 99

References 99

7 Multidimensional Models 101

7.1 Introduction. 101

7.2 2-D Model: Estimation of Frequencies 104

7.2.1	LSEs	104
7.2.2	Sequential Method	106
7.2.3	Periodogram Estimators	107
7.2.4	Nandi–Prasad–Kundu Algorithm	108
7.2.5	Noise Space Decomposition Method	109
7.3	2-D Model: Estimating the Number of Components.	111
7.4	Conclusions.	111
	References	112
8	Related Models	113
8.1	Introduction.	113
8.2	Damped Sinusoidal Model	113
8.3	Amplitude Modulated Model.	115
8.4	Fundamental Frequency Model	116
8.4.1	Test for Harmonics	120
8.5	Generalized Fundamental Frequency Model	121
8.6	Partially Sinusoidal Frequency Model.	122
8.7	Burst-Type Model	124
8.8	Discussion/Remarks	125
	References	126
Index		129

Abbreviations

1-D	One-dimensional
2-D	Two-dimensional
3-D	Three-dimensional
AIC	Akaike's information criterion
ALSE(s)	Approximate least squares estimator(s)
AM	Amplitude modulated
AR	Autoregressive
ARIMA	Autoregressive integrated moving average
ARMA	Autoregressive moving average
BIC	Bayesian information criterion
CV	Cross-validation
ECG	Electrocardiograph
EDC	Efficient detection criterion
EM	Expectation maximization
ESPRIT	Estimation of signal parameters via rotational invariance technique
EVLP	Equivariance linear prediction method
i.i.d.	Independent and identically distributed
Im(z)	Imaginary part of a complex number z
ITC	Information theoretic criterion
IQML	Iterative quadratic maximum likelihood
KIC	Kundu's information criterion
LSE(s)	Least squares estimator(s)
MA	Moving average
MDL	Minimum description length
MFBLP	Modified forward backward linear prediction method
MLE(s)	Maximum likelihood estimator(s)
NSD	Noise space decomposition
PE(s)	Periodogram estimator(s)
QIC	Quinn's information criterion
QT	Quinn and Thomson

Re(z)	Real part of a complex number z
RGB	Red-green-blue
SIC	Sakai's information criterion
TLS-ESPRIT	Total least squares estimation of signal parameters via rotational invariance technique
WIC	Wang's information criterion
WLSE(s)	Weighted least squares estimator(s)

Symbols

$\arg(z)$	$\tan^{-1}\theta$ where $z = r e^{i\theta}$
$\mathcal{N}(a, b)^2$	Univariate normal distribution with mean a and variance b^2
$\mathcal{N}_p(\mathbf{0}, \Sigma)$	p -variate normal distribution with mean vector $\mathbf{0}$ and dispersion matrix Σ
$X_n \xrightarrow{d} X$	X_n converges to X in distribution
$X_n \xrightarrow{p} X$	X_n converges to X in probability
a.s.	Almost surely
i.o.	Infinitely often
a.e.	Almost everywhere
$X_n \xrightarrow{a.s.} X$	X_n converges to X almost surely
$o(a_n)$	$(1/a_n)o(a_n) \rightarrow 0$ as $n \rightarrow \infty$
$O(a_n)$	$(1/a_n)O(a_n)$ is bounded as $n \rightarrow \infty$
$X_n = o_p(a_n)$	$\lim_{n \rightarrow \infty} P(X_n/a_n \geq \epsilon) = 0$, for every positive ϵ
$X_n = O_p(a_n)$	X_n/a_n is bounded in probability as $n \rightarrow \infty$
$\ \mathbf{x}\ $	$\sqrt{x_1^2 + \dots + x_n^2}$, $\mathbf{x} = (x_1, x_2, \dots, x_n)$
\mathbb{R}	Set of real numbers
\mathbb{R}^d	d -Dimensional Euclidean space
\mathbb{C}	Set of complex numbers
$ \mathbf{A} $	Determinant of matrix \mathbf{A}
$X \stackrel{d}{=} Y$	X and Y are identically distributed

Chapter 1

Introduction

Signal processing may broadly be considered to involve the recovery of information from physical observations. The received signal is usually disturbed by thermal, electrical, atmospheric, or intentional interferences. Due to random nature of the signal, statistical techniques play important roles in analyzing the signal. Statistics is also used in the formulation of appropriate models to describe the behavior of the system, the development of an appropriate technique for the estimation of model parameters, and the assessment of the model performances. Statistical signal processing basically refers to the analysis of random signals using appropriate statistical techniques.

The main aim of this monograph is to introduce different signal processing models which have been used in analyzing periodic data, and different statistical and computational issues associated with them. We observe periodic phenomena everyday in our lives. The daily temperature of Delhi or the number of tourists visiting the famous Taj Mahal everyday or the ECG signal of a normal human being, clearly follow periodic nature. Sometimes, the observations/signals may not be exactly periodic on account of different reasons, but they may be nearly periodic. It should be clear from the following examples, where the observations are obtained from different disciplines, that they are nearly periodic. In Fig. 1.1, we provide the ECG signal of a healthy person. In Fig. 1.2, we present an astronomical data set which represents the daily brightness of a variable star on 600 successive midnights. Figure 1.3 represents a classical data set of the monthly international airline passengers from January 1953 to December 1960 and is collected from the Time Series Data Library <http://www.robhyndman.info/TDSL>.

The simplest periodic function is the sinusoidal function and it can be written in the following form:

$$y(t) = A \cos(\omega t) + B \sin(\omega t), \quad (1.1)$$

where $A^2 + B^2$ is known as the amplitude of $y(t)$ and ω is the frequency. In general, a *smooth* mean zero periodic function $y(t)$ can always be written in the form