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Quantum Field Theory and Gravity

Conceptual and Mathematical Advances in the Search for a Unified Framework

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Preface

The present volume arose from the conference on *"Quantum field theory and gravity – Conceptual and mathematical advances in the search for a unified framework"*, held at the University of Regensburg (Germany) from September 28 to October 1, 2010. This conference was the successor of similar conferences which took place at the *Heinrich Fabri Institut* in Blaubeuren in 2003 and 2005 and at the *Max Planck Institute for Mathematics in the Sciences* in Leipzig in 2007. The intention of this series of conferences is to bring together mathematicians and physicists to discuss profound questions within the non-empty intersection of mathematics and physics. More specifically, the series aims at discussing conceptual ideas behind different mathematical and physical approaches to quantum field theory and (quantum) gravity.

As its title states, the Regensburg conference was devoted to the search for a unified framework of quantum field theory and general relativity. On the one hand, the standard model of particle physics – which describes all physical interactions except gravitation – is formulated as a quantum field theory on a fixed Minkowski-space background. The affine structure of this background makes it possible for instance to interpret interacting quantum fields as asymptotically "free particles". On the other hand, the gravitational interaction has the peculiar property that all kinds of energy couple to it. Furthermore, since Einstein developed general relativity theory, gravity is considered as a dynamical property of space-time itself. Hence space-time does not provide a fixed background, and a back-reaction of quantum fields to gravity, i.e. to the curvature of space-time, must be taken into account. It is widely believed that such a back-reaction can be described consistently only by a (yet to be found) quantum version of general relativity, commonly called *quantum gravity*. Quantum gravity is expected to radically change our ideas about the structure of space-time. To find this theory, it might even be necessary to question the basic principles of quantum theory as well.

Similar to the third conference of this series, the intention of the conference held at the University of Regensburg was to provide a forum to discuss different mathematical and conceptual approaches to a quantum (field) theory including gravitational back-reactions. Besides the two well-known paths laid out by string theory and loop quantum gravity, also other ideas were presented. In particular, various functorial approaches were discussed, as well as the possibility that space-time emerges from discrete structures.

The present volume provides an appropriate cross-section of the conference. The refereed articles are intended to appeal to experts working in different fields of mathematics and physics who are interested in the subject of quantum field theory and (quantum) gravity. Together they give the reader some overview of new approaches to develop a quantum (field) theory taking a dynamical background into account.

As a complement to the invited talks which the articles in this volume are based on, discussion sessions were held on the second and the last day of the conference. We list some of the questions raised in these sessions:

1. **Can we expect to obtain a quantum theory of gravity by purely mathematical considerations?** What are the physical requirements to expect from a unified field theory? How can these be formulated mathematically? Are the present mathematical notions sufficient to formulate quantum gravity, or are new mathematical concepts needed? Are the criteria of mathematical consistency and simplicity promising guiding principles for finding a physical theory? Considering the wide variety of existing approaches, the use of gedanken experiments as guiding paradigms seems indispensable even for pure mathematicians in the field.
2. **Evolution or revolution?** Should we expect progress rather by small steps or by big steps? By "small steps" we mean a conservative approach towards a unified theory where one tries to keep the conventional terminology as far as possible. In contrast, proceeding in "big steps" often entails to replace the usual terminology and the conventional physical objects by completely new ones.

   In the discussion, the possibilities for giving up the following conventional structures were considered:
   - **Causality:** In what sense should it hold in quantum gravity?
   - **Superposition principle:** Should it hold in a unified field theory? More specifically, do we have to give up the Hilbert space formalism and its probabilistic interpretation?

   A related question is:
3. **Can we quantize gravity separately?** That is, does it make physical sense to formulate a quantum theory of pure gravity? Can such a formulation be mathematically consistent? Or is it necessary to include all other interactions to obtain a consistent theory?
4. **Background independence**: How essential is it, and which of the present approaches implement it? Which basic mathematical structure would be physically acceptable as implementing background independence?
5. **What are the relevant open problems in classical field theory?** One problem is the concept of charged point particles in classical electrodynamics (infinite self-energy). Other problems concern the notion of quasi-local mass in general relativity and the cosmic censorship conjectures.

6. **(How) can we test quantum gravity?** Can one hope to test quantum gravity in experiments whose initial conditions are controlled by humans, similar to tests of the standard model in particle accelerators? Or does one need to rely on astronomical observations (of events like supernovae or black hole mergers)?

Having listed some of the basic questions, we will now give brief summaries of the articles in this volume. They are presented in chronological order of the corresponding conference talks. Unfortunately, not all the topics discussed at the conference are covered in this volume, because a few speakers were unable to contribute; see also pp. xii–xiii below.

The volume begins with an overview by **Claus Kiefer** on the main roads towards quantum gravity. After a brief motivation why one should search for a quantum theory of gravitation, he discusses canonical approaches, covariant approaches like loop quantum gravity, and string theory. As two main problems that a theory of quantum gravity should solve, he singles out a statistical explanation of the Bekenstein–Hawking entropy and a description of the final stage of black-hole evaporation. He summarizes what the previously discussed approaches have found out about the first question so far.

Locally covariant quantum field theory is a framework proposed by Brunetti–Fredenhagen–Verch that replaces the Haag–Kastler axioms for a quantum field theory on a fixed Minkowski background, by axioms for a functor which describes the theory on a large class of curved backgrounds simultaneously. After reviewing this framework, **Klaus Fredenhagen*** [1] and **Katarzyna Rejzner** suggest that quantum gravity can be obtained from it via perturbative renormalization à la Epstein–Glaser of the Einstein–Hilbert action. One of the technical problems one encounters is the need for a global version of BRST cohomology related to diffeomorphism invariance. As a preliminary step, the authors discuss the classical analog of this quantum problem in terms of infinite-dimensional differential geometry.

Based on his work with Joel Smoller, **Blake Temple** suggests an alternative reason for the observed increase in the expansion rate of the universe, which in the standard model of cosmology is explained in terms of "dark energy" and usually assumed to be caused by a positive cosmological constant. He argues that since the moment when radiation decoupled from matter 379000 years after the big bang, the universe should be modelled by a wave-like perturbation of a Friedmann–Robertson–Walker space-time, according to the mathematical theory of Lax–Glimm on how solutions of conservation laws decay to self-similar wave patterns. The possible perturbations form a 1-parameter family. Temple proposes that a suitable member of this family describes the observed anomalous acceleration of the galaxies (without invoking a cosmological constant). He points out that his hypothesis makes testable predictions.

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[1] In the cases where articles have several authors, the star marks the author who delivered the corresponding talk at the conference.

The term *"third quantization"* refers to the idea of quantum gravity as a quantum field theory on the space of geometries (rather than on space-time), which includes a dynamical description of topology change. **Steffen Gielen** and **Daniele Oriti*** explain how matrix models implement the third-quantization program for 2-dimensional Riemannian quantum gravity, via a rigorous continuum limit of discretized geometries. Group field theory (GFT) models, which originated in loop quantum gravity (LQG) but are also relevant in other contexts, implement third quantization for 3-dimensional Riemannian quantum gravity – but only in the discrete setting, without taking a continuum limit. The authors compare the GFT approach to the LQG-motivated idea of constructing, at least on a formal level, a continuum third quantization on the space of connections rather than geometries. They argue that the continuum situation should be regarded only as an effective description of a physically more fundamental GFT.

Andreas Döring* and **Rui Soares Barbosa** present the topos approach to quantum theory, an attempt to overcome some conceptual problems with the interpretation of quantum theory by using the language of category theory. One aspect is that physical quantities take their values not simply in the real numbers; rather, the values are families of real intervals. The authors describe a connection between the topos approach, noncommutative operator algebras and domain theory.

Many problems in general relativity, as well as the formulation of the AdS/CFT correspondence, involve assigning a suitable boundary to a given space-time. A popular choice is Penrose's *conformal boundary*, but it does not always exist, and it depends on non-canonical data and is therefore not always unique. **José Luis Flores**, **Jónatan Herrera** and **Miguel Sánchez*** explain the construction of a *causal boundary of space-time* which does not suffer from these problems. They describe its properties and the relation to the conformal boundary. Several examples are discussed, in particular pp-waves.

Dietrich Häfner gives a mathematically rigorous description of the Hawking effect for second-quantized spin-$\frac{1}{2}$ fields in the setting of the collapse of a rotating charged star. The result, which confirms physical expectations, is stated and proved using the language and methods of scattering theory.

One problem in constructing a background-free quantum theory is that the standard quantum formalism depends on a background metric: its operational meaning involves a background time, and its ability to describe physics *locally* in field theory arises dynamically, via metric concepts like causality and cluster decomposition. In his *general boundary formulation (GBF)* of quantum theory, **Robert Oeckl** tries to overcome this problem by using, instead of spacelike hypersurfaces, boundaries of arbitrary spacetime regions as carriers of quantum states. His article lists the basic GBF objects and the axioms they have to satisfy, and describes how the usual quantum states, observables and probabilities are recovered from a GBF setting. He proposes various quantization schemes to produce GBF theories from classical theories.

Felix Finster, **Andreas Grotz*** and **Daniela Schiefeneder** introduce causal fermion systems as a general mathematical framework for formulating relativistic quantum theory. A particular feature is that space-time is a secondary object which emerges by minimizing an action for the so-called universal measure. The setup provides a proposal for a "quantum geometry" in the Lorentzian setting. Moreover, numerical and analytical results on the support of minimizers of causal variational principles are reviewed which reveal a "quantization effect" resulting in a discreteness of space-time. A brief survey is given on the correspondence to quantum field theory and gauge theories.

Christian Bär* and **Nicolas Ginoux** present a systematic construction of bosonic and fermionic locally covariant quantum field theories on curved backgrounds in the case of free fields. In particular, they give precise mathematical conditions under which bosonic resp. fermionic quantization is possible. It turns out that fermionic quantization requires much more restrictive assumptions than bosonic quantization.

Christopher J. Fewster asks whether every locally covariant quantum field theory (cf. the article by Fredenhagen and Rejzner described above) represents "the same physics in all space-times". In order to give this phrase a rigorous meaning, he defines the "SPASs" property for families of locally covariant QFTs, which intuitively should hold whenever each member of the family represents the same physics in all space-times. But not every family of locally covariant QFTs has the SPASs property. However, for a "dynamical locality" condition saying that kinematical and dynamical descriptions of local physics coincide, every family of dynamically local locally covariant QFTs has SPASs.

Rainer Verch extends the concept of *local thermal equilibrium (LTE) states*, i.e. quantum states which are not in global thermal equilibrium but possess local thermodynamical parameters like temperature, to quantum field theory on curved space-times. He describes the ambiguities and anomalies that afflict the definition of the stress-energy tensor of QFT on curved space-times and reviews the work of Dappiaggi–Fredenhagen–Pinamonti which, in the setting of the semi-classical Einstein equation, relates a certain fixing of these ambiguities to cosmology. In this context, he applies LTE states and shows that the temperature behavior of a massless scalar quantum field in the very early history of the universe is more singular than the behavior of the usually considered model of classical radiation.

Inspired by a version of Mach's principle, **Julian Barbour** presents a framework for the construction of background-independent theories which aims at quantum gravity, but whose present culmination is a theory of classical gravitation called *shape dynamics*. Its dynamical variables are the elements of the set of compact 3-dimensional Riemannian manifolds divided by isometries and volume-preserving conformal transformations. It "eliminates time", involves a procedure called *conformal best matching*, and is equivalent to general relativity for space-times which admit a foliation by compact spacelike hypersurfaces of constant mean curvature.

Michael K.-H. Kiessling considers the old problem of finding the correct laws of motion for a joint evolution of electromagnetic fields and their point-charge sources. After reviewing the long history of proposals, he reports on recent steps towards a solution by coupling the Einstein–Maxwell–Born–Infeld theory for an electromagnetic space-time with point defects to a Hamilton–Jacobi theory of motion for these defects. He also discusses how to construct a "first quantization with spin" of the sources in this classical theory by replacing the Hamilton–Jacobi law with a de Broglie–Bohm–Dirac quantum law of motion.

Several theories related to quantum gravity postulate (large- or small-sized) extra dimensions of space-time. **Stefan Hollands**' contribution investigates a consequence of such scenarios, the possible existence of higher-dimensional black holes, in particular of stationary ones. Because of their large number, the possible types of such stationary black holes are much harder to classify than their 4-dimensional analogs. Hollands reviews some partial uniqueness results.

Since properties of general relativity, for instance the *Einstein equivalence principle (EEP)*, could conceivably fail to apply to quantum systems, experimental tests of these properties are important. **Domenico Giulini**'s article explains carefully which subprinciples constitute the EEP, how they apply to quantum systems, and to which accuracy they have been tested. In 2010, Müller–Peters–Chu claimed that the least well-tested of the EEP subprinciples, the *universality of gravitational redshift*, had already been verified with very high precision in some older atom-interferometry experiments. Giulini argues that this claim is unwarranted.

Besides the talks summarized above there were also presentations covering the "main roads" to quantum gravity and other topics related to quantum theory and gravity. PDF files of these presentations can be found at *www.uni-regensburg.de/qft2010.*

Dieter Lüst (LMU München) gave a talk with the title *The landscape of multiverses and strings: Is string theory testable?*. He argued that, despite the huge number of vacua that superstring/M-theory produces after compactification, it might still yield experimentally testable predictions. If the string mass scale, which can a priori assume arbitrary values in brane-world scenarios, is not much larger than 5 TeV, then effects like string Regge excitations will be seen at the Large Hadron Collider.

Christian Fleischhack from the University of Paderborn gave an overview of loop quantum gravity, emphasizing its achievements – e.g. the construction of geometric operators for area and volume, and the derivation of black hole entropy – but also its problems, in particular the still widely unknown dynamics of the quantum theory.

In her talk *New 'best hope' for quantum gravity*, **Renate Loll** from the University of Utrecht presented the motivation, the status and perspectives of "Quantum Gravity from Causal Dynamical Triangulation (CDT)" and how it is related to other approaches to a non-perturbative and mathematically rigorous formulation of quantum gravity.

Mu-Tao Wang from Columbia University gave a talk *On the notion of quasilocal mass in general relativity.* After explaining why it is difficult to define a satisfying notion of quasilocal mass, he presented a new proposal due to him and Shing-Tung Yau. This mass is defined via isometric embeddings into Minkowski space and has several desired properties, in particular a vanishing property that previous definitions were lacking.

Motivated by the question – asked by ’t Hooft and others – whether quantum mechanics could be an emergent phenomenon that occurs on length scales sufficiently larger than the Planck scale but arises from different dynamics at shorter scales, **Thomas Elze** from the University of Pisa discussed in the talk *General linear dynamics: quantum, classical or hybrid* a path-integral representation of classical Hamiltonian dynamics which allows to consider direct couplings of classical and quantum objects. Quantum dynamics turns out to be rather special within the class of such general linear evolution laws.

In his talk on *Massive quantum gauge models without Higgs mechanism*, **Michael Dütsch** explained how to construct the S-matrix of a non-abelian gauge theory in Epstein–Glaser style, via the requirements of renormalizability and causal gauge invariance. These properties imply already the occurrence of Higgs fields in massive non-abelian models; the Higgs fields do not have to be put in by hand. He discussed the relation of this approach to model building via spontaneous symmetry breaking.

Jerzy Kijowski from the University of Warszawa spoke about *Field quantization via discrete approximations: problems and perspectives.* He explained how the set of discrete approximations of a physical theory is partially ordered, and that the observable algebras form an inductive system for this partially ordered set, whereas the states form a projective system. Then he argued that loop quantum gravity is the best existing proposal for a quantum gravity theory, but suffers from the unphysical property that its states form instead an inductive system.

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It is a great pleasure for us to thank all participants for their contributions, which have made the conference so successful. We are very grateful to the staff of the Department of Mathematics of the University of Regensburg, especially to Eva Rütz, who managed the administrative work before, during and after the conference excellently.

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Bonn, Leipzig, Regensburg
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Quantum Gravity: Whence, Whither?

Claus Kiefer

Abstract. I give a brief summary of the main approaches to quantum gravity and highlight some of the recent developments.

Mathematics Subject Classification (2010). Primary 83-02; Secondary 83C45, 83C47, 83E05, 83E30.

Keywords. Quantum gravity, string theory, quantum geometrodynamics, loop quantum gravity, black holes, quantum cosmology.

1. Why quantum gravity?

Quantum theory provides a universal framework that encompasses so far all particular interactions – with one exception: gravitation. The question whether gravity must also be described by a quantum theory at the most fundamental level and, if yes, how such a theory can be constructed, is perhaps the deepest unsolved problem of theoretical physics. In my contribution I shall try to give a general motivation and a brief overview of the main approaches as well as of some recent developments and applications. A comprehensive presentation can be found in [1], where also many references are given; an earlier short overview is [2].

The main obstacle so far in constructing a theory of quantum gravity is the lack of experimental support. Physics is an empirical science, and it is illusory to expect that a new fundamental physical theory can be found without the help of data. This difficulty is connected with the fact that the fundamental quantum-gravity scale – the Planck scale – is far from being directly accessible. The Planck scale (Planck length, Planck time, and Planck mass or energy) follows upon combining the gravitational constant, G, the

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speed of light, c, and the quantum of action, $\hbar$,

$$l_{\rm P} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33} \text{ cm} , \tag{1}$$

$$t_{\rm P} = \frac{l_{\rm P}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.39 \times 10^{-44} \text{ s} , \tag{2}$$

$$m_{\rm P} = \frac{\hbar}{l_{\rm P} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.18 \times 10^{-5} \text{ g} \approx 1.22 \times 10^{19} \text{ GeV}/c^2 . \tag{3}$$

To probe the Planck scale with present technology, for example, one would need a storage ring of galactic size, something beyond any imagination. So why should one be interested in looking for a quantum theory of gravity?

The reasons are of conceptual nature. The current edifice of theoretical physics cannot be complete. First, Einstein's theory of general relativity (GR) breaks down in certain situations, as can be inferred from the singularity theorems. Such situations include the important cases of big bang (or a singularity in the future) and the interior of black holes. The hope is that a quantum theory can successfully deal with such situations and cure the singularities. Second, present quantum (field) theory and GR use concepts of time (and spacetime) that are incompatible with each other. Whereas current quantum theory can only be formulated with a rigid external spacetime structure, spacetime in GR is dynamical; in fact, even the simplest features of GR (such as the gravitational redshift implemented e.g. in the GPS system) cannot be understood without a dynamical spacetime. This is often called the *problem of time*, since non-relativistic quantum mechanics is characterized by the absolute Newtonian time t as opposed to the dynamical configuration space. A fundamental quantum theory of gravity is therefore assumed to be fully background-independent. And third, the hope that all interactions of Nature can be unified into one conceptual framework will only be fulfilled if the present hybrid character of the theoretical structure is overcome.

In the following, I shall first review the situations where quantum effects are important in a gravitational context. I shall then give an overview of the main approaches and end with some applications.

2. Steps towards quantum gravity

The first level of connection between gravity and quantum theory is quantum mechanics in an external Newtonian gravitational field. This is the only level where experiments exist so far. The quantum-mechanical systems are mostly neutrons or atoms. Neutrons, like any spin-1/2 system, are described by the Dirac equation, which for the experimental purposes is investigated in a non-relativistic approximation ('Foldy–Wouthuysen approximation'). One thereby arrives at

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} \approx H_{\rm FW} \psi$$

with (in a standard notation)

$$
\begin{aligned}
H_{\rm FW} &= \underbrace{\beta mc^2}_{\text{rest mass}} + \underbrace{\frac{\beta}{2m}\mathbf{p}^2}_{\text{kinetic energy}} - \underbrace{\frac{\beta}{8m^3c^2}\mathbf{p}^4}_{\text{SR correction}} + \underbrace{\beta m(\mathbf{a}\,\mathbf{x})}_{\text{COW}} \\
&\quad - \underbrace{\boldsymbol{\omega}\mathbf{L}}_{\text{Sagnac effect}} - \underbrace{\boldsymbol{\omega}\mathbf{S}}_{\text{Mashhoon effect}} \\
&\quad + \frac{\beta}{2m}\mathbf{p}\frac{\mathbf{a}\,\mathbf{x}}{c^2}\mathbf{p} + \frac{\beta\hbar}{4mc^2}\vec{\Sigma}(\mathbf{a}\times\mathbf{p}) + \mathcal{O}\left(\frac{1}{c^3}\right) .
\end{aligned} \tag{4}
$$

The underbraced terms have been experimentally tested directly or indirectly. (‘COW’ stands for the classic neutron interferometry experiment performed by Colella, Overhauser, and Werner in 1975.)

The next level on the way to quantum gravity is quantum field theory in an external curved spacetime (or, alternatively, in a non-inertial system in Minkowski spacetime). Although no experimental tests exist so far, there are definite predictions.

One is the Hawking effect for black holes. Black holes radiate with a temperature proportional to $\hbar$,

$$
T_{\rm BH} = \frac{\hbar\kappa}{2\pi k_{\rm B}c} , \tag{5}
$$

where κ is the surface gravity. In the important special case of a Schwarzschild black hole with mass M, one has for the Hawking temperature,

$$
\begin{aligned}
T_{\rm BH} &= \frac{\hbar c^3}{8\pi k_{\rm B}GM} \\
&\approx 6.17\times 10^{-8}\left(\frac{M_\odot}{M}\right) \text{ K} .
\end{aligned}
$$

Due to the smallness of this temperature, the Hawking effect cannot be observed for astrophysical black holes. One would need for this purpose primordial black holes or small black holes generated in accelerators.

Since black holes are thermodynamical systems, one can associate with them an entropy, the Bekenstein–Hawking entropy

$$
S_{\rm BH} = k_{\rm B}\frac{A}{4l_{\rm P}^2} \overset{\text{Schwarzschild}}{\approx} 1.07\times 10^{77}k_{\rm B}\left(\frac{M}{M_\odot}\right)^2 . \tag{6}
$$

Among the many questions for a quantum theory of gravity is the microscopic foundation of $S_{\rm BH}$ in the sense of Boltzmann.

There exists an effect analogous to (5) in flat spacetime. An observer linearly accelerated with acceleration a experiences a temperature

$$
T_{\rm DU} = \frac{\hbar a}{2\pi k_{\rm B}c} \approx 4.05\times 10^{-23}\ a\left[\frac{\rm cm}{\rm s^2}\right] \text{ K} , \tag{7}
$$

the ‘Unruh’ or ‘Davies–Unruh’ temperature. The analogy to (5) is more than obvious. An experimental confirmation of (7) is envisaged with higher-power, short-pulse lasers [3].

The fact that black holes behave like thermodynamical systems has led to speculations that the gravitational field might not be fundamental, but is instead an effective macroscopic variable like in hydrodynamics, see e.g. [4] for a discussion. If this were true, the search for a quantum theory of the gravitational field would be misleading, since one never attempts to quantize effective (e.g. hydrodynamic) variables. So far, however, no concrete 'hydrodynamic' theory of gravity leading to a new prediction has been formulated.

The third, and highest, level is full quantum gravity. At present, there exist various approaches about which no consensus is in sight. The most conservative class of approaches is quantum general relativity, that is, the direct application of quantization rules to GR. Methodologically, one distinguishes between covariant and canonical approaches. A more radical approach is string theory (or M-theory), which starts with the assumption that a quantum description of gravity can only be obtained within a unified quantum theory of all interactions. Out of these approaches have grown many other ones, most of them building on discrete structures. Among them are quantum topology, causal sets, group field theory, spin-foam models, and models implementing non-commutative geometry. In the following, I shall restrict myself to quantum general relativity and to string theory. More details on discrete approaches can be found in [5] and in other contributions to this volume.

3. Covariant quantum gravity

The first, and historically oldest, approach is covariant perturbation theory. For this purpose one expands the four-dimensional metric $g_{\mu\nu}$ around a classical background given by $\bar{g}_{\mu\nu}$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{\frac{32\pi G}{c^4}} f_{\mu\nu} \ , \tag{8}$$

where $f_{\mu\nu}$ denotes the perturbation. This is similar to the treatment of weak gravitational waves in GR. Associated with $f_{\mu\nu}$ is a massless 'particle' of spin 2, the graviton. The strongest observational constraint on the mass of the graviton comes from investigating gravity over the size of galaxy clusters and leads to $m_{\rm g} \lesssim 10^{-29}$ eV, cf. [6] for a discussion of this and other constraints. This mass limit would correspond to a Compton wavelength of 2×10^{22} m.

One can now insert the expansion (8) into the Einstein–Hilbert action and develop Feynman rules as usual. This can be done [1], but compared to Yang–Mills theories an important difference occurs: perturbative quantum gravity is non-renormalizable, that is, one would need infinitely many parameters to absorb the divergences. As has been shown by explicit calculations, the expected divergences indeed occur from two loops on. Recent progress in this direction was made in the context of $N = 8$ supergravity [7], see also [8]. $N = 8$ supergravity, which has maximal supersymmetry, is *finite* up to four loops, as was shown by an explicit calculation using powerful new methods. There are arguments that it is finite even at five and six loops and perhaps up to eight loops. If this is true, the question will arise whether there exists a hitherto unknown symmetry that prevents the occurrence of divergences at all.

Independent of this situation, one must emphasize that there exist theories at the non-perturbative level that are perturbatively non-renormalizable. One example is the non-linear σ model for dimension $D > 2$, which exhibits a non-trivial UV fixed point at some coupling g_{c} ('phase transition'). An expansion in $D - 2$ and use of renormalization-group (RG) techniques gives information about the behaviour in the vicinity of the non-trivial fixed point. The specific heat exponent of superfluid helium as described by this model was measured in a space shuttle experiment, and the results are in accordance with the calculations; the details are described, for example, in [9].

Another covariant approach that makes heavy use of RG techniques is asymptotic safety. A theory is called asymptotically safe if all essential coupling parameters g_i of the theory approach for $k \to \infty$ a non-trivial (i.e. non-vanishing) fixed point. This approach has recently attracted a lot of attention, see, for example, [9, 10] and the references therein. The paper [10] puts particular emphasis on the role of background independence in this approach.

Most modern covariant approaches make use of path integrals. Formally, one has to integrate over all four-dimensional metrics,

$$Z[g] = \int \mathcal{D}g_{\mu\nu}(x)\ \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}(x)]/\hbar} \ ,$$

and, if needed, non-gravitational fields. The expression is formal, since for a rigorous definition one would have to specify the details of the measure and the regularization procedure. Except for general manipulations, the path integral has therefore been used mainly in a semiclassical expansion or for discretized approaches. An example for the first is Hawking's use of the Euclidean path integral in quantum cosmology, while examples for the second application are Regge calculus and dynamical triangulation. In dynamical triangulation, for example, one decomposes spacetime into simplices whose edge lengths remain fixed. The sum in the path integral is then performed over all possible combinations with equilateral simplices, and heavy use of Monte-Carlo simulations is made, see, for example [11] for a review. Among the many interesting results of this approach, I want to mention here the fact that the (expected) four-dimensionality of spacetime emerges at macroscopic scales, but that spacetime appears two-dimensional at small scales. Surprisingly, this microscopic two-dimensionality is also a result of the asymptotic-safety approach.

In spite of being perturbatively non-renormalizable, quantum general relativity can be used in the limit of small energies as an effective field theory. One can obtain, for example, one-loop corrections to non-relativistic potentials from the scattering amplitude by calculating the non-analytic terms in the momentum transfer. In this way one can find one-loop corrections to the

Newton potential [12],

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right) ,$$

as well as to the Coulomb potential [13],

$$V(r) = \frac{Q_1Q_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{6}{\pi}\frac{G\hbar}{r^2c^3}\right) + \dots ,$$

The first correction terms, which do not contain $\hbar$, describe, in fact, effects of classical GR. The quantum gravitational corrections themselves are too small to be measurable in the laboratory, but they are at least definite predictions from quantum gravity.

4. Canonical quantum gravity

Canonical quantum gravity starts from a Hamiltonian formulation for GR and uses quantization rules to arrive at a wave functional Ψ that depends on the configuration space of the theory [1]. A central feature of all canonical theories is the presence of constraints,

$$\hat{H}\Psi = 0 , \tag{9}$$

where (9) stands for both the Hamiltonian and the diffeomorphism (momentum) constraints, which arise as a consequence of the presence of redundancies ('coordinate freedom') in GR. The various canonical versions of GR can be distinguished by the choice of canonical variables. The main approaches are

Geometrodynamics. The canonical variables are the 3-dimensional metric h_{ab} and a linear combination p^{cd} of the components of the extrinsic curvature.

Connection dynamics. The canonical variables are a connection A_a^i and a coloured electric field E_i^a.

Loop dynamics. The canonical variables are a holonomy constructed from A_a^i and the flux of E_i^a through a two-dimensional surface.

I shall give a brief review of the first and the third approach.

4.1. Quantum geometrodynamics

Quantum geometrodynamics is a very conservative approach [14]. One arrives inevitably at the relevant equations if one proceeds analogously to Schrödinger in 1926. In that year Schrödinger found his famous equation by looking for a wave equation that leads to the Hamilton–Jacobi equation in the (as we now say) semiclassical limit. As already discussed by Peres in 1962, the Hamilton–Jacobi equation(s)[1] for GR reads (here presented for

[1]The second equation states that S be invariant under infinitesimal three-dimensional coordinate transformations.

simplicity in the vacuum case)

$$16\pi G\, G_{abcd}\frac{\delta S}{\delta h_{ab}}\frac{\delta S}{\delta h_{cd}} - \frac{\sqrt{h}}{16\pi G}({}^{(3)}R - 2\Lambda) \;=\; 0\;, \tag{10}$$

$$D_a\frac{\delta S}{\delta h_{ab}} \;=\; 0\;, \tag{11}$$

where G_{abcd} is a local function of the three-metric and is called 'DeWitt metric', since it plays the role of a metric on the space of all three-metrics.

The task is now to find a functional wave equation that yields the Hamilton–Jacobi equation(s) in the semiclassical limit given by

$$\Psi[h_{ab}] = C[h_{ab}]\exp\left(\frac{\mathrm{i}}{\hbar}S[h_{ab}]\right)\;,$$

where the variation of the prefactor C with respect to the three-metric is much smaller than the corresponding variation of S. From (10) one then finds the Wheeler–DeWitt equation (Hamiltonian constraint)

$$\hat{H}\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (16\pi G)^{-1}\sqrt{h}\big({}^{(3)}R - 2\Lambda\big)\right)\Psi = 0, \tag{12}$$

and from (11) the quantum diffeomorphism (momentum) constraints

$$\hat{D}^a\Psi \equiv -2\nabla_b\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0\;. \tag{13}$$

The latter equations guarantee that the wave functional Ψ is independent of infinitesimal three-dimensional coordinate transformations.

A detailed discussion of this equation and its applications can be found in [1]. We emphasize here only a central conceptual issue: the wave functional does not depend on any external time parameter. This is a direct consequence of the quantization procedure, which treats the three-metric and the extrinsic curvature (which can be imagined as the 'velocity' of the three-metric) as canonically conjugated, similar to position and momentum in quantum mechanics. By its local hyperbolic form, however, one can introduce an intrinsic timelike variable that is constructed out of the three-metric itself; in simple quantum cosmological models, the role of intrinsic time is played by the scale factor a of the Universe.

By its very construction, it is obvious that one can recover quantum field theory in an external spacetime from (12) and (13) in an appropriate limit [1]. The corresponding approximation scheme is similar to the Born–Oppenheimer approximation in molecular physics. In this way one finds the equations (10) and (11) together with a functional Schrödinger equation for non-gravitational fields on the background defined by the Hamilton–Jacobi equation. The time parameter in this Schrödinger equation is a many-fingered time and emerges from the chosen solution S.

The next order in this Born–Oppenheimer approximation gives corrections to the Hamiltonian $\hat{H}^{\mathrm{m}}$ that occurs in the Schrödinger equation for the non-gravitational fields. They are of the form

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2}\,(\text{various terms})\ ,$$

see [1] for details. From this one can calculate, for example, the quantum gravitational correction to the trace anomaly in de Sitter space. The result is [15]

$$\delta\epsilon \approx -\frac{2G\hbar^2 H_{\mathrm{dS}}^6}{3(1440)^2\pi^3 c^8}\ .$$

A more recent example is the calculation of a possible contribution to the CMB anisotropy spectrum [16]. The terms lead to an enhancement of power at small scales; from the non-observation of such an enhancement one can then get a weak upper limit on the Hubble parameter of inflation, $H_{\mathrm{dS}} \lesssim 10^{17}$ GeV.

One may ask whether there is a connection between the canonical and the covariant approach. Such a connection exists at least at a formal level: the path integral satisfies the Wheeler–DeWitt equation and the diffeomorphism constraints. At the one-loop level, this connection was shown in a more explicit manner. This means that the full path integral with the Einstein–Hilbert action (if defined rigorously) should be equivalent to the constraint equations of canonical quantum gravity.

4.2. Loop quantum gravity

An alternative and inequivalent version of canonical quantum gravity is loop quantum gravity [17]. The development started with the introduction of Ashtekar's New Variables in 1986, which are defined as follows. The new momentum variable is the densitized version of the triad,

$$E_i^a(x) := \sqrt{h(x)}e_i^a(x)\ ,$$

and the new configuration variable is the connection defined by

$$GA_a^i(x) := \Gamma_a^i(x) + \beta K_a^i(x)\ ,$$

where $\Gamma_a^i(x)$ is the spin connection, and $K_a^i(x)$ is related to the extrinsic curvature. The variable β is called the Barbero–Immirzi parameter and constitutes an ambiguity of the theory; its meaning is still mysterious. The variables are canonically conjugated,

$$\{A_a^i(x), E_j^b(y)\} = 8\pi\beta\delta_j^i\delta_a^b\delta(x,y)\ ,$$

and define the connection representation mentioned above.

In loop gravity, one uses instead the following variables derived from them. The new configuration variable is the holonomy around a loop (giving the theory its name),

$$U[A,\alpha] := \mathcal{P}\exp\left(G\int_\alpha A\right)\ ,$$

and the new momentum variable is the densitized triad flux through the surface $\mathcal{S}$ enclosed by the loop,

$$E_i[\mathcal{S}] := \int_{\mathcal{S}} \mathrm{d}\sigma_a \, E_i^a \ .$$

In the quantum theory, these variables obey canonical commutation rules. It was possible to prove a theorem analogous to the Stone–von Neumann theorem in quantum mechanics [18]: under some mild assumption, the holonomy–flux representation is unique. The kinematical structure of loop quantum gravity is thus essentially unique. As in quantum geometrodynamics, one finds a Hamiltonian constraint and a diffeomorphism constraint, although their explicit forms are different from there. In addition, a new constraint appears in connection with the use of triads instead of metrics ('Gauss constraint').

A thorough presentation of the many formal developments of loop quantum gravity can be found in [17], see also [19] for a critical review. A main feature is certainly the discrete spectrum of geometric operators. One can associate, for example, an operator $\hat{A}$ with the surface area of a classical two-dimensional surface $\mathcal{S}$. Within the well-defined and essentially unique Hilbert space structure at the kinematical level one can find the spectrum

$$\hat{A}(\mathcal{S})\Psi_S[A] = 8\pi\beta l_{\mathrm{P}}^2 \sum_{P\in S\cap\mathcal{S}} \sqrt{j_P(j_P+1)}\Psi_S[A] \ ,$$

where the j_P denote integer multiples of $1/2$, and P denotes an intersection point between the fundamental discrete structures of the theory (the 'spin networks') and $\mathcal{S}$. Area is thus quantized and occurs as a multiple of a fundamental quantum of area proportional to l_{P}^2. It must be emphasized that this (and related) results are found at the kinematical level, that is, before all quantum constraints are solved. It is thus an open problem whether they survive the solution of the constraints, which would be needed in order to guarantee physical meaning. Moreover, in contrast to quantum geometrodynamics, it is not yet clear whether loop quantum gravity has the correct semiclassical limit.

5. String theory

String theory is fundamentally different from the approaches described above. The aim is not to perform a direct quantization of GR, but to construct a quantum theory of all interactions (a 'theory of everything') from where quantum gravity can be recovered in an appropriate limit. The inclusion of gravity in string theory is, in fact, unavoidable, since no consistent theory can be constructed without the presence of the graviton.

String theory has many important features such as the presence of gauge invariance, supersymmetry, and higher dimensions. Its structure is thus much more rigid than that of quantum GR which allows but does not demand these features. The hope with string theory is that perturbation theory is finite at all orders, although the sum diverges. The theory contains only three fundamental dimensionful constants, $\hbar, c, l_{\rm s}$, where $l_{\rm s}$ is the string length. The expectation is (or was) that all other parameters (couplings, masses, ...) can be derived from these constants, once the path from the higher-dimensional (10- or 11-dimensional) spacetime to four dimensions is found. Whether this goal can ever be reached is far from clear. It is even claimed that there are so many possibilities available that a sensible selection can only be made on the basis of the anthropic principle. This is the idea of the 'string landscape' in which at least 10^{500} 'vacua' corresponding to a possible world are supposed to exist, cf. [20]. If this were true, much of the original motivation for string theory would have gone.

Since string theory contains GR in some limit, the above arguments that lead to the Wheeler–DeWitt equation remain true, that is, this equation should also follow as an approximate equation in string theory if one is away from the Planck (or string) scale. The disappearance of external time should thus also hold in string theory, but has not yet been made explicit.

It is not the place here to give a discussion of string theory. An accessible introduction is, for example, [21]; some recent developments can be found in [5] as well as in many other sources. In fact, current research focuses on issues such as AdS/CFT correspondence and holographic principle, which are motivated by string theory but go far beyond it [22]. Roughly, this correspondence states that non-perturbative string theory in a background spacetime that is asymptotically anti-de Sitter (AdS) is dual to a conformal field theory (CFT) defined in a flat spacetime of one less dimension, a conjecture made by Maldacena in 1998. This is often considered as a mostly background-independent definition of string theory, since information about the background metric enters only through boundary conditions at infinity.[2]

AdS/CFT correspondence is considered to be a realization of the holographic principle which states that all the information needed for the description of a spacetime region is already contained in its boundary. If the Maldacena conjecture is true, laws including gravity in three space dimensions will be equivalent to laws excluding gravity in two dimensions. In a sense, space has then vanished, too. It is, however, not clear whether this equivalence is a statement about the reality of Nature or only (as I suspect) about the formal properties of two descriptions describing a world with gravity.

6. Black holes and cosmology

As we have seen, effects of quantum gravity in the laboratory are expected to be too small to be observable. The main applications of quantum gravity should thus be found in the astrophysical realm – in cosmology and the physics of black holes.

[2]But it is also claimed that string *field* theory is the only truly background-independent approach to string theory, see the article by Taylor in [5].

As for black holes, at least two questions should be answered by quantum gravity. First, it should provide a statistical description of the Bekenstein–Hawking entropy (6). And second, it should be able to describe the final stage of black-hole evaporation when the semiclassical approximation used by Hawking breaks down.

The first problem should be easier to tackle because its solution should be possible for black holes of arbitrary size, that is, also for situations where the quantum effects of the final evaporation are negligible. In fact, preliminary results have been found in all of the above approaches and can be summarized as follows:

Loop quantum gravity. The microscopic degrees of freedom are the spin networks; $S_{\rm BH}$ only follows for a specific choice of the Barbero–Immirzi parameter β: $\beta = 0.237532\ldots$ [23].

String theory. The microscopic degrees of freedom are the 'D-branes'; $S_{\rm BH}$ follows for special (extremal or near-extremal) black holes. More generally, the result follows for black holes characterized by a near-horizon region with an AdS_3-factor [24].

Quantum geometrodynamics. One can find $S \propto A$ in various models, for example the LTB model describing a self-gravitating spherically-symmetric dust cloud [25].

A crucial feature is the choice of the correct state counting [26]. One must treat the fundamental degrees of freedom either as distinguishable (e.g. loop quantum gravity) or indistinguishable (e.g. string theory) in order to reproduce (6).

The second problem (final evaporation phase) is much more difficult, since the full quantum theory of gravity is in principle needed. At the level of the Wheeler–DeWitt equation, one can consider oversimplified models such as the one presented in [27], but whether the results have anything in common with the results of the full theory is open.

The second field of application is cosmology. Again, it is not the place here to give an introduction to quantum cosmology and its many applications, see, for example, [1, 28] and the references therein. Most work in this field is done in the context of canonical quantum gravity. For example, the Wheeler–DeWitt equation assumes the following form for a Friedmann universe with scale factor a and homogeneous scalar field ϕ,

$$\frac{1}{2}\left(\frac{G\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2} - G^{-1}a + G^{-1}\frac{\Lambda a^3}{3} + m^2a^3\phi^2\right)\psi(a,\phi) = 0\ .$$

In loop quantum cosmology, the Wheeler–DeWitt equation is replaced by a difference equation [29]. Important issues include the possibility of singularity avoidance, the semiclassical limit including decoherence, the justification of an inflationary phase in the early Universe, the possibility of observational confirmation, and the origin of the arrow of time.

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Local Covariance and Background Independence

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Abstract. One of the many conceptual difficulties in the development of quantum gravity is the role of a background geometry for the structure of quantum field theory. To some extent the problem can be solved by the principle of local covariance. The principle of local covariance was originally imposed in order to restrict the renormalization freedom for quantum field theories on generic spacetimes. It turned out that it can also be used to implement the request of background independence. Locally covariant fields then arise as background-independent entities.

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1. Introduction

The formulation of a theory of quantum gravity is one of the most important unsolved problems in physics. It faces not only technical but, above all, conceptual problems. The main one arises from the fact that, in quantum physics, space and time are a priori structures which enter the definition of the theory as well as its interpretation in a crucial way. On the other hand, in general relativity, spacetime is a dynamical object, determined by classical observables. To solve this apparent discrepancy, radical new approaches were developed. Among these the best-known are string theory and loop quantum gravity. Up to now all these approaches meet the same problem: It is extremely difficult to establish the connection to actual physics.

Instead of following the standard approaches to quantum gravity we propose a more conservative one. We concentrate on the situation when the influence of the gravitational field is weak. This idealization is justified in a large scope of physical situations. Under this assumption one can approach the problem of quantum gravity from the field-theoretic side. In the first step we consider spacetime to be a given Lorentzian manifold, on which quantum

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fields live. In the second step gravitation is quantized around a given background. This is where the technical problems start. The resulting theory is nonrenormalizable, in the sense that infinitely many counterterms arise in the process of renormalization. Furthermore, the causal structure of the theory is determined by the background metric. Before discussing these difficulties we want to point out that also the first step is by no means trivial. Namely, the standard formalism of quantum field theory is based on the symmetries of Minkowski space. Its generalization even to the most symmetric spacetimes (de Sitter, anti-de Sitter) poses problems. There is no vacuum, no particles, no S-matrix, etc. A solution to these difficulties is provided by concepts of algebraic quantum field theory and methods from microlocal analysis.

One starts with generalizing the Haag-Kastler axioms to generic spacetimes. We consider algebras $\mathfrak{A}(\mathcal{O})$ of observables which can be measured within the spacetime region $\mathcal{O}$, satisfying the axioms of isotony, locality (commutativity at spacelike distances) and covariance. Stability is formulated as the existence of a vacuum state (spectrum condition). The existence of a dynamical law (field equation) is understood as fulfilling the timeslice axiom (primitive causality) which says that the algebra of a timeslice is already the algebra of the full spacetime. This algebraic framework, when applied to generic Lorentzian manifolds, still meets a difficulty. The causal structure is well defined, but the absence of nontrivial symmetries raises the question: What is the meaning of repeating an experiment? This is a crucial point if one wants to keep the probability interpretation of quantum theory. A related issue is the need of a generally covariant version of the spectrum condition. These problems can be solved within *locally covariant quantum field theory*, a new framework for QFT on generic spacetimes proposed in [8].

2. Locally covariant quantum field theory

The framework of locally covariant quantum field theory was developed in [8, 17, 18]. The idea is to construct the theory simultaneously on all spacetimes (of a given class) in a coherent way. Let $\mathcal{M}$ be a globally hyperbolic, oriented, time-oriented Lorentzian 4d spacetime. Global hyperbolicity means that $\mathcal{M}$ is diffeomorphic to $\mathbb{R} \times \Sigma$, where Σ is a Cauchy surface of $\mathcal{M}$. Between spacetimes one considers a class of admissible embeddings. An embedding $\chi : \mathcal{N} \to \mathcal{M}$ is called admissible if it is isometric, time-orientation and orientation preserving, and causally convex in the following sense: If γ is a causal curve in $\mathcal{M}$ with endpoints $p, q \in \chi(\mathcal{N})$, then $\gamma = \chi \circ \gamma'$ with a causal curve γ' in $\mathcal{N}$. A locally covariant QFT is defined by assigning to spacetimes $\mathcal{M}$ corresponding unital C^*-algebras $\mathfrak{A}(\mathcal{M})$. This assignment has to fulfill a set of axioms, which generalize the Haag-Kastler axioms:

1. $\mathcal{M} \mapsto \mathfrak{A}(\mathcal{M})$ unital C^*-algebra (*local observables*).
2. If $\chi : \mathcal{N} \to \mathcal{M}$ is an admissible embedding, then $\alpha_\chi : \mathfrak{A}(\mathcal{N}) \to \mathfrak{A}(\mathcal{M})$ is a unit-preserving C^*-homomorphism (*subsystems*).