Ralf Der Georg Marti<u>us</u>

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The Playful Machine

Theoretical Foundation and Practical Realization of Self-Organizing Robots

Foreword by Rolf Pfeifer





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To our families

Foreword

This book is about intelligence, embodied intelligence—and it is about a paradigm shift, a shift from a computational view of intelligence, thinking and cognition, to one that takes the complete organism—brain, body and environment—into account. As we know from Thomas Kuhn's famous book, "The structure of scientific revolutions", paradigm shifts do not occur instantaneously, but the process is a long and tedious one that happens at different levels and it takes a lot of time before a new consensus can be reached. There have been a large number of books and publications arguing, essentially from a conceptual, philosophical, or a biological stance, why computation is in many ways inappropriate to explain the behavior of systems in the real world and that completely novel approaches are needed. 25 years after the appearance of Rodney Brooks' seminal article with the innocuous title "A robust layered control system for a mobile robot," in a sense the manifesto for the famous 'subsumption architecture,' which, in the fields of Artificial Intelligence and robotics, marked the starting point of the "embodied turn," there is increasing consensus that embodied intelligence is the new paradigm, not only in AI, but in psychology, philosophy, and the neurosciences.

While most of the arguments in favor of an embodied perspective on intelligence are intuitively plausible and easy to understand, there is a definite lack of scientific theory, rigor, and methodology. Take, for example, the concept of sensory-motor contingencies as introduced by O'Regan and Noë a decade ago, the idea that there are law-like relations between the actions of an agent and associated changes in sensory stimulation (which depend on the environment as well as the morphological and material properties of the organism), which is intuitively very plausible and is by now generally accepted. However, at this point, it is still open how an in-depth mathematical treatment that captures all these relationships could be developed. Moreover, in the literature, in particular in robotics and artificial intelligence, rather superficial notions of embodiment are floating around that might be characterized by the slogan "intelligence requires a body." Many researchers were and still are convinced that if you simply use a robot into which you embed your otherwise traditional control algorithms, you have an embodied system. But embodiment is much more than that: it is not merely about having a body, but it is about the interrelationship between body, interaction with the environment, and information. After all, the body with its motor and sensory systems is the only way in which we can learn something about the environment, in other words, any interaction with the world is mediated by the body. In addition, rather than having to be controlled by a centralized "brain" or computational system, the body can be exploited for movement and locomotion: the elasticity in the muscles, and the compliance in a robotic device can take over part of the functionality of coping with impact in walking or running. Or the morphology and the material properties of the hand-arm-shoulder system, in particular the soft, deformable tissue in the palm and the finger tips, account for easy grasping of hard objects like glasses and cups in front of us with little control. We could go on for a long time.

But now that we are convinced that there is a real need for an "embodied turn," we have to think about how to proceed from here, how to develop a better understanding and a methodology of how to design and build embodied systems. Here, we can draw on knowledge in the areas of biology, in particular evolutionary theory, biomechanics, behavioral ethology, and neuroscience. Powerful concepts such as self-organization, complex systems, emergence, and the idea of sensory-motor contingencies mentioned above, promise to enhance our understanding of biological systems and bear the potential of being transferred to the design of artificial ones. However, the application of these principles to the design of robots and other devices, has proven much more difficult than initially anticipated. One of the big conundrums of research in the cognitive sciences and artificial intelligence has been and still is how creatures are motivated to do things and alternatively how we can design artificial systems that behave in seemingly goal-directed ways, without having to program the goal-directed behavior explicitly into the artifacts (as we would have done in the traditional approach). In the late 1980s, Luc Steels coined the term "design for emergence": Given a particular set of desired functionalities, e.g. a group of agents behaving in a swarm-like fashion, how can we design the system such that its behavior emerges as it interacts with its environment, including other agents? Guided self-organization is a potent principle that enables the exploitation of phenomena of self-organization to achieve desired functionalities in robotic devices, an efficient and robust engineering method-one solution to "designing for emergence."

Ralf Der and Georg Martius with their book "The playful machine" target precisely these issues by mustering the entire gamut of concepts and formal mathematical "machinery" like complex dynamics, self-organization, embodiment, homeostasis—and its later development, homeokinesis which, in addition to stability, provides a notion of intrinsic motivation—bifurcations, and various oscillator regimes, in order to design and build robots that are self-exploratory, self-motivated, situated, and adaptive. These machines are shown to display many surprising behaviors that are truly emergent from control, morphology, and environment, in the sense that they have not been programmed into their behavioral algorithms. The spirit of designing robots in this way, is completely different from the standard way where we have a particular goal, a set of tasks, in mind and we design the devices such that they are most likely to actually perform the tasks in an efficient, cheap and robust way: it is indeed the playful robots. And why playful robots? There are many hypotheses, but one of the main purposes of play seems to be to practice and explore novel skills in a protected environment so that they can be used in the real – serious – world at a later point in time. Because of the missing top-down goal-directedness normally imposed by the engineers, the robots need to be designed such that they are self-motivated to engage in play and that they can learn novel behaviors. In order for the playful robot scenario to work, the robots themselves must be motivated to engage in tasks and behaviors of ever increasing complexity. If done right, the competences acquired during playful behavior will generalize and transfer to other environments precisely because they are not task-specific. Note that this is very different from merely avoiding being damaged or coping with external perturbations; in this latter case, often the best strategy is to simply do nothing, a phenomenon called the "lazy robot" effect—to be prevented by all means.

In a sense, the book reflects its title: it is itself playful and self-motivating. In addition to the theoretical framework and conceptual discussions, the mathematical background and many examples of model robots, of real devices, and simulated creatures are presented. But even more, the whole book comes with lots of software with simulations, examples, and case studies that invite the reader to play around in order to get into the spirit of the "playful machine." With their seminal contribution, Der and Martius have not only substantially advanced the field of embodied intelligence, design for emergence, and robotics in general, but they have given it the rigor which it deserves as a scientific discipline: through their work, they have on the one hand accelerated the paradigm shift towards embodied intelligence, and on the other hand they have made inroads by demonstrating how existing methods in mathematics, dynamical systems, and engineering design, can be applied to this new and exciting perspective. We may in fact be at the beginning of a new revolution: What Bill Gates forcefully requested in his famous article in Scientific American in 2007: "A robot in every home!" (just as he demanded 20 years ago: "A computer in every home"), may in fact materialize sooner than we think. Der and Martius have decisively moved the field in this direction. What we, the community, need to do now, is to popularize it and make it accessible to a broad audience not only of engineering and computer science experts, but of biologists, neuroscientists, psychologists, philosophers, teachers at all levels, and to people simply interested in novel ideas and technological developments. I'm sure that you, the reader, will be thoroughly enjoying the book and that you will draw intellectual benefit and satisfaction from it. But beware-it may change the way you have been thinking about the world and yourself so far in unanticipated ways-and this process will be irreversible!

Someo, Valle Maggia, Switzerland, September 2011

Rolf Pfeifer

Preface

Imagine a world of artificial creatures living in a kind of paradise. These creatures would have unlimited access to resources (electrical current), they may live in a richly structured environment, are potentially immortal, and are not subject to any external pressure for development. The cushy situation in this robotic "paradise" gives rise to a nasty question that is the actual seed for this book: without any given task, goal, purpose, or other external pressures, why should such a creature do any-thing at all? Moreover, if there is no goal, no purpose, no plan, what can we expect the system to do? Will the resulting behaviors (if there ever is one) be arbitrary or will they relate to the specific nature of the physical system?

When trying to find an answer many routes are possible. You may focus on philosophical questions regarding free will or the existence of a machine-self; you may come across important challenges of modern robotics concerning intrinsic motivation, self-learning, or artificial curiosity; and you may end up with thinking about the very roots of autonomy and self-determination. The book faces the problem in a practical way by formulating a general principle—homeokinesis—that is unspecific, unbiased, surprisingly simple, completely internal to the agent, and fully operational on concrete robotic systems of high complexity.

As you will see when reading the book or doing the experiments, this general principle makes machines discover their behavioral variety in a playful individual development. The emerging activities, while being many and varied, are seen to be related to the physical properties of the body and environment so that the robot discovers the most natural modes that are accessible to him by minimal control. While this completely self-determined behavior generation may also help in the future to answer the more philosophical questions, it will in the short term provide very down-to-earth consequences. In particular, as we have learned from the field of embodied AI, behavior generation in complex robotic objects is improved and stabilized by taking brain, body, and environment as a whole. The playful unfolding of behavioral patterns offers a new way of getting the embodiment of the agent involved.

In a wider context, potential benefits are foreseeable when taking a new attitude to robotics, leaving the fixation on a specific goal aside in the beginning of a behavioral design process. Instead, comparable to a liberalist concept of education, we let the machines play freely, giving them a chance to demonstrate their potential capabilities. As a second step, we may try to gently guide the free play into desired directions, and it is only in a third step that the emerging behaviors could be valued and stored for later use in behavioral architectures following prescribed goals. The methods for autonomous development presented in this book are a first step towards this very practical goal and they might help in the future to overcome the enormous difficulties in the behavior generation of complex systems.

Given the practical orientation of the book, the reader is invited to enjoy the numerous videos at http://playfulmachines.com demonstrating the specific applications and/or doing experiments using the simulator that comes with the book. You will find nearly 30 experiments ranging from simple systems in low-dimensional sensorimotor loops to the robotic zoo with creatures such as snakes, dogs, and humanoids in a highly complex virtual, but physically realistic world. Hopefully this will also help to understand the theoretical considerations based on dynamical systems theory.

The book is intended for students and researchers interested in self-organization on both the practical and theoretical level. It should also be of interest for direct practical application since the controller with the homeokinetic learning algorithm can easily be connected to any real robot, helping it to a playful self-exploration of its behavioral variety. In combination with the experiments, the book also offers itself as an undergraduate or graduate course on self-organization and the dynamical systems approach to robotics.

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Contents

1	Intro	oduction	1
2	Self-	Organization in Nature and Machines	9
	2.1	Self-Organization — The Physical Perspective	10
		2.1.1 Phase Transitions	10
		2.1.2 Convection Patterns	10
		2.1.3 Reaction-Diffusion Systems	13
		2.1.4 Examples from Biology	14
	2.2	Self-Organization in Machines	16
		2.2.1 Swarms and Multi-Robot Systems	17
		2.2.2 Behavioral Self-Organization	18
	2.3	Exploiting Self-Organization for Robot Control	20
		2.3.1 Mechanisms of Self-Organization	20
		2.3.2 Self-Organization of Robot Behavior	20
3	The	Sensorimotor Loop	23
	3.1	Sensorimotor Loop — The General Case	24
		3.1.1 The Controller	24
	2.2	3.1.2 Forward Model and Sensorimotor Dynamics	25
	3.2	Dominated by Embodiment: The BARREL	27
		3.2.1 Properties	28
		3.2.2 Open Loop Control	28
	2.2	3.2.3 Closed Loop Control	30
	3.3	Analyzing the Loop	33
		3.3.1 Feedback Strength	33
		3.3.2 Fixed Points	30
		2.2.4 The Effective Differentian Date	3/
		2.2.5 Effective Differentian Deint and Europeration Debuties	58
	2.4	5.5.5 Effective Bifurcation Point and Explorative Behavior	41
	3.4	Extending the Parameter Space	45
		D.4.1 Biturcauon Scenario.	43

		3.4.2 Application: Biasing the Behavior 4
	3.5	Expanding the Body 4
	3.6	Realization by Neural Networks 5
		3.6.1 Rate-Coding Neuron Model 5
		3.6.2 Supervised Learning 5
		3.6.3 Feed-Forward Networks 5
		3.6.4 Recurrent Networks
	3.7	Summary 5
	App	endix 3.A Mathematical Details
		3.A.1 Stability Analysis
		3.A.2 Determining the Effective Bifurcation Point
4	Derte	sinks of Salf Description Homospheric
4	Pri	<u>Icipies of Self-Regulation — Homeostasis</u>
	4.1	History and Development of Homeostasis
		4.1.1 Ashby's Approach
		4.1.2 Ine Homeostat
		4.1.3 Learning Homeostat
	4.0	4.1.4 Relating Time Scales
	4.2	Self-Regulation
		$4.2.1 \text{First Ideas} \qquad \qquad$
		4.2.2 Learning the Controller from Specialized Models
		4.2.3 Homeostasis: Self-Regulated Stability
		4.2.4 Conclusions and Outlook
	App	endix 4.A Learning a Behavior from its Forward Model
	App	endix 4.B A Bootstrapping Scenario
5	AG	eneral Approach to Self-Organization — Homeokinesis
	5.1	Homeokinesis — Introduction
		5.1.1 Time-Loop Error
		5.1.2 Discussion
		5.1.2 Discussion
		5.1.5 Surfault Setting
	52	Homeokinetic Learning
	<u> </u>	5.2.1 Canonical Learning Rule
		5.2.2 Explicit Learning Rules
		5.2.3 Self-Actualization Adaptivity and Sensitivity — Example
		5.2.4 The Homeokinetic BARREI
	53	Key Features of Homeokinesis
	5.5	5.3.1 Homeokinesis as a Self Referential Dynamical System
		5.3.2 Sensitization — The Driving Force of Homeokinesid
		5.3.2 State Parameter Internlay
		5.3.4 Temperature Effect
		5.2.5 Summatry Crown of Attractors
		5.2.6 Spontonoous Symmetry Droubles 1
		D.S.O. Spontaneous Symmetry Breaking
	App	endix J.A. Formalizations 9

	Appendix 5.B Effective States	100
	Appendix 5.C Average Gradient	101
	Appendix 5.D Remarks on Reconstruction vs. Postdiction	103
	Appendix 5.E Derivation of Eq. (5.16)	103
	Appendix 5.F Derivation of Eqs. (5.22, 5.23)	104
	Appendix 5.G Moore-Penrose Pseudoinverse	105
6	From Fixed-Point Flows to Hysteresis Oscillators	. 107
	6.1 Time-Loop Error in One-Dimensional Systems	108
	6.2 Explicit Learning Rules and Fixed Point Flows	108
	6.2.1 Learning Rule	109
	6.2.2 Learning under Real World Conditions	110
	6.2.3 Relation to the Effective Bifurcation Point*	111
	6.3 Extending the Parameter Space — The Hysteresis Oscillator	112
	6.4 Embodiment and Situatedness — Robotic Experiments	115
	6.4.1 Wheeled Robots	115
	6.4.2 Spontaneous Cooperation in a Chain of Wheeled Robots	116
	6.4.3 Emergent Locomotion of the SLIDER ARMBAND	119
	6.5 Relation to Other Systems	120
	Appendix 6.A Fixed-Point Flow	122
	Appendix 6.B Towards the Effective Bifurcation Point	123
	Appendix 6.C Frequency of Hysteresis Oscillations	124
7	Symmetries Resonances and Second Order Hysteresis	127
<u>u</u>	7.1 Properties of the Two-Dimensional Sensorimotor Loop	127
	7.1.1 Two-Dimensional Loop	127
	7.1.2 Bifurcation Scenario	128
	7.1.3 Properties of $SO(2)$ Dynamics	129
	7.1.4 The Effect of the Bias	131
	7.2 Homeokinetic Learning	131
	7.2.1 State-Parameter Dynamics	132
	7.2.2 Oscillatory Behavior	134
	7.2.3 Low Frequency Oscillations	136
	7.2.4 Theory of Oscillatory Modes*	139
	7.3 Second Order Hysteresis	140
	7.3.1 Bias Dynamics and SO(2) Oscillations	140
	7.3.2 Frequency Sweeping and Hysteresis in Frequency Space	142
	7.3.3 Frequency Sweeping in Real Systems	143
	7.3.4 Frequency Sweeping in 3D	145
	7.4 Resonances — A Case for Embodiment	146
	Appendix 7.A Properties of Oscillatory Modes	149
	7.A.1 Derivation of Eq. (7.16).	150
	7.A.2 The Period-4 Cycle	150

8	Low	Dimensional Robotic Systems	. 153
	8.1	Semni	154
		8.1.1 Construction	154
		8.1.2 Experiments	156
	8.2	SPHERICAL	162
		8.2.1 Construction	162
		8.2.2 Self-Exploration of Rolling Modes	163
		8.2.3 Situatedness — How the Environment Shapes the Behavior	165
		8.2.4 Situatedness — Adapting to the Environment	167
	8.3	SLINGING SNAKE	168
		8.3.1 Construction	169
		8.3.2 Experiments	170
	8.4	ROCKING STAMPER	172
		8.4.1 Hardware	173
	0.7	8.4.2 Experiments	175
	8.5	BARREL	178
		8.5.1 Creativity in New Situations	178
	0.6	8.5.2 Creativity Using the Real BARREL	180
	8.6	Summary	182
0	Mod	lel Learning	183
	9 1	Cognitive Deprivation and Informative Actions	185
	2.1	9.1.1 Demonstration by the TwoWHEFLED	185
		9.1.2 Deprivation Effect	187
		9.1.3 Homeokinetic Learning and Bootstrapping	188
		9.1.4 Planar SNAKE	190
		9.1.5 SPHERICAL Robot in a Basin	192
		9.1.6 Discussion	193
	9.2	Extending the Forward Model	193
		9.2.1 Getting Misled by the Forward Model	194
		9.2.2 Including the Sensor Branch	195
		9.2.3 Ambiguity of the Extended Model	195
		9.2.4 Increasing Awareness of Causality	196
		9.2.5 Experiment with the SPHERICAL Robot	197
	9.3	<u>Summary</u>	199
	App	endix 9.A Ambiguities in the Sensorimotor Loop	199
10			• • •
10	High	<u>n-Dimensional Robotic Systems</u>	. 201
	10.1	Underactuated and Compliant	203
	10.2	DOG and HIPPODOG	207
	10.3	HUMANOID	209
	10.4	Snakes — Adaptation and Spontaneity	216
	10.5	Self-Kescue Scenario	218
	10.5	Discussion	219
	110./		$\angle \angle U$

11	Facing the Unknown — Homeokinesis in a New Representation [*]	. 223
	11.1 Interaction Representation of Sensorimotor Dynamics	224
	11.2 Extending the Time Horizon	226
	11.2.1 Preliminaries: Orbits in Dynamical Systems	226
	11.2.2 Interaction Representation of a Time Series	228
	11.2.3 Relation to the Time-Loop Error	229
	11.2.4 General Multiple-Step Learning Rule	230
	11.3 Homeokinesis as a Flow of Lyapunov Exponents	230
	Appendix 11.A Proof of Eq. (11.13)	232
	Appendix 11.B Derivation of the Interaction Representation	233
10		225
12	Guided Self-Organization — A First Realization	. 235
	12.1 Integration of Problem Specific Error Functions	236
	12.2 Guidance by Jeaching	237
	12.3 Direct Motor reaching	238
	12.5.1 Experiment	238
	12.4 Direct Sensor Teaching and Distal Learning	240
		241
13	Channeling Self-Organization	. 243
	13.1 From Spontaneous to Guided Symmetry Breaking	244
	13.2 Multiple Motor Relations	246
	13.2.1 Guiding Towards Directed Locomotion	246
	13.2.2 Scaling Properties	250
	13.3 Summary	252
14	Reward-Driven Self-Organization	. 253
	14.1 Reinforcing Speed	254
	14.2 Reinforcing Spin	256
	14.3 Discussion	257
15	Algorithmia Ingelon ontotion	261
15	15.1 The new Algorithm	. 201
	15.1 The Sox Algonium	201
	15.1.1 Extensions and Corresponding Meta-Parameters	203
	15.2 Cookbook for Diauful Machined	209
	15.2 COORDOOK TOL FlayIul Machines	271
	15.2.1 Type of Robots — Mechanical Setup	271
	15.2.2 Initialization Flocedures — Motor Dabbing	272
	15.2.5 Choosing the Meta-Lataneters	273
	15.3. Increasing Internal Complexity	274
	15.3.1 Controller Architecture	274
	15.3.2 Learning Rule	275
	15.4 Generalized Pseudoinverse*	275
	15.4.1 Examples	279
	15.5. Simplified Algorithms	280
	new emphanes and enance	200

15.5.1 Avoiding Matrix Inversion	281		
15.5.2 Matrix Inversion by Iteration	281		
15.6 Motor-Space Approach	282		
15.6.1 Reconstructing Previous Motor Values	283		
15.6.2 Learning Rules	283		
15.6.3 Using the Generalized Pseudoinverse	284		
Appendix 15.A Learning with Generalized Pseudoinverse	285		
Appendix 15.B Stability of the Gradient Descent	286		
15.B.1 Regularization of g' Terms	287		
Appendix 15.C Learning Rules for Arbitrary Neuron Types	287		
Appendix 15.D Derivation of the Multilayer Learning Rule	288		
15.D.1 Algorithmic Realization	290		
16 The LPZROBOTS Simulator	. 293		
16.1 Structure	294		
16.2 Controller Framework	294		
16.3 Matrix Library	295		
16.4 Physics Simulator	296		
16.4.1 Rigid Body Dynamics — Open Dynamics Engine	297		
16.4.2 Efficient Collision Detection	298		
16.4.3 Material and Surface Properties	298		
16.4.4 Motors and Sensors	301		
16.4.5 User Interaction	303		
16.4.6 Creating the Virtual World	304		
16.5 Highlights	307		
16.6 Summary	308		
	308		
17 Discussion and Perspectives	. 309		
List of Figures	. 313		
List of Videos	317		
	. 517		
List of Experiments	. 319		
References	. 321		
Index	331		
111112			

Chapter 1 Introduction

Robots and their relation to mankind have taken a long and diversified development. Starting from the romantic desire to have a workmate and/or playmate centuries ago, the modern history of robot control starts with the birth of artificial intelligence (AI) about 50 years ago. In the hype of that time, robots were considered as machines under total control of an artificial intelligence thought to understand the world and the physics of the body well enough in order to control the robot by a set of rules defining its behavior.

Subsequent developments have generated on the one hand the marvelous machines welding, mounting, and painting cars, but, on the other hand, also the bitter insight that the complexity of the world exceeds by far the scope of the internal models necessary for the AI approach. Even now, after 50 years, behavioral skills of the most advanced robots are far behind that of any insect. The rethinking began more than two decades ago, prompting a drastic change of paradigms in the control of autonomous robots [25, 26]. The new or embodied AI recognizes the role of the body as an equal partner in the control process. The exploitation of the specific properties of the body, sometimes called morphological computation [132], not only reduces the computational load on the controller in specific tasks like walking or swimming but also leads to smoother and more natural motions of the robot, see [131, 136] for a comprehensive and inspiring presentation of these ideas. The research has produced many successful and inspiring results. Of particular importance is the attempt to understand more about intelligence when using the embodiment approach.

Although quite successful, the method seems to be restricted to specific tasks and requires an inspired designer for the "morphological computer," instead of the intelligent programmer of the classical AI systems. Moreover, a general theoretical foundation is still missing. In our view, the achieved results suggest a next step on the way that gives machines more and more autonomy. Our ambition is to make the machines discover their behavioral options in a playful individual development in the first place and to look only afterwards for uses of the emerging behavioral options. Different from developmental robotics [97, 179], our approach is not focused so much on mental development but on the playful unfolding of the sensorimotor contingencies that form the basis for mental development.

Why Activity

In order to make this idea more concrete, let us formulate it in a casual way. As introduced in the Preface, let us consider robots in a kind of paradise, instead of a world driven by needs. With unlimited access to resources and without any given task, goal, purpose, or other pressures why should such a creature do anything at all? Borrowing from psychology, we may call this the problem of self-actualization.

Besides being of practical interest, self-actualization (at any level of the behavioral hierarchy) is also of principal interest in modern biology, artificial intelligence, and robotics. For instance, robotics and embodied AI both are in quest of features ranging from artificial curiosity to internal motivation to the very emergence of a machine-self. In biology this problem is related to the origin and realization of behavioral variability under identical circumstances (a question that is discussed much in connection with establishing free will as a biological trait, see [24] for a review).

There are some approaches known from the literature. One is making use of information theory. Considering the robot with its brain as an information processing system, the optimization of information flows might be an option. So far, different information measures have been shown to drive the robot to self-induced activity. Examples are empowerment [83, 87, 148] or predictive information [19] in the flow of sensor values the robot induces by its behavior [10, 162, [191]]. While of high generality, the application of these principles is hindered by excessive sampling costs. Information theory also allows for the formalization of conceptual terms like autonomy [17, 18], which may help in developing truly autonomous systems. Other general paradigms like Autopoiesis [109], although very intellectually appealing and helpful in describing the fundamental nature of living systems, are not constructive enough so that they are difficult to operationalize.

More concrete approaches address the question of artificial curiosity or intrinsic motivation in reinforcement learning. Pioneering work has been done by Schmidhuber using the prediction error as a reward signal in order to make the robot curious for new experiences [150]. The approach has been further developed in a number of papers, see e.g. [151], [64]. Related ideas have been put forward in the so called playground experiment by Kaplan and Oudeyer [85, [117], using the learning progress as a reward signal. Steels [157] proposes the Autotelic Principle, i.e. the balance of skill and challenge of behavioral components as the motivation for open ended development. This are interesting and encouraging developments towards agents with an internal, self-determined drive for activity. However, so far the developments are rather application related and require a convenient pre-structuring of state-action spaces whenever the systems are getting more complex. Information theory, while being domain invariant, is restricted in applicability by its tremendous sampling costs.

¹ However, different from psychology we will use this term not at the mental but at the sensorimotor level, meaning the self-determined unfolding of sensorimotor contingencies instead of the strive for personal growth and fulfillment in the sense of Maslow's "what a man can be, he must be" [108].

The Machines

Our approach to self-actualization is not philosophical but practical, aiming at the creation of a universal concept that can be operationalized and tested in complex robotic objects. Our approach is machine driven. So, before formulating the general principle let us have a closer look at the kind of machines we are interested in. The different strands of development in robotics and AI have produced very different kinds of robots. On the one side of the spectrum, there is the highly sophisticated machine for the reliable execution of motion plans like complicated dance moves, see Fig. [1.1(a)].



Fig. 1.1: Two different worlds of robots. (a) Sony robot QRIO dancing. These robots in an ideal manner realize the dream of having a machine that is authentic in executing a given motion plan. (b) Anthropomimetic robot ECCEROBOT. Instead of just mimicking the shape of the human body the compliantly designed robot tries to copy also the inner structure and mechanisms made up by bones, joints, muscles and tendons. In this way, it has the potential for human-like action and interaction with the world. This is considered the main prerequisite for the emergence of human-like intelligence. Images included with kind permission of (a) PC Watch [126] and (b) R. Knight [169].

On the other side, we find machines, like for instance the anthropomimetic EC-CEROBOT in Fig. [1.1[b]] designed to reflect the morphology of the human body as closely as possible. Copying the internal structure and mechanisms made up of bones, joints, muscles and tendons, these machines are compliant to the influence of external forces and intra-body couplings in a similar way as humans are. There are severe practical reasons for considering such machines. For instance in health care, service robots are to behave human-like and in particular should not react rigidly in encounters with humans. Moreover, they also have a high intellectual appeal, cumulating in the question whether the human-like body shapes cognitive processing into human-like dimensions. It is in these compliant machines where the principles of embodied robotics find their real playground. In fact, robots of that kind are a nightmare to any classical control approach and there is no chance for anything like classical AI realization, based on planning and a concrete world model, under these circumstances. Instead, the controller is challenged to maximally exploit the physical peculiarities of the body in its interaction with the environment. However, even in the embodied AI approach, the control of such systems is still in its infancy.

Compliant machines are the target group of this book, using however an extended concept of compliance. Compliance can be extended to the way the robot is related to its environment. In fact, we will introduce the notion of an extended body so that we can have a "rigid" robot (like Sony's Qrio) surrounded virtually by a compliant sphere provided by its sensors.

The robots studied in this book are simpler than the ECCEROBOT but we claim that they share many features and fundamental challenges with the latter. Let us demonstrate the parallels by one of our machines, the SPHERICAL, or its simpler variant, the BARREL, both being driven by an internal mechanism for shifting the robot's center of gravity, see Fig. [1.2]. Rolling is the most natural but by far not the only form of motion of this physical system. As demonstrated in experiments, while stable rolling modes can be excited by very simple closed loop controllers even on structured grounds, the execution of a motion plan, formulated in the space of the motor commands, may become quite complicated.

In this kind of machine, obviously there are specific patterns of behavior that, while singled out by a certain complexity, are achievable with minimal control. So, the idea is not to force the body into a specific behavior but to come into a kind of



Fig. 1.2: Robots dominated by embodiment. Two robots arguably matching the conceptual level of the anthropomimetic robots. The robots locomote by shifting their internal masses defining the center of gravity. The only sensor values are the inclination of the axes. As with ECCEROBOT, there are no single actions with definite consequences, even in the probabilistic sense. Instead, each action, a motion of the internal weights, produces a whole body reaction that depends on the current physical state of the mechanical system. (a) The BARREL, a physical system with 8 degrees of freedom. If simply rolling, the number of degrees of freedom reduces by 4. (b) The SPHERICAL has only one additional degree of freedom, the third internal mass, but displays much more complex motion patterns due to its different physical properties.

functional resonance with its specific physical properties. This is the point where the different machines pose the same challenge: feeling the body instead of forcing it is a indispensable prerequisite for any successful control paradigm. Of course, there are differences in the level of complexity and in the scientific questions. In particular, when looking for the roots of human like intelligence, the SPHERICAL probably can not make a real contribution.

Facing the Unknown

Now we can return to our original question. Keeping to the example of the sphere, why should a controller try at all to start shifting the internal weights so that the robot is driven into moving? And what can a general principle look like that drives the brain to find out about the most natural modes of behavior and the ways it can excite them? This book presents a solution that is based on the dynamical systems approach, thereby working with continuous state-action spaces right from the outset. We realize the brain of the robot by two neural networks, one for control and the second one acting as an internal forward model, predicting the next sensor state based on the present sensor and motor values. However, in contrast to classical AI, we do not use the model for planning ahead. Instead, by its prediction error, the model just quantifies the ability of the brain to look ahead, separating in this way the knowable components of the sensorimotor dynamics from what we call the unknown.

In these terms, a self-determined and explorative behavior can be understood as ways of facing the unknown. A naive way of doing so would be to "get things under control," i. e. adapt behavior in order to reduce the influence of the unknown on the future evolution of the system. However, in our robot paradise, this principle generates in most cases systems that self-regulate into a "do nothing" behavior. This "lazy robot effect" is easily understood by the fact that the consequences of doing nothing are perfectly predictable, in a static environment at least.

The astonishingly simple solution to that problem is found in a new representation of the unknown. Technically this amounts to replacing the prediction error with its time inverted counterpart, the reconstruction or time-loop error.

Homeokinesis

Facing the unknown is now defined as a continual process of adaptation directed towards reducing the size of the time-loop error. This book will show that this method not only solves the lazy robot problem but also provides a systematic approach to the self-determined individual development of embodied and compliant robots. The minimization of the time-loop error is shown to generate a common kinetic regime, called "homeokinesis", jointly involving the physical, the neural, and the synaptic processes of the artificial brain-body system formed by the robot and its controlling unit.

We show both theoretically and in many applications that homeokinesis is a fruitful solution to the problem of self-actualization and realizes in a systematic way the playful self-exploration of complex robotic objects. In the example of the spherical robot we observe that the homeokinetic brain excites the most stable rolling modes if running on a level surface, changes between modes in a self-determined manner, and finds another adequate rolling mode if in a spherical basin. These results propagate through to other robots with always the same findings—emerging search and the playful exploration of complex motion patterns with high sensorimotor coordination. Surprise is one of the inherent features of homeokinesis. In many cases we were witnessing the emergence of quite unexpected behaviors, e. g. various wrestling scenarios if two humanoid robots come into closer contact.

The approach to a self-organized, playful exploration of robot behavior is, in its concrete realization, based on many novelties making the principle applicable to a broad variety of robots with only minor changes. In particular, we realize a real-time application to high dimensional systems of many active degrees of freedom.

Guided Self-Organization

Without any purpose or goal, the emerging motion patterns are contingent, meaning many and varied, and transient by nature. This is vital for a developing system but less attractive from the practical point of view. The book also introduces a new research field, called guided self-organization, showing how external influences can be integrated in order to guide the self-organization into given directions, like the emergence of specific locomotion patterns. We introduce and investigate in practical applications several guidance principles, giving guidance for instance by directly influencing the motor patterns, by phase relations inducing symmetry breaking, or by rewards.

How to Use the Book

Besides conveying conceptual and theoretical foundations for the world of playful machines the book aims at making the reader interested in doing their own experiments. Therefore, the book comes with a demonstration software using our fully fledged robot simulator called LPZROBOTS and numerous suggestions for experiments scattered throughout the book. The software can be downloaded for free from http://www.playfulmachines.com Furthermore, we demonstrate our results by a large number of videos that can be watched at the same site. For illustration the videos are referenced in the book with a single image or a series of frames, however the captions are referring to the entire video clips.

The book is written on different levels of detail. Most chapters are organized such that they start with the more basic content and dive deeper and deeper into the details. So you may choose your own level by skipping later parts of the chapters. To better digest the theoretical content you can also at any time procrastinate your study by playing around with the experiments. They can run for hours while you read the book and you may come back to see what happened.

The mathematics may appear a bit hard at the beginning, but **don't panic**! Particularly hard sections are marked with a * sign and are intended for advanced study.

Organization of the Book

We start in Chap. 2 with a condensed introduction to self-organization phenomena in nature and machines as they are known from the literature. We have included this chapter in order to give the reader some background on the basic mechanisms of self-organization, since the latter are helpful for understanding the specific properties of homeokinetic systems.

Special to our approach is what we call the externalization of complexity. While using extremely simple structures for both internal model and controller, the behavioral patterns generated by our approach are of a highly complex nature. The very basis of that phenomenon is the use of closed loop control in a tight sensorimotor coupling. This is explained in detail in Chap. 3 Moreover, we use extremely fast learning procedures so that both model and controller are relearning if situations change, i. e. we replace internal complexity with flexibility.

Chapter d gives some theory and illustrative examples for the interplay between model and controller, demonstrating thereby the "lazy robot effect" that appears here in a kind of self-regulated stability. This phenomenon is considered in the historical context of homeostasis.

The subsequent Chap. Sintroduces homeokinesis on the basis of the time-loop error and its use for the self-organization of control. Homeokinetic learning by gradient descending the time-loop error is shown to generate a specific dynamical regime of the brain-body system that gives the synaptic dynamics a primary functional role in behavior generation.

In Chaps. 6 and 7 we give an analytical investigation of homeokinesis as a selfreferential dynamical system. Chapter 8 gives first applications focusing on specific features like the role of symmetries, spontaneity, and the emergence of specific embodiment effects in both simulated and real robots with only two motors but many physical degrees of freedom. The often surprising effects may be interpreted as a kind of arousal of the most natural motion patterns latent in the physical system.

Chapter D is concerned with another unique feature of our approach, namely the bootstrapping of both model and controller from scratch, i.e. if starting from our general initialization of "do nothing" and "know nothing." Moreover, it introduces the cognitive deprivation effect and shows how systems can recover from the deprivation by the homeokinetic learning process.

The specific phenomena observed in the low-dimensional systems, the excitation of latent whole body modes in particular, find their counterpart also in the high-dimensional examples considered in Chap. [10]. These complex robots are the actual target group of homeokinesis. Therefore, we try to present a broad spectrum of different robotic systems subject to homeokinetic control. In all examples we observe complex motion patterns with high sensorimotor coordination emerging, so to say, out of nothing, demonstrating the playfulness of these machines under the homeokinetic control paradigm.

This chapter is followed by an alternative approach to homeokinesis, based on a novel representation of the sensorimotor dynamics that we call the interaction representation. Besides giving an additional motivation for homeokinesis, this chapter will extend the considerations to the case of several time steps and will eventually consider infinite time horizons making contact with the global Lyapunov exponents and chaos theory. The chapter is a little more mathematically demanding, which is the reason for locating it after the concrete applications.

As mentioned above, a serious practical concern is to find ways for guiding selforganization towards specific goals. The results obtained so far in this novel direction of research are presented in Chap. 12 through Chap. 14, giving both the methodological ideas, concrete realizations, and a first analysis of concrete applications.

Details on the realization and several extensions of the algorithm are presented in Chap. [15] Although many of the algorithmic details are already referenced in earlier chapters, we have pushed this chapter towards the end of the book in order not to burden the reader with too much details for a first reading. The same is true for Chap. [16] devoted to a detailed description of the LPZROBOTS simulator that is the basis for all experiments in the virtual worlds proposed in the book.

Chapter 2 Self-Organization in Nature and Machines

Abstract: Self-organization in the sense used in natural sciences means the spontaneous creation of patterns in space and/or time in dissipative systems consisting of many individual components. Central in this context is the notion of emergence meaning the spontaneous creation of structures or functions that are not directly explainable from the interactions between the constituents of the system. This chapter presents at first several examples of prominent self-organizing systems in nature with the aim to identify the underlying mechanisms. While self-organization in natural systems shares a common scheme, self-organization in machines is more diversified. An exception is swarm robotics because of the similarity to a system of many constituents interacting via local laws as encountered in physics (particles), biology (insects), and technology (robots). This chapter aims at providing a common basis for a translation of self-organization effects to **single** robots considered as complex physical systems consisting of many constituents that are constraining each other in an intensive manner.

Self-organization is a ubiquitous phenomenon observed in many complex systems in the fields of physics, chemistry, computer science, economics, and biology. While synergetics provides a general theoretical framework for the wide field of such phenomena [60, 62, 182] there are many examples that are controversial and difficult to fit into a quantitative explanatory system [139]. This chapter will not try to shed new light onto this research field but instead work out by way of examples the most prominent features and underlying mechanisms of self-organizing systems in nature and machines. Once identified, these mechanisms may serve as a guiding principle for autonomous robot development.

After identifying one cornerstone—self-amplification—by the example of spontaneous magnetization (Sect. 2.1.1), we are going to study systems as different as convection patterns (Sect. 2.1.2), reaction diffusion systems (Sect. 2.1.3), and Turing patterns (Sect. 2.1.2) in order to understand the role of symmetries and spontaneous symmetry breaking, the second corner stone of self-organization.

After giving examples from biology (Sect. 2.1.4) that may help to understand the comprehensive nature of the general scheme, we will consider swarm robotics (Sect. 2.2.1) because of its close relationship to biology. Further fields of robotic research are related to self-organization only in a more distant way. We discuss in particular the research in artificial evolution in Sect. 2.2.2

Let us start with the self-organization phenomena in physics since they are very clear cut.

2.1 Self-Organization — The Physical Perspective

There are common features of self-organization that are shared by many examples in physics. The most essential ingredient in a self-organization scenario is the effect of self-amplification of small perturbations. Let us work out this in a simple case.

2.1.1 Phase Transitions

Many self-organization phenomena happen at phase transitions. A prominent example is the spontaneous magnetization in ferromagnetic materials [30]. These materials consist of a field of spins, which may be consider as little magnets. At high temperatures their orientation is random due to thermal fluctuations. The system could be given an order from outside by applying an external magnetic field that forces the magnets to orient.

However, even without the external field, when lowering the temperature below a critical value T_c , there is a phase transition to a magnetic domain structure consisting of clusters of aligned magnets. How can this be without any external guidance? The explanation rests on then "willingness" of the magnets to align along a given field. The thermal fluctuations are still active also below T_c and may create small clusters of aligned magnets by chance. However, such a micro-cluster acts like a little magnet that may force neighboring magnets to align so that the cluster increases in size and influence on its neighborhood. In this way, an originally small region will increase by this self-amplification mechanism so that the new order can expand over macroscopically large regions. The competition between clusters growing at different places in the substance leads eventually to the mentioned magnetic domain structure.

The magnetization direction of each of the domains is essentially random since it is the result of a thermal fluctuation. The space, however, is invariant against rotations of the magnets in any direction so that we have a breaking of this spatial symmetry. By the described scenario the original symmetry is broken spontaneously. This is one of the central effects in self-organization.

2.1.2 Convection Patterns

The structures created by the above effect are quite irregular and thus display only one part of the self-organization phenomena we are interested in. Of more interest are the regular structures observed in open physical systems driven by external influences into a steady state far from equilibrium. Well known is for instance the emergence of convection patterns in fluids under a heat gradient. A famous example are the Bénard cells, named after the French physicist Henri Bénard who discovered the phenomenon in 1900, see Fig. [2.2] for a specific example. He studied thin layers of water on a homogeneous surface heated from below such that the heat gradient is constant throughout the liquid, see Fig. [2.1].



Fig. 2.1: Convection patterns. Heat transport in a layer of water with small (top) and large heat gradient (middle). The interaction between the particles depends on the distance alone (bottom) so that there is no relation to the global structure.

As long as the gradient is not too large, heat transport is realized by conduction, which is based on energy transfer by collisions between particles of different kinetic energy. This is a random process, which is stable against perturbations but not very effective. Instead, heat transport is realized more effectively by convection instead of conduction. An example is given by the radiator driving a convection pattern in a room since air is ascending over the heat source and descending after cooling at the ceiling and the walls. The reason for the emerging convection pattern is the different distribution of heat sources and sinks in the room.

We could organize the Bénard system to develop a convection pattern, too, by using an inhomogeneous heat source, but this is not necessary. Instead, once the gradient exceeds a certain critical value, a phase transition is taking place towards a surprisingly regular convection pattern, a hexagonal structure in the concrete case. The emergence of that pattern may be considered as self-organized, since no external help has been given. How can this be? The explanation has two steps. First, with a large gradient, the system becomes unstable against thermal fluctuations. If somewhere a higher concentration of hot molecules is emerging, this acts as a kind of mini-radiator causing a tiny upward convection with a subsequent counter-con-



Fig. 2.2: Bénard cells. Macroscopic regular structures as a ubiquitous phenomenon of self-organization: Heated fluid forms regular hexagonal convection patterns. On the **right** is a small zoomed in part of the **left**. To illustrate the flow elongated particles have been added. ⓒ Herbert Oertel [18]].

vection of cooler molecules in the surroundings. Such a mini-convection cell can induce further convection cells in the vicinity since the downward current will facilitate other upward currents close-by. In this way, one convection cell acts like a germ for further cells in the neighborhood so that, by the instability of the system, a small thermal fluctuation can self-amplify to a macroscopic pattern, which may spread over the whole system (or parts of it). In this way the original symmetry is broken spontaneously, without any guidance from outside.

So, the first step in the explanation is given by the ability of the system to amplify microscopic thermal fluctuations due to the emerging instability. But how can we explain the observed regularities. And why are the patterns hexagonal? Well, the concrete form of the patterns in such phenomena depends on circumstances but there is a "weak" rule of thumb. Coarsely speaking, one can say that spontaneous symmetry breaking is in a sense economical, i.e. the symmetry is broken in the least possible way. In other words, the emerging structures are as simple as possible. This is explained by the fact that the establishment of the ordered structure is a complicated process that involves the reorganization of the many constituents of the system, the re-organizational efforts being the smaller, the simpler the structure. Hexagonal patterns are preferred if the system (under equilibrium conditions) is isotropic and homogeneous since they preserve the original invariance against translation and rotation, in a restricted sense, at least. This principle of "least commitment" is a general phenomenon that gives a qualitative understanding of the observed patterns.

This is interesting and intriguing. But there is more. An essential point is the tremendous reduction in dimensionality. The system actually consists of roughly 10^{23} components whereas the observed pattern is a regular macroscopic structure that essentially can be described by a few so called order parameters. Moreover, the patterns reflect a new dynamical organization of the system which is neither inscribed explicitly into the microscopic dynamics nor into the external conditions. This is why we may say that the system created this organization "out of nothing" by itself.

2.1.3 Reaction-Diffusion Systems

Further generic examples for self-organization are reaction-diffusion systems. Let us take this as another example in order to work out the fundamental principles of self-organizing systems in nature. As we have seen in the Bénard example the emergence of a pattern is the result of two opposing drives, a constraining one, which tries to conserve the overall symmetry of the system and some self-amplification mechanism that tries to destroy symmetries by amplifying the effects of local fluctuations. In a reaction-diffusion system the tendency towards symmetry is realized by diffusion, which is destroying any local density fluctuations. The self-amplification in reaction-diffusion systems is caused by an autocatalytic chemical reaction like

$$A \rightarrow A + A$$

This generates the desired self-amplification. If, by a fluctuation, the concentration of A is locally enhanced, the concentration of A is increasing exponentially in this region. This process is counteracted by the diffusion and the presence of other processes, which are responsible for the supply of raw and the disposal of waste material. Both are indispensable, since the autocatalytic reaction can not exist by itself but needs a support system.

2.1.3.1 Turing Patterns

There are many different reaction-diffusion systems leading to different kinds of patterns by a skillful combination of the components of the process. Alan Turing [173] was probably the first to investigate systems of the above kind in order to understand morphogenesis as a pattern formation process guided by the interplay of reaction and diffusion. He was far ahead of his time and it took more than 40 years until the first reaction proposed by Turing could be realized. Nowadays Turing patterns are very popular since they can be easily simulated by cellular automata.

Reaction-diffusion processes play also an important role in modern biology. For instance, they are postulated to cause the pigmentation of animal coats during morphogenesis. An illustration is given in Fig. 2.3 where the fur of a jaguar and two different results of a reaction-diffusion system are displayed.

2.1.3.2 Self-Organization in Space and Time

Classical Turing patterns are stationary. Even more interesting are reaction-diffusion systems, which realize a self-organization not only in space but also in time. The most famous example is the Belousov-Zhabotinsky reaction [192]. The competing processes, diffusion and chemical reactions, need time in order to be executed. By a skillful combination of the time scales and the nature of the involved reactions, the interplay between diffusion and reaction can be designed such that the respective