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Rudrapatna V. Ramnath

Computation and Asymptotics



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*Dedicated to the fond memory of my parents,
Sri R. Venkataramaiya and Smt. Seethamma*

Foreword

The monograph *Computation and Asymptotics* is published in the series of SpringerBriefs in Applied Sciences and Technology—Computational Mechanics, which features cutting-edge research and practical applications presented and introduced by leading experts.

The author of this contribution, Professor Rudrapatna V. Ramnath, received a M.Sc. degree in Electrical Engineering (Flight Control) from Cranfield Institute of Technology (U.K.), M.S.E., M.A. and Ph.D. degrees in Aerospace and Mechanical Engineering from Princeton University. He generalized the multiple scales theory and significantly contributed to the theory and applications of asymptotic analysis. Using multiple scales theory, he solved a large class of problems in advanced aircraft and spacecraft dynamics and control including: hover-forward flight transitions of VTOL aircrafts, analytical theory of high angle-of-attack flight dynamics (for 1, 2, and 3 degree-of-freedom wing rock), re-entry vehicle dynamics, stability and parameter sensitivity of time-varying systems applied to the Generic Hypersonic Aerodynamic Model of NASA, etc. Furthermore, he developed a new handling quality theory of advanced aircrafts through variable flight conditions. In spacecraft he developed new and useful approaches for such as new attitude control design for single spin and dual spin satellites, dynamics, stability and control of large space structures with deformable reflectors, and mode shape determination for Heliogyro spacecraft, etc. Professor Ramnath's work also includes the development of rapid computational algorithms and reduced order models which were applied to NASA and Defense Department systems. He applied his theory to many aerospace systems at The Charles Stark Draper Laboratory and SPARTA, Inc., and also designed guidance laws for tactical missiles at Raytheon Missile Systems. He founded the company Vimanic Systems for performing research contracts for NASA centers including Dryden Flight Research Center and U.S. Department of Defense Laboratories.

For many years he taught undergraduate and graduate courses at Massachusetts Institute of Technology (MIT) on advanced aircraft and spacecraft dynamics and control and systems modeling. His scientific achievements are highly recognized through numerous technical papers and books on nonlinear control (published by

ASME), *Multiple Scales Theory and Aerospace Applications* (published by AIAA) and book chapters. In addition at MIT, Professor Ramnath conceived and developed the instrumentation, test methodology and procedures for performance evaluation of sports equipment in tennis and racket sports and golf clubs. He served as the Technical Editor of *Tennis* magazine and as Technical Advisor of ATP (Association of Tennis Professionals).

The monograph *Computation and Asymptotics* is a state-of-the-art presentation of the pioneering works of Professor Ramnath in applying and developing asymptotic analysis and the generalized multiple scales theory for efficient computations. The novelty of the approach is to introduce these two concepts in demanding computational problems in order to facilitate and to speed-up the solution procedure while ensuring the required accuracy. This work is well balanced between theoretical concepts and computation-intensive applications from the aerospace field. It serves as an excellent introduction to the power of combining asymptotic analysis and multiple scales approaches to complex computational problems to enable the aspiring engineer, interested in applied mathematical approaches, to achieve greater rewards in enhanced insight and computational efficiency in dynamic analysis, design and simulation. As such, I would highly recommend this work as a valuable addition to the technical library of a serious engineering analyst and practitioner.

December 2011

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Preface

In contrast to the traditional technical literature dealing with numerical computation and analytical asymptotic solution approaches, this book aims at presenting a methodology and philosophy that combine both aspects in a common synergistic combination. The purpose is to utilize the best features of each approach in an effort to facilitate efficient computation and glean insight into the nature of the system under study. In order to do this, it must be recognized that some degree of clever insight and understanding of the system behavior is needed in addition to implementing the optimal computational algorithms. In this sense the philosophy of this book marks a departure or a variation from the traditional and independent roles of computation and analytical solutions. The purpose is to combine the power of each approach and achieve a result that is greater than the sum of the parts. This is the view pursued and presented in this book.

This book is addressed to the dynamics and control systems analysts and designers who deal with computational issues of complex problems involving advanced scientific and engineering systems. In particular, engineers and scientists working with the computational aspects of sophisticated systems would find it very useful. Several important applications illustrating the benefits of a unified approach are presented. In addition, the techniques and methodology should be of interest to analysts and designers in mechanical, electrical and chemical engineering, and also some areas of physics, chemistry and biology.

The aim of this work is to present the powerful concepts and techniques of asymptotics and multiple scales approach in the context of computations of the dynamics of modern high performance scientific and engineering systems. Although the presentation is intended to be systematic and rigorous, the emphasis is on the concept, applicability and usefulness of these methods rather than on deep mathematical rigor. Reflecting my own interests, I have attempted to motivate the reader and appeal to a need for completeness, connectedness and philosophical abstraction in developing the theoretical framework. Recent research has culminated in a wealth of useful information and techniques which are generally unfamiliar to the practising engineer. I feel that substantial rewards are to be

gained by applying these techniques. The book intended to be used as a monograph reflecting the above spirit and philosophy.

Approximate solutions to complex physical and mathematical problems have been in use for a long time. This work concerns an area of approximations in applied mathematics known as asymptotic analysis and perturbation theory which deal with the study of applied mathematical systems in limiting cases. There are a number of good books on asymptotic analysis and perturbation theory, which form the basis upon which the multiple scales method rests. They are cited in the references and the reader would benefit by consulting them. In the technical literature the treatment has ranged from simple constructs to highly theoretical topics. However, the subject of multiple scales has been only briefly outlined in some books until recently. This powerful technique is relatively recent in its development, which is still continuing. It has led to a number of general and useful results which have been applied to a large number of diverse advanced engineering systems. Much of this body of knowledge, however, resides in research papers or is only partially treated in a small number of specialized mathematical books. A full and general development of the technique in available books appears to be lacking. Consequently, the power and usefulness of the technique is not well known among practising engineers. The value of this approach is being appreciated more and more with time as new applications are seen. A comprehensive development of this technique in its own right seems to be warranted by the rapid growth of the theory and the range of its applications covering a rather broad spectrum of engineering and scientific disciplines. It is in this spirit that this book is written. Engineering analysts and designers should derive much benefit from the simplicity of the concept and the general applicability of the method. The book is aimed at filling this gap and covering the middle ground between an entirely heuristic treatment and one of deep rigor and sophistication. It is intended as a bridge between esoteric mathematical theory and practical applications in the “real world”.

The early chapters present the basic concept, foundations and the techniques of asymptotic analysis, perturbation theory, multiple scales and an outline of standard numerical methods. As the basic ideas and concepts of asymptotic analysis are essential to a proper development of the multiple scales theory, a brief discussion of asymptotic analysis is first presented. Next, elements of perturbation theory are discussed, mainly as relevant to multiple scaling. Deeper insight into perturbation theory may be gained by the interested reader by consulting many well known works on the subject cited in the references.

The computational advantages of asymptotics is illustrated through applications. First, a classical problem of computing the effective nuclear charge near a singularity in the Thomas-Fermi problem in atomic physics is solved by multiple scales approach ([Chap. 6](#)). Next the problem of computational speed-up is demonstrated in the case of the satellite attitude prediction ([Chap. 7](#)) with gravity gradient and geomagnetic torques for earth satellites. [Chapter 8](#) presents the satellite attitude control problem for which the multiple scales approach facilitates the task of control design and computational efficiency by enabling the use of large

step sizes without loss of accuracy. All these problems utilize the multiple scales technique, which leads to solutions which are easily calculable and are accurate when compared to the conventional numerical solutions. Also included is an appendix on earth's environmental gravity gradient torque and geomagnetic torques and Floquet theory which is mainly of reference value.

The selection and presentation of the subjects reflects my own interests and experience. The material is an outgrowth of the lectures and courses taught by me at Princeton University and Massachusetts Institute of Technology and includes much of the research carried out by me with my students.

It is with pleasure that I record here that the original motivation, interest and inspiration came through meetings with Professor M.J. Lighthill first in India and later in the U.K. when I was a student. Later as a student at Princeton University, I was initiated into the subject by my Professors W.D. Hayes and M.D. Kruskal, my friend Dr. G.V. Ramanathan and by Dr. G. Sandri. I wish to record the interest and insight into engineering applications that I received from Professor D. Graham and Professor D. Seckel at Princeton and Dr. D.C. Fraser and Dr. R.H. Battin at the C.S. Draper Laboratory. Further, I wish to recognize the vigorous interaction and participation by my students in this research leading to many useful and important results.

Finally, I recognize with great appreciation the encouragement and support from my wife Vijaya, in motivating and helping me greatly in preparing many figures and the final manuscript. I also acknowledge the considerable joy that filled me from my wife and my children,—my son Venkatesh, and daughters Seetha and Leela.

Lexington, MA, December 2011

Rudrapatna V. Ramnath

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