AUTOMATION, COLLABORATION, AND E-SERVICES A C E S



Ways of Thinking, Ways of Seeing

Mathematical and Other Modelling in Engineering and Technology



Chris Bissell and Chris Dillon (Eds.)

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Editors

Dr. Chris Bissell The Open University Faculty of Mathematics, Computing and Technology Walton Hall Milton Keynes, MK7 6AA UK E-mail: c.c.bissell@open.ac.uk Dr. Chris Dillon The Open University Faculty of Mathematics, Computing and Technology Walton Hall Milton Keynes, MK7 6AA UK E-mail: c.r.dillon@open.ac.uk

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Preface

Chris Bissell and Chris Dillon

This is a book about 'models' and 'modelling', concepts that are extraordinarily difficult to pin down. As pointed out by a number of authors of this volume, a model could be a scale model, a mathematical model, a sketch, a segment of computer code, an analogy, a working device, or many other things. A model can be used to describe or explain some aspect of the natural world, it can be used as part of the design of an artefact, it can be part of an attempt to convince someone of some argument or ideology, or to determine public or corporate policy. The objective of this book is to investigate, in both a historical and a contemporary context, such aspects of 'modelling'.

Turning first to engineering and technology, these areas, like science, often use mathematical descriptions of the world. Because engineering models use many of the same mathematical techniques as scientific models (differential equations, Fourier and Laplace transformations, vectors, tensors, for example) it is easy to assume that they are one and the same in essence. Yet in the case of engineering (and technology in general) such models are much more likely to be used to design devices, equipment or industrial plant than for representing and analysing natural objects or phenomena. This means that historically there has been just as great – if not greater – emphasis on rules-of-thumb, charts, and empirical models as there has been on analytical models (although the latter have also been vitally important in areas such as electronics, mechanics, chemical and civil engineering, and so on). Other important tools have been scale models of various types, analogue simulators, and analogue and digital computation. A characteristic of all these approaches has been to develop new ways of seeing, thinking and talking about natural and artificial objects and systems.

Much of what has been written about mathematical modelling has been:

- The casting of traditional mathematics in 'modelling' dress, but still very theoretical, such as Gershenfeld (1999).
- A detailed study of historical and/or contemporary practice in the natural sciences or economics, such as Morgan & Morrison (1999).
- Highly theoretical and specialised examinations of engineering modelling, such as Lucertini et al (2004).
- The formal use of computers in converting well-specified modelling assumptions into computer code, see, for example Kramer (2007).

This book aims to redress the balance by studying primarily modelling in technological practice (although some chapters address essentially non-technological areas where there is an important link to engineering and technology). It is worth noting that some of the models considered in this book are not always highly valued in formal education at university level, which often takes an 'applied science' approach close to that of the natural sciences (something that can result in disaffection on the part of students). Yet in an informal context, such as design departments, laboratories, industrial placements, policy making, and so on, a very different situation obtains. A number of chapters will consider such epistemological aspects, as well as the status of different types of models within practitioner communities.

Professional training in many areas often includes a great deal of mathematics, conventionally said to 'underpin' the various disciplines. Yet practitioners often claim never to have used the majority of the mathematics they were taught. If by this they mean, for example, that they rarely if ever solve differential equations, invert matrices, or use vector calculus then – unless they are working in highly specialised research and development – they are almost certainly correct. This apparent paradox is best understood by examining the social context of the use of mathematics by the vast majority of professionals. Consider, for example, information engineering – disciplines such as telecommunications, electrical/electronics engineering, control engineering and signal processing that form the focus of several chapters of this book. Information engineers have developed visual or pictorial ways of representing systems that not only avoid the use of complex mathematics (although the techniques may well be isomorphic with the conventional formulations taught in universities), but have enabled ways of seeing and talking about systems that draw on the graphical features of the models.

This approach develops within a community of engineering practice where the interpretation and understanding of these visual representations of systems behaviour are learnt, shared and become part of the normal way of talking. Engineers put models to work by using them as the focal point for a story or conversation about how a system behaves and how that behaviour can be changed. It is by mediating in this process – acting to focus language by stressing some features of the real system while ignoring others – that models contribute to new shared understandings in a community of engineering practice. Interestingly, modern computer tools continue to exploit many much earlier information engineering techniques – techniques originally designed to eliminate computation but now used primarily to facilitate communication and human-machine interaction.

Very similar arguments apply to the use of mathematics and modelling in areas outside engineering. Models deriving originally from scientific or engineering principles may have to inform policy making, or be used to develop robust computer software, and the way this is best done is not always clear or straightforward; very often it is highly contested. A number of chapters consider this aspect of modelling in the context of economics, climate change, epidemiology and software development. Themes that are highlighted in this volume include:

- The role of language: the models developed for engineering design have resulted in new ways of talking about technological systems.
- Communities of practice: related to the previous point, particular engineering communities have particular ways of sharing and developing knowledge.
- Graphical (re)presentation: engineers have developed many ways of reducing quite complex mathematical models to more simple representations.
- Reification: highly abstract mathematical models are turned into 'objects' that can be manipulated almost like components of a physical system.
- Machines: not only the currently ubiquitous digital computer, but also older analogue devices – slide rules, physical models, wind tunnels and other small-scale simulators, as well as mechanical, electrical and electronic analogue computers.
- Mathematics and modelling as a bridging tool between disciplines.
- Modelling in large-scale socio-technological contexts, such as climate change and epidemiology.
- A move away from rigid formalism in software engineering.

The wide-ranging first chapter, by John Monk, looks at historical and philosophical aspects of modelling. Beginning with the nineteenth century, and the work of Lodge, Maxwell, Thomson, Kirchhoff, Mach and others – all predominantly physicists, but all of whom were enormously influential on electrical and mechanical technologies – Monk examines the use in particular of hydraulic and mechanical analogies and physical models. The consideration of Mach's philosophical bent then leads naturally to a study of a number of twentieth century philosophers, who have been intrigued by the notion of a model, including Wittgenstein, Foucault and Rorty. This initial chapter sets the scene for much that follows.

Chapter 2, by John Bissell, moves from the wide-ranging to the highly specific: a variety of approaches in science and technology based on the notions of dimensional analysis and dimensional reasoning. Like many of the themes of this book, the dimensional approach is one which is of enormous utility for scientific and technological practice, but one which finds only a minor place in the professional education of scientists and engineers. Bissell gives a number of examples of the power of this approach – from scale modelling in engineering design to estimating parameters of nuclear explosions from limited data.

The following two chapters, by Chris Dillon and Chris Bissell, can be usefully considered together, as both of them examine the role of language, communities of practice, and graphical modelling tools in information engineering. One important theoretical 'underpinning' of these disciplines (although the authors of these two chapters might well contest such a notion of 'underpinning') is the mathematics of complex numbers and vector calculus. From the turn of the twentieth century up to the 1950s information engineers invented a range of 'meta-mathematical' techniques to enable them to free electronics, telecommunications and control engineering design practice from the difficulties engendered by such a mathematical basis of their models. Furthermore, such tools that enabled mid-twentieth century engineers to avoid complicated calculation have now become an essential element in the human computer interface design of CAD software.

If the previous two chapters concentrated on 'meta-mathematical' tools in the forms of tables, maps, graphs and charts, then Charles Care's Chapter 5 turns to the physical devices used in such modelling before the days of the digital computer: physical models, electrical analogies and analogue computers. There are echoes of both Chapter 1 and Chapter 4 here, as Care looks in more detail at some of the work of Lord Kelvin (William Thomson) as well as analogue devices used in control engineering and related areas. But he also examines direct analogues such as soap films, electrolytic tanks and wax models, topics which have been very much under-researched in the history of modelling (Care, 2010).

Chapter 6 represents something of a turning point in the book, as it documents some of the ways that the technological modelling tradition centred on cybernetics and even control engineering – closely related to several earlier chapters – was transferred to the human domain through systems thinking (Ramage and Shipp, 2009). Magnus Ramage and Karen Shipp discuss the systems dynamics approach deriving from Jay Forrester and others; the work of Stafford Beer on organisations; Howard Odum on ecological systems; and the unique systems diagramming approach of the former Faculty of Technology at the UK Open University (now incorporated into the OU's Faculty of Mathematics, Computing and Technology.)

The remaining three chapters, in a sense, focus more on the human element – although this is not to underestimate the human factors in the book as a whole. In Chapter 7 Marcel Boumans situates Dutch computer-based economic modelling of the 1980s in the context of a history of the analogue modelling of economic systems. It is a particularly interesting case study, as the system concerned, FYSIO-EN, is a *computer* model of a *hydraulic* model of an economy – that itself looks back to a famous physical hydraulic model of the 1950s (Bissell, 2007). Ultimately FYSIOEN was not particularly successful, yet there are many lessons to be learned from the attempt.

Chapter 8, by Gabriele Gramelsberger and Erika Mansnerus, turns to contemporary policy issues, looking at how models of infectious disease transmission and climate change can inform decision making. The chapter covers the philosophical framework of such modelling and contrasts the inner world (empirical knowledge, computational framework, etc) and the outer world (predictive and prognostic power) of such modelling. It also considers the 'story telling power' of models, something that also emerges in a number of other chapters.

Finally, Chapter 9, by Meurig Beynon, describes the 'Empirical Modelling' approach to software construction developed over a number of years at the University of Warwick, UK, which draws particularly on the philosophy of William James and the sociology of Bruno Latour. Essentially this is a call to move away from an excessively reductionist and algorithmic approach to software development. All three of Chapters 7 - 9 echo the ideas about models as analogues and the pragmatic philosophy of Chapter 1.

These essays will be of interest to scientists, mathematicians and engineers, but also to sociologists, historians and philosophers of science and technology. And, although one or two of the chapters include significant mathematical content, understanding the fine details of this is not necessary in order to appreciate the general thrust of the argument.

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Contributors

MEURIG BEYNON is Emeritus Reader at the University of Warwick, UK. His major research interests are in the empirical modelling area outlined in his chapter in this volume.

CHRIS BISSELL is Professor of Telematics at The Open University, UK, where he teaches many aspects of ICT and researches primarily the history of technology.

JOHN BISSELL, at the time of writing his chapter, was completing his PhD in plasma physics at Imperial College of Science, Technology and Medicine, UK.

MARCEL BOUMANS is Associate Professor in the Faculty of Business and Economics at the University of Amsterdam, Co-Director of the faculty research programme 'Methodology and History of Economics' and Co-Editor of the Journal of Economic Thought.

CHARLES CARE completed a PhD on the history of analogue simulation at the University of Warwick, UK, and has subsequently worked for British Telecom as a Senior Software Engineer while retaining an Associate Fellowship at Warwick.

CHRIS DILLON was, until his recent retirement, Director of the Centre for Outcomes-Based Education at The Open University, UK. Before then he was Senior Lecturer in Electronics at the Open University and an author of course materials in control engineering, signal processing and other areas.

GABRIELE GRAMELSBERGER is a philosopher at the Freie Universität, Berlin, who is interested in the influence of computation on science and society. She has conducted an extensive study on the influence of computer based simulation on science, in particular in climate research and cell biology.

ERIKA MANSNERUS is a Postdoctoral Research Fellow at the London School of Economics. She studies the use of computational tools in public health interventions, such as vaccination planning or pandemic preparedness work, particularly the benefits and limitations of these tools and to how to interpret the evidence produced by them.

JOHN MONK is Emeritus Professor of Digital Electronics at The Open University, where he contributed to many courses in electronics, control engineering, robotics, and related areas. MAGUS RAMAGE is Lecturer in Systems at The Open University, UK. He has researched the history of systems thinking and is currently investigating the notion of 'information' in an interdisciplinary context.

KAREN SHIPP recently retired as Lecturer in Systems at The Open University, UK. Before this she was an educational software developer and designed numerous suites of computer aided learning for OU courses.

Chapter 1 Creating Reality

John Monk

The Open University, UK

Abstract. Analogues in the nineteenth century provided experimenters such as Lodge, Maxwell, Kirchhoff, Mach and Hertz with inspiration for mechanical descriptions of hidden physical processes that had, for example, electrical or magnetic properties, and suggested mechanical models that could illustrate their developing theories to a wider audience. Models and theories intertwine, since any confirmation or test of a theory has to show its predictive power in a specific situation. It is tempting to imagine that a model or theory is an accurate reflection of what takes place in reality; however prominent nineteenth century physicists and latterly pragmatist philosophers have insisted that our descriptions of reality are of our own making and are a product of our institutions and customs. Models as part of our descriptive practices, therefore, make a contribution to the construction of reality. This chapter discusses some of the nineteenth-century analogical models that offered ways of seeing and understanding physical phenomena, and goes on to discuss how philosophers have explored ways of thinking about the relationship between models and reality.

1.1 Modelling as a Discursive Practice

In an article entitled *Models*, Ludwig Boltzmann (1902) described a model as a 'representation ... of an object'. He noted that 'real objects' can be modelled in thought, and that 'objects in thought' can be represented by writing in a notebook. Kühne (2005), answering the question 'What is a model?' unpromisingly pointed out, 'there is little consensus about what exactly a model is' but he also claimed that 'All ... models are linguistic in nature' and that in software engineering, for example, a model is 'formulated in a modelling language' which employs diagrammatic and graphical notations¹. Similarly Kulakowski et al. (2007, 54) regarded system models as being articulated as 'verbal text, plots and graphs, tables of relevant numerical data, or mathematical equations'. In a theological paper that, most unusually, made reference to the physical sciences, Cosgrave (1983) saw

¹ See Beynon Chapter 9 in this volume for further discussion of software engineering models.

models as images, metaphors, analogies or symbols that are familiar and 'taken from ordinary life' which, for theology at least, intermingle with references to 'particular religious or theological objects or realities', an approach that has resonances with some of the philosophical perspectives discussed later in this chapter. Hesse (1953) likewise observed that 'most physicists do not regard models as literal descriptions of nature, but as standing in a relation of analogy to nature'. The emphasis on metaphor and analogy suggests that models and modelling have strong connections with other kinds of imaginative descriptive practices. Indeed Frigg (2010) argued that modelling has strong similarities with fiction writing and Vorms (2011) declared that '[m]odels such as the simple pendulum, isolated populations, and perfectly rational agents, play a central role in theorizing' and are 'imaginary ...fictional or abstract entities'.

It is tempting to imagine that a model or theory is an accurate reflection of what takes place in reality; however, prominent nineteenth century physicists and latterly pragmatist philosophers have insisted that our descriptions of reality are of our own making and are a product of our institutions and customs. Models as part of our descriptive practices, therefore, make a contribution to the construction of reality. This chapter discusses some of the nineteenth-century analogical models that offered ways of seeing and understanding physical phenomena, and goes on to discuss how philosophers such as Peirce, Wittgenstein, Foucault, Fleck and Rorty have explored ways of thinking about the relationship between models and reality.

1.2 Ways of Seeing: Some Nineteenth-Century Analogues

For nineteenth century physicists the term 'model' often referred primarily to mechanical constructions. Boltzmann (1892), for example, described an exhibition of mathematical models, many of which were made from 'plaster casts, models with fixed and movable strings, links, and all kinds of joints' but he also took the opportunity to mention 'mechanical fictions' – models or analogies that were 'dynamical illustrations in the fancy' that aided the development of mathematical statements of theories without necessarily ever being built. While theories were intended to be universal, any demonstration of a theory or hypothesis demanded descriptions of particular situations. The textbooks of the time described numerous examples of simple situations, for instance, of a single swinging pendulum (Maxwell 1876a, 100). Such descriptions of situations or artefacts used to illustrate a physical theory might today be called models.

The emergence of theories about electromagnetic phenomena provided a rich environment for the development of modelling practices (Hesse 1953). Phenomena such as sparks, electric shocks and forces between electromagnets and pieces of iron, attributed to electricity and magnetism, hint at out-of-sight physical activity. A demonstration may show what happens in a particular situation but does not comprehensively expose the relationships between such phenomena. How, then, are these relationships to be explained? One option is to speculate about the form of microscopic, or otherwise imperceptible, processes that bind phenomena together; another is to circumvent any explanation and systematically record observations about what happens and the conditions under which events occur, and then try to summarize a way of calculating what will happen from knowledge of the prevalent conditions; a third option is to construct an analogue – a mechanism that is not intended to represent the hidden physical workings but has visible or visualizable parts that, in some way, bear a parallel relationship to observable electrical phenomena.

Analogues in the nineteenth century provided physicists such as Lodge, Maxwell, Kirchhoff, Mach and Hertz with inspiration for mechanical descriptions of hidden physical processes that resulted in, for example, observable electrical or magnetic phenomena, and suggested mechanical models that could illustrate their developing theories to a wider audience. Models and theories intertwine, since any confirmation or illustration of a theory has to show its predictive power in a specific situation, while avoiding misleading or over-elaborate explanations that serve to obfuscate rather then illuminate the phenomena. The advantage of having a plausible mechanism in mind is that it breaks down the relationships between phenomena into components. If these correspond to familiar mechanical components in well-defined configurations, then the model helps to suggest and visualize relationships between them. The mechanism, or plan, or diagram of a mechanism, then acts as a systematic record and a familiar reminder and, by nature of the implied constraints on the physical behaviour of the mechanism, restricts the grammar of accounts of what can happen.

1.2.1 Oliver Lodge

In his book entitled *Modern Views on Electricity* and published in 1889, Oliver Lodge aimed to 'explain without technicalities ... the position of thinkers on electrical subjects' and he chose to do this with 'mechanical models and analogies' (Lodge 1889, v). In the text he avoided mathematics, but in a handful of places and in the appendix he told something about contemporary developments 'in less popular language than in the body of the book' (Lodge 1889, 387). His primary audience, however, was those who had 'some difficulty' with the published theories (Lodge 1889, v).

To avoid ignorance of the state of knowledge of electricity, one option, Lodge proposed, was to accept an analogy. He anticipated that his readers would 'get a more real grasp of the subject and insight into the actual processes occurring in Nature' by becoming familiar with analogies (Lodge 1889, 61), and he advised that the alternative was to use mathematics and dispense with 'pictorial images' (Lodge 1889, 13). Although some mathematicians rejected 'mental imagery', it was nevertheless helpful to have 'some mental picture' alongside the 'hard and rigid mathematical equations' (Lodge 1889, 61).

A promotional page at the front of Lodge's book was explicit about the text developing the 'incompressible-fluid' idea of electricity'' (Lodge 1889, v), and in the subsequent text Lodge suggested that electricity 'behaves like a perfect and allpermeating liquid' and 'obeys the same laws' (Lodge 1889, 12). But he warned his readers against becoming too attached to the analogy and inferring that 'because electricity obeys the laws of a liquid therefore it is one' (Lodge 1889 p.12). He considered the possibility that electricity and fluids may be 'really identical' to be a 'fancy', and counseled vigilance for discrepancies between the behavior of the two that would undermine their apparent similarity.

An important electrical device at the time – and still a vital part of electronics, but now predominantly at the microscopic level – was the condenser, or Leyden jar (now known as a capacitor), used to store electrical charge. Lodge exploited several analogues to explain the operation of Leyden jars. These devices were made of glass and had separate conducting coatings on their inside and outside. Connections were made to the interior and exterior coatings of a jar, which could retain an electric charge. One analogue used elastic, cords and beads. Another was a hydraulic model built from parts that could be bought from a plumbers' shop. As well as including an etching of the hydraulic model Lodge's book showed a 'skeleton diagram', reproduced here in Figure 1.1, and provided instructions for making the model. These began by telling the reader to 'procure a thin india-rubber bag, such as are distended with gas at toy-shops'. The bag, or balloon, was set inside a globular glass flask and the whole apparatus filled with water (Lodge 1889, 54–55).



Fig. 1.1 Lodge's hydraulic analogue of a Leyden jar. Source: Lodge (1889, 55).

When stopcock C was closed the rubber bag or balloon could be inflated with water by a pump connected through the open stopcock at A. The water displaced by the distending bag flowed out to a tank through the open stopcock B. In the analogue the water pump represents a generator of electricity, the inside of the inflatable rubber bag corresponds to the inner conducting surface of the Leyden jar, and the outer surface of the bag corresponds to the outer conducting surface of the jar. Having established an analogical connection with an electrical device, the behaviour of the hydraulic model could be translated speculatively into electrical activity. On their own, the diagrams, pictures and descriptions of the construction

of the hydraulic model did not indicate how the analogue might be operated; nevertheless, the diagrams, pictures and prose imposed constraints on any narrative of the model's operation, and offered ways of thinking about the Leyden jar's behaviour.

Lodge himself described a number of possible hydraulic experiments that had parallels with demonstrations carried out on a Leyden jar. For instance, the hydraulic pump connected to the open stopcock A could be turned on and the rise in pressure indicated by a rise in the height of water in the narrow vertical tube, a. When the stopcock B was open to a tank of water 'the pump can be steadily worked, so as to distend the bag and raise the gauge a to its full height, b remaining at zero all the time'. If stop-cock A was then closed the balloon would remain inflated unless the valve C was opened when the stretched balloon would force water out via C and suck water in to the globular glass flask through the opening next to B (Lodge 1889, 56).

When Lodge described the discharge he wrote, 'by the use of the discharger C the fluid can be transferred' and, although he was describing the experiment on the hydraulic model, he continued his sentence using terms that would normally be applied to a Leyden jar and adds 'from inner to outer coat'. He then reverted to terms appropriate to the apparatus constructed from the water-filled rubber bag to conclude 'the strain relieved, and the gauges equalized'. In Lodge's account the two sides of the analogy become entangled and the vocabulary and grammar of one domain is used in the other.

The Leyden jar, Lodge openly declared, has a 'hydrostatic analogue', and parts of the hydraulic equipment could be identified with parts of the Leyden jar. It would be easy to see, and it is easy to imagine, the rubber bag inflating and deflating and, when inflated, exerting pressure on the water so as to force water out on one side and draw it in on the other. The apparatus is unnecessary; a picture and a few words are sufficient for readers acquainted with balloons.

The electrical mechanism of the Leyden jar is inaccessible to human senses, nonetheless the hydraulic model offers figures of speech that can be a part of a plausible explanation of the electrical phenomena. However, caution is needed for not all electrical phenomena will have parallels in the field of hydraulics. For example, Lodge listed a number of effects of electrical currents and voltages that occurred outside of the wires carrying them. One such is the effect that an electric current has on a magnetic compass, which cannot be portrayed by any analogous effect occurring outside pipework carrying water (Lodge 1889, 91–92).

Limited by the hydraulic analogy Lodge introduced a new model for electromagnetism and electromagnetic fields. Fig. 1.2 is one of his diagrams. The rack is supposed to be an analogue of an electrical conductor with the linear movement of the rack mirroring the movement of electricity in a wire. In the analogue, the rack engages with an array of cogs that rotate when the rack moves, with the speed of rotation of the cogs standing for the strength of the magnetic field. All the cogs with plus signs on one side of the rack would rotate in the same direction, and all those with plus signs on the other side would rotate in the opposite direction; those cogs with minus signs made the mechanism plausible but unfortunately for the analogy would rotate in the opposite direction to those with the plus signs.



Fig. 1.2 Lodge's model of an electromagnetic field using a rack and meshing cogs. Source: Lodge (1889, 186).

To overcome the difficulty Lodge suggested the cogs with minus signs were related to 'negative electricity' (Lodge 1889, 264).

Lodge exploited this analogue of rotating wheels to explain 'electric inertia', or 'self-induction' (Lodge 1889, 186–187). He viewed the cogs as flywheels, so that to accelerate the enmeshed cogs some effort is necessary to move the rack. If the effort is no longer applied the 'motion is prolonged for a short time by the inertia'. In materials such as iron the magnetic effect is pronounced, and to emulate such substances, Lodge declared, they 'have their ... wheel-work exceedingly massive'. Lodge added various modifications to the model to refine the analogy, such as introducing wheels that can slip, leading him to write 'A magnetized medium ... is thus to be regarded as full of spinning wheels ... imperfectly cogged together'. Lodge's mechanical analogue illustrated a radical theory of electromagnetism that was, at the same time, a simple model of a straight, current-carrying wire.

Electrical activity is often invisible but, through various analogies, Lodge created fantastical explanations that could predict electrical effects in a convincing way. His choices of mechanical and hydraulic analogues were plausible although they did not validate the use of the analogy as an explanation of electrical phenomena. Instead the analogues served as useful and effective rhetorical devices. To help his readers understand and visualise what was going on Lodge suggested that they should think of:

electrical phenomena as produced by an all-permeating liquid embedded in a jelly; think of conductors as holes and pipes in this jelly, of an electrical machine as a pump, of charge as excess or defect, of attraction as due to strain, of discharge as bursting, of the discharge of a Leyden jar as a springing back or recoil, oscillating till its energy has gone. (Lodge 1889, 61–62).

Such fantasies can become so remote from personal experience that they cease to be persuasive or, as Lodge pointed out in the case of the hydraulic analogy for the Leyden jar, they risk being pressed too far. In some instances, however, analogies can offer an acceptable way of accommodating and debating hidden relationships between observable phenomena.

1.2.2 James Clerk Maxwell

Lodge was not the first to consider rotational elements in the explanation of electromagnetic phenomena. Helmholtz (1867), for example, observed 'a remarkable analogy between the vortex-motion of fluids and the electro-magnetic action of electric currents'. William Thomson (1872) endorsed 'Helmholtz's exquisite theory of vortex-motion' especially because the analogy showed how a theory might be developed – a challenge that Maxwell (1873a, 416) took up by looking at electromagnetic fields from a mechanical viewpoint and supposing that they were 'occupied with innumerable vortices of revolving matter' (Maxwell 1862). But he 'found great difficulty in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction' (Maxwell 1861, 469) until, like Lodge after him, he likened the relationship between the flow of electric current and magnetic lines of force 'to the relation of a toothed wheel or rack to wheels which it drives (Maxwell 1861).

Lodge clearly drew on Maxwell's analogy but offered a slightly different interpretation. Lodge initially described an analogy that included wheels connected by rubber bands and then introduced Maxwell's approach reporting: "It consisted of a series of massive wheels, connected together ... by a row of elastic particles or 'idle wheels'" (Lodge 1889, 263). Maxwell, like Lodge, had introduced two sets of wheels and explained: "when two wheels are intended to revolve in the same direction, a wheel is placed between them so as to be in gear with both, and this wheel is called an 'idle wheel'". But unlike Lodge, Maxwell, whose focus was on a vortex model and its mathematical expression, avoided mentioning negative electricity by not placing an interpretation on the 'idle wheels', writing:

The hypothesis about the vortices which I have to suggest is that a layer of particles, acting as idle wheels, is interposed between each vortex and the next, so that each vortex has a tendency to make the neighbouring vortices revolve in the same direction with itself. (Maxwell 1861)

In Maxwell's case, therefore, the layer of counter-rotating wheels were included to keep the analogy intact but the idle wheels played no part in the electrical interpretation. Lodge's mention of negative electricity did not invalidate his descriptions; however, it added an additional term to the theory that Maxwell had eliminated. In his attempt to follow the mechanical analogue closely Lodge had complicated the theory. As Hesse (1953) observed, redundant elements can also crop up in mathematical models. Referring to fragments of mathematics that might form a model she noted that a mathematical model is:

not an isolated collection of equations ..., but is a recognisable part of the whole structure of abstract mathematics, and this is true whether the symbols employed have any concrete physical interpretation or not. (Hesse, 1953)

In doing so she revealed that some mathematical fragments in a model may not be related to anything observable, but are included to ensure the model fits an established mathematical form. To maintain a consistent description these instrumental elements of the model could be treated as references to hypothetical objects or quantities. Without a physical counterpart, however, the new object or quantity has to remain metaphysical, and is vulnerable to a reformulation of the model that might render the hypothetical object or quantity redundant.



Fig. 1.3 Tyndall's analogue for sound propagation. Source: Tyndall (1867, 3).

In John Tyndall's book, Maxwell (1872, vi) admiringly noted, 'the doctrines of science are forcibly impressed on the mind by well-chosen illustrative experiments'. He was similarly complimentary about Tyndall's metaphoric imagery which illustrated the inscrutability of the physical world as 'a sanctuary of minuteness and power where molecules [build] up in secret the forms of visible things' (Maxwell 1890). In a controversial address, Tyndall (1874) explained that the senses are all we have to experience the world; all we can do is make inferences and treat what we sense as 'symbols'.

Accordingly he saw the outcome of a science education as an ability 'to picture with the eye of the mind those operations which entirely elude the eye of the body' and used analogy to encourage this. For example, he used a 'row of glass balls', illustrated in Fig. 1.3 as an image for explaining the propagation of the sound produced by a bursting balloon. Nudging the first ball would cause a movement which propagated along the row until the 'last ball only of the row flies away'. He anticipated this analogy would create an image of sound propagating from 'particle to particle through the air' (Tyndall 1867, 3–5).

Maxwell used imagery effectively to prompt questions. He suggested 'One kind of motion of the æther is evidently a wave-motion'. Deploying an analogy, he asked 'How will such waves affect an atom? Will they propel it forward like the driftwood which is flung upon the shore, or will they draw it back like the shingle which is carried out by the returning wave?' (Maxwell 1873b). What was needed, Maxwell (1864) emphasized, was 'a clear physical conception' in the analogue. In outlining the connection between calculus and mechanics he proposed a well defined mechanism 'free from friction, destitute of inertia, and incapable of being strained by the action of the applied forces'. This mechanism was not to be built but was 'to assist the imagination' and provide an alternative to algebra (Maxwell 1873a, 185).

There was, however, a psychological difficulty which Maxwell illustrated with 'the analogy between the phenomena of self-induction and those of the motion of material bodies'. He warned that it becomes difficult to abandon the mechanical analogy, or recognize that it is misleading, because our familiarity with the movement of material objects 'is so interwoven with our forms of thought that, when ever we catch a glimpse of it ..., we feel that a path is before us leading ... to the complete understanding' (Maxwell 1873a, 181). Consequently he was concerned about assumptions that are 'not warranted by experimental evidence' and cautioned about concluding, for instance, 'the electric current is really a current of a material substance, or a double current, or whether its velocity is great or small' (Maxwell 1873a, 202).

However, Maxwell (1873a, 201) applauded the 'many analogies between the electric current and a current of a material fluid'. Faraday's speculations on why an interrupted electric current should give an electric shock 'when we consider one particular wire only', Maxwell concluded, brought to bear phenomena 'exactly analogous to those of a pipe full of water flowing in a continued stream' (Maxwell 1873a, 180) and with pride announced 'the analogy between statical electricity and fluid motion turns out more perfect than we might have supposed' (Maxwell 1864).

But Maxwell's interests were far wider than the study of electricity; in his work on colour he assumed colours can be 'represented in quantity and quality by the magnitude and direction of straight lines' then, he concluded, 'the rule for the composition of colours is identical with that for the composition of forces in mechanics' (Maxwell 1860). He took the 'diffusion of heat from a part of the medium initially hotter or colder than the rest' as an analogy for the 'diffusion and decay' of an electric current induced in one circuit by a current in another. The result was that a calculation involving forces was transformed into a calculation involving heat (Maxwell 1873a, 397–398).

In the field of properties of materials Maxwell (1878) made the observation that a twisted wire 'creeps back towards its original position', and that such a wire exposed to twisting first one way and then another will also exhibit creep. He illustrated what happens with a series of different analogies: one involved the fall in temperature of 'a very large ball of iron' exposed to a series of temperature changes; another referred to the decay of electrical potential in a Leyden jar that was repeatedly charged and discharged; and the final illustration employed the decline of magnetism in iron and steel after a succession of changes in magnetisation.

Analogies transpose words and phrases into new settings. Maxwell suggested 'Scientific Metaphor' was a suitable phrase for describing a figure of speech that

is transferred from 'the language ... of a familiar science to one with which we are less acquainted' (Maxwell 1890). When studying moving bodies and profiting from the resources of the mathematicians, Maxwell encouraged the retranslation 'from the language of the calculus into the language of dynamics, so that our words may call up the mental image, ... of some property of moving bodies ... intelligible without the use of symbols.' (Maxwell 1873a, 185–194) If the language is to be scientific, Maxwell wrote, 'each term in its metaphorical use retains all the formal relations to the other terms of the system which it had in its original use' and, for example, should help those familiar with dynamics to become acquainted with electrical theories (Maxwell 1890).

In commenting on tackling the incompleteness of theories about electricity Maxwell (1864) considered that an early step was to provide 'simplification and reduction of the results of previous investigation'. He illustrated one kind of simplification with the identification of a theoretical particle which he described as 'A body so small that, for the purposes of our investigation, the distances between its different parts may be neglected', although the term particle may be applied to a planet, or even the Sun, when 'the actions of different parts of these bodies does not come under our notice'. On the other hand 'Even an atom, when we consider it as capable of rotation, must be regarded as consisting of many material particles'. (Maxwell 1876a, 11–12).

One form of simplification in a 'scientific procedure [involves] ... marking out a certain region or subject as the field of our investigations' and then, Maxwell proposed, ignoring 'the rest of the universe'. In a physical science, therefore, we identify a physical system 'which we make the subject of our statements'. This system can be as simple or as complex as we choose, and involve just a few particles or bodies, or the entire material universe (Maxwell 1876a, 10). Such a simplification is evident in construction of diagrams where 'no attempt is made to represent those features of the actual material system which are not the special object of our study' (Maxwell 1876b).

Abstractions and simplifications have profound implications. A generalization can allow, for example, a single model to relate to a number of different objects by stressing particular characteristics while ignoring details of individual peculiarities and weaker associations. Further, abstraction suggests considering something apart from any specific material embodiment or conventional ways of thinking. Thus when Rubenstein (1974, 192) wrote 'A model is an abstract description of the real world' he implied that the material form of the model was not necessarily significant; the model could have a role in deliberations in a number of conceivable realities.

Another approach is to provide an analogy which contains an object, or system, and a grossly simplified environment; for example, a pendulum suspended under constant gravity (the system) within an environment that provides only the initial push to start the pendulum swinging. Interactions are then divided into two sorts: the inputs, such as the push, which originate outside the system but affect its behaviour, and the outputs, such as the pendulum's resulting motion, which are generated by the system (Kulakowski et al. 2007, 1-2). This simplification introduces causation; the inputs cause the model's behaviour and the outputs are the resulting

behaviour. As with any simplification, however, the results can be misleading if the model or analogy is expected to give a good account of behaviour when the simplifying assumptions about the system or the environment do not obtain.

Maxwell was a profligate user of analogies. His strategy was to seek understanding of a collection of relationships in a poorly formulated field by adopting an analogy from another familiar and thoroughly explored domain. This was an intermediate step towards introducing and adapting a mathematical formulation which was, therefore, analogical. A set of phenomena might be explained by one or more different analogies; hence a range of analogies offers alternative ways of thinking about constructing a theory.

1.2.3 William Thomson (Lord Kelvin)

William Thomson (1824–1907), later Lord Kelvin (Anon 1892), seldom used the word 'model' in his technical papers. However, he often illustrated his popular lectures with mechanical models and he famously suggested that a measure of understanding in physics is 'Can we make a mechanical model of it?' (Thomson 1910, 830). What Thomson was seeking was an explanation of puzzling phenomena in terms of familiar objects. He wished to understand the phenomena of light, for example, 'without introducing things that we understand even less of' and the lack of a familiar analogue was why he believed he could not grasp electromagnetics (Thompson 1910, 835–836).

Thomson saw models, diagrams and examples as ways of illuminating the workings of the physical world. He expressed 'admiration for Maxwell's mechanical model of electromagnetic induction' because it was 'immensely instructive' and would assist the development of electromagnetic theory. And, in commenting on an insistence on the existence of a fixed relation between compressibility and rigidity (which is evidently flouted by a material such as jelly) Thomson berated Laplace, Lagrange and Poisson for their 'vicious habit ... of not using examples and diagrams'. For Thomson, models were pedagogical but did not necessarily represent reality; for example, although he viewed a mechanical illustration of 'the molecular constitution of solids' as 'undoubtedly instructive' the model was 'not to be accepted as true in nature' (Thompson 1910, 830).

Occasionally, however, he sought structures to satisfy his presumption that '[w]e cannot suppose all dead matter to be without form and void, and without any structure' and, he insisted, the use of a model was not 'merely playing at theory' but suggested possibilities for how molecules might be arranged (Thomson 1889).

In a lecture entitled *The Size of Atoms*, Thomson demonstrated wave propagation using a physical model made with wood and wire. This apparatus was also depicted in a full page woodcut in the report of the lecture, together with a footnote giving details of its construction. It was apparently made of a 'series of equal and similar bars ... of which the ends represent molecules of the medium'. The dimensions of the components and their materials were given and additional constructional details were noted, such as the way each wooden bar was attached to the supporting wire (Thomson 1894a). There is no doubt in this case, therefore, that when Thomson referred to the model he was referring to the physical apparatus. Although the constructional details were relevant primarily for someone wanting to replicate Thomson's apparatus, it was the demonstrations showing the behaviour of the model that provided the visualisations of propagation. The woodcut was a satisfactory alternative for a reader who was not present at the lecture but who could imagine the model's behaviour and integrate the image with the transcript of the lecture. Through its associations with common components such as lengths of wood and wire, the material model (or its image) conjured up constraints on what might happen, as well as offering familiar terms to describe the microscopic wave phenomenon that Thomson addressed.

However, Thomson's references to models did not always indicate a model that was to be built. In his IEE President's address on the ether, he showed his audience a 'skeleton model' which was a gyrostat² mounted on a square frame. Next he described a web of similar 'rigid squares with their neighbouring corners joined by endless flexible inextensible threads'. The impracticality of showing the infinite network led him to ask his audience to 'imagine mounted in each one of the rigid squares of this web a gyrostat'. This loosely coupled web of framed gyrostats, he claimed, was a model of an incompressible fluid (Thomson 1889). Similarly, in speculating on the propagation of light through a substance Thomson proposed 'a model with all needful accuracy' and asked his audience to 'suppose particles of real matter arranged in the cubic order, and six steel wire spiral springs, or elastic indiarubber bands, to be hooked on to each particle and stretched between it and its six nearest neighbours'. Although this description appears to be part of a plan for a practical construction, his proposition became openly fantastical when, to eliminate gravitational effects, he suggested transporting 'the theatre of the Royal Institution ... to the centre of the Earth' so he could show 'a model of an elastic solid' with a wave propagating through it (Thomson 1894a).

In contrast, some of Thomson's proposed analogues were plausible but impractical because they would be time consuming or expensive to construct. Nevertheless they were intended to stir the imagination and introduce the metaphors implied by the analogue. Thomson offered a story to explain the 'benefit' of 'electro-magnetic induction' which he compares to the 'benefit that mass is to a body shoved along against a viscous resistance'. His bizarre (but arguably plausible) allegory begins: 'Suppose, for instance, you had a railway carriage travelling through a viscous fluid'. He then switched his reference from a railway carriage to that of a boat on wheels in a viscous liquid and continued,

We will shove off two boats with a certain velocity ... but let one of them be loaded to ten times the mass of the other: it will take greater force to give it its impulse, but it will go further ...

and then, to relate the story to electrical phenomena, Thomson explained:

... [i]t requires more electric force to produce a certain amount of current, but the current goes further. (Thomson 1889).

² A gyrostat is a form of gyroscope.