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ISSN 2191-530X ISSN 2191-5318 (electronic)
ISBN 978-3-642-29233-0 ISBN 978-3-642-29234-7 (eBook)
DOI 10.1007/978-3-642-29234-7
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2012935677

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Preface

This monograph concerns with application of the upper bound theorem to finding the limit load for welded structures including structures with cracks. The presentation of the introductory material and the theoretical developments appear in a text of six chapters. The topics chosen are primarily of interest to engineers as postgraduates and practitioners but they should also serve to capture a readership from among applied mathematicians. The monograph provides both a collection of limit load solutions for welded structures and a description of general approaches to finding the limit load for a class of structures. Many solutions are represented by formulae. Such solutions are immediately ready for practical use. Other solutions are illustrated by diagrams. These diagrams demonstrate most important tendencies in solutions behavior. It is however evident that they cannot be used for practical calculation of the limit load. Therefore, most of such solutions are described in great detail, including possible difficulties with application of numerical methods, and quantitative results can be easily reproduced. In most cases, numerical techniques are only necessary to evaluate integrals and minimize functions of one variable. As a rule, approximations of solutions by elementary functions are not given in the monograph. Although such approximations are widely used in the literature, it is believed that they are not efficient when there are several essential input parameters. For reasons of space, the main focus is on various highly undermatched tensile specimens, though undermatched and overmatched cases are briefly discussed as well.

Among the topics that are either new or presented in greater detail than would be found in similar texts are the following:

1. An approach to modifying upper bound solutions for a class of structures with no crack to account for the presence of a crack.
2. An approach to using singular velocity fields for constructing accurate upper bound solutions for highly undermatched joints.
3. The effect of the thickness of specimens on the limit load.
4. The effect of plastic anisotropy on the limit load.

5. A discussion of difficulties with application of numerical techniques in conjunction with simple kinematically admissible velocity fields.

Chapter 1 concerns with the upper bound theorem for rigid perfectly plastic materials. A formal proof is not given because it can be found in any text on plasticity theory. Instead, original and efficient approaches to finding upper bound limit load solutions for welded joints with and with no cracks are introduced and explained. These approaches are used in subsequent chapters.

Chapters 2–4 deal with highly undermatched specimens subject to tension. Firstly, in Chap. 2, two solutions for the center cracked specimen under plane strain conditions are presented. Each of these solutions illustrates one of the general approaches introduced in Chap. 1. The solutions found are generalized to scarf joint specimens in Chap. 3, also under plane strain conditions. Axisymmetric solutions are given in Chap. 4.

In Chap. 5 two solutions for pure bending of highly undermatched panels under plane strain conditions are discussed. One of these solutions is based on an exact solution of plasticity theory. The other solution is obtained with the use of one of the universal methods proposed in Chap. 1. Comparison of the solutions determines the ranges in parametric space where each of them should be adopted.

Chapter 6 includes a brief discussion of several topics. Firstly, the effect of the thickness of panels on the limit load is illustrated. To this end, the solution for the center cracked specimen presented in Chap. 2 is compared to a new three-dimensional solution. The effect of the mis-match ratio is discussed next. Solutions for the undermatched and overmatched center cracked specimen are given and the definition for the highly undermatched case is clarified. Finally, it is shown that the effect of plastic anisotropy on the limit load is very significant and this material property should not be ignored in the development of flaw assessment procedures.

Contents

Symbols

The intention within the various theoretical developments given in this monograph has been to define each new symbol where it first appears in the text. A number of symbols are introduced in the abstract to individual chapters. These symbols re-appear consistently throughout the chapter. In this regards each chapter should be treated as self-contained in its symbol content. There are, however, certain symbols that re-appear consistently throughout the text. These symbols are given in the following list.

F	Tensile force
F_u	Upper bound on F
f_u	Dimensionless representation of F_u
G	Bending moment
G_u	Upper bound on G
g_u	Dimensionless representation of G_u
u_x, u_{y}, u_z	Components of kinematically admissible velocity field in Cartesian coordinates
$\lvert[u_\tau]\rvert$	Amount of velocity jump
x, y, z	Cartesian coordinates
ζ_{eq}	Equivalent strain rate found using kinematically admissible velocity field
ζ_{xx}, ζ_{yy}, ζ_{zz}, ζ_{xy}, ζ_{xz}, ζ_{yz}	Components of kinematically admissible strain rate field in Cartesian coordinates
ξ_{eq}	Equivalent strain rate
σ_0	Yield stress in tension
$\mathbf{n}$	Unit normal vector
$\mathbf{U}$	Velocity vector in rigid zone
$\mathbf{u}$	Velocity vector in plastic zone

Chapter 1
Upper Bound Theorem

Plastic limit analysis is a convenient tool to find approximate solutions of boundary value problems. In general, this analysis is based on two principles associated with the lower bound and upper bound theorems. The latter is used in the present monograph to estimate the limit load for welded structures with and with no crack. A proof of the upper bound theorem for a wide class of material models has been given by Hill (1956). The only reliable output of upper bound solutions is the load required to initiate the process of plastic deformation. Any upper bound limit load is higher than or equal to the actual load. This statement becomes more complicated in the case of multiple load parameters. Upper bound solutions are not unique and their accuracy significantly depends on the kinematically admissible velocity field chosen. Therefore, the development of methods for constructing kinematically admissible velocity fields accounting for some mathematical features of real velocity fields is of great importance for successful applications of the method. In addition to the methods described in the present monograph, original approaches have been proposed in Sawczuk and Hodge (1968), Wilson (1977), and Yeh and Yang (1996) among others.

Upper bound solutions overestimate the load carrying capacity of structures. Therefore, such solutions may be associated with one possible failure mechanism of structures. However, a more important application of limit load solutions to structures with cracks is that the limit load is an essential input parameter of flaw assessment procedures. Therefore, the accuracy of the limit load found has a great effect of the accuracy of predictions made with the use of these procedures. A review of flaw assessment procedures can be found in Zerbst et al. (2000). Reviews of limit load solutions for structures with cracks are available in Miller (1988) and Alexandrov (2011).

S. Alexandrov, *Upper Bound Limit Load Solutions for Welded Joints with Cracks*, SpringerBriefs in Computational Mechanics, DOI: 10.1007/978-3-642-29234-7_1,
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1.1 Basic Assumptions and Equations

The present monograph concerns with rigid perfectly plastic material. This means that the elastic portion of the strain rate tensor is neglected and all yield stresses are material constants. It is worthy of note here that the former assumption is not essential since the limit load is independent of elastic properties (Drucker et al. 1952). Strain hardening has no effect on the limit load as well, as long as the initial configuration is of concern. The constitutive equations of the model chosen consist of the yield criterion and its associated flow rule. A great account on this model is given in Hill (1950). Extension of the theory to piece-wise homogeneous materials, as it is required for welded structures, is in general straightforward. It is presented in Rychlewski (1966) under plane strain conditions. The Mises yield criterion is adopted throughout this monograph (except anisotropic solutions given in Chap. 6). This criterion can be written in the form

$$\sqrt{\frac{3}{2}\tau_{ij}\tau_{ij}} = \sigma_0. \tag{1.1}$$

Here and in what follows the summation convention, according to which a recurring letter suffix indicates that the sum must be formed of all terms obtainable by assigning to the suffix the values 1, 2, and 3, is adopted. Similarly, in a quantity containing two repeated suffixes, say i and j, the summation must be carried out for all values 1, 2, 3 of both i and j. Also, $\tau_{ii} = \sigma_{ij} - \sigma\delta_{ij}$, σ_{ij} are the components of the stress tensor, $\sigma = \sigma_{ij}\delta_{ij}/3$, δ_{ij} is the Kroneker symbol, and σ_0 is the yield stress in tension.

A proof of the upper bound theorem can be found, for example, in Hill (1950). When the yield criterion (1.1) is adopted, the theorem reads

$$\iint\limits_{S_v} (t_i v_i)\, dS \le \sigma_0 \iiint\limits_{V} \zeta_{eq}\, dV - \iint\limits_{S_f} (t_i u_i)\, dS + \frac{\sigma_0}{\sqrt{3}} \iint\limits_{S_d} |[u_\tau]|\, dS \tag{1.2}$$

where V is the volume of material loaded by prescribed external stresses t_i over a part S_f of its surface, and by prescribed velocities over the remainder S_v. Also, v_i are the components of the real velocity vector, u_i are the components of a kinematically admissible velocity vector, ζ_{eq} is the equivalent strain rate, and $|[u_\tau]|$ is the amount of velocity jump across the velocity discontinuity surface S_d. The velocity component u_τ should be found using the kinematically admissible velocity field. The normal velocity must be continuous across the velocity discontinuity surface. The equivalent strain rate involved in Eq. (1.2) is defined by

$$\zeta_{eq} = \sqrt{\frac{2}{3}\zeta_{ij}\zeta_{ij}} \tag{1.3}$$

where ζ_{ij} are the strain rate components. These components should be found using the kinematically admissible velocity field u_i. The left hand side of the inequality

(1.2) can be evaluated using any kinematically admissible velocity field. The boundary value problems considered in the present monograph contain just one unknown load. Therefore, this load can be evaluated using Eq. (1.2). Useful results for boundary value problems containing several independent load parameters can be found in Hodge and Sun (1968). The second integral on the right hand side of Eq. (1.2) usually include traction free and frictional surfaces. There are no frictional surfaces in the boundary value problems considered in the present monograph. Moreover, $t_i = 0$ over any traction free surface. Therefore, the second integral on the right hand side of Eq. (1.2) vanishes and the inequality simplifies to

$$\iint\limits_{S_v} (t_i v_i)\, dS \leq \sigma_0 \iiint\limits_{V} \zeta_{eq}\, dV + \frac{\sigma_0}{\sqrt{3}} \iint\limits_{S_d} |[u_\tau]|\, dS. \tag{1.4}$$

Many simple upper bound solutions are obtained by assuming that the kinematically admissible velocity field consists of rigid blocks. In this case $\zeta_{eq} = 0$ in V and the inequality (1.4) further simplifies to

$$\iint\limits_{S_v} (t_i v_i) dS \leq \frac{\sigma_0}{\sqrt{3}} \iint\limits_{S_d} |[u_\tau]| dS. \tag{1.5}$$

The physical meaning of the left hand side of Eq. (1.4) is the rate at which external forces do work. The physical meaning of the first term on its right hand side is the rate of work dissipation in plastic zones and the physical meaning of the second term is the rate of work dissipation at velocity discontinuity surfaces.

By definition, any velocity field satisfying the velocity boundary conditions and the equation of incompressibility is a kinematically admissible velocity field. The equation of incompressibity can be written as

$$\zeta_{ii} = 0. \tag{1.6}$$

The condition that the normal velocity must be continuous across velocity discontinuity surfaces can be represented as the following scalar product of two vectors.

$$(\mathbf{u_1} - \mathbf{u_2}) \cdot \mathbf{n} = 0 \tag{1.7}$$

where $\mathbf{u_1}$ and $\mathbf{u}_2$ are the velocity vectors on sides 1 and 2 of the velocity discontinuity surface, respectively, and $\mathbf{n}$ is the unit normal vector to this surface. Then, the amount of velocity jump can be found as

$$|[u_\tau]| = \sqrt{(\mathbf{u_1} - \mathbf{u_2}) \cdot (\mathbf{u_1} - \mathbf{u_2})}. \tag{1.8}$$

It is obvious that it is possible to propose any number of kinematically admissible velocity fields for any problem. If a kinematically admissible velocity field coincides with the real velocity field then Eq. (1.2) gives the exact value of the limit load. In most cases however, kinematically admissible velocity fields result in upper bounds on the limit load. If a kinematically admissible velocity field chosen contains no free parameters, its substitution into Eq. (1.2) immediately gives an upper bound on the limit load. A better prediction can be indeed achieved when a kinematically admissible velocity field contains free parameters. In such cases, substituting this velocity field into Eq. (1.2) transforms the functional on its right hand side into a function of one or several variables. It is obvious from the structure of the inequality (1.2) that its right hand side should be minimized with respect to these variables to find the best upper bound limit load based on the kinematically admissible velocity field chosen. In general, there are two main approaches to handle this problem: (1) numerical methods based on finite element approximation, and (2) analytical and semi-analytical methods. The former have been developed, for example, in Chang and Bramley (2000) and Bramley (2001). The original approaches have been developed for metal forming simulation. However, they are also applicable for structural analysis. The present monograph is devoted to analytical and semi-analytical methods of plastic limit analysis. Such methods are very useful for engineering applications (Schwalbe 2010).

There is the companion theorem to the upper bound theorem to find the lower bound limit load (see, for example, Hill 1950). This theorem is not considered in the present monograph and all limit loads in subsequent chapters should be understood as the upper bound limit loads.

1.2 Vicinity of the Bi-Material Interface

Even though any kinematically admissible velocity field results in an upper bound on the limit load, it is advantageous to choose a kinematically admissible velocity field which takes into account behavior of the real velocity field that must exist near some surfaces. For welded joints, the bi-material interface is such a surface when it is also a velocity discontinuity surface. The latter is a typical situation for highly undermatched joints. In such joints, the weld is much softer than the base material and plastic deformation is localized within the weld whereas the base material is rigid. Such compositions of materials are of practical interest (Hao et al. 1997). According to the general theory, the shear stress at any velocity discontinuity surface coinciding with a bi-material surface must be equal to the shear yield stress of the softer material. In particular, Rychlewski (1966) has considered such a distribution of stresses in the case of plane strain deformation. Alexandrov and Richmond (2001) have shown that the real velocity field must be singular near surfaces on which the shear stress is equal to the shear yield stress (there are exceptions to this rule but those are not significant in most cases of practical interest). They have also proposed a conceptual approach to use this property of the real velocity field in plastic limit analysis of structures (Alexandrov and Richmond 2000). The main result obtained in Alexandrov and Richmond (2001) can be represented by the following equation