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Editors

Topics in Numerical Methods for Finance

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Topics in Numerical Methods for Finance

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Preface

The papers in this volume were presented at the Numerical Methods for Finance Conference 2011, which was held at the University of Limerick, Ireland. All of the papers, with the exception of those of the keynote speakers, have been subjected to a rigorous refereeing procedure by the Editorial Committee. There is one additional paper, which was presented and accepted for publication in, but was accidentally omitted from, the published proceedings of the 2006 conference.

The aim of the conference series Numerical Methods for Finance is to attract leading international researchers from both academia and industry to discuss new research advances in, and applications of, numerical methods relevant to the solution of real problems in finance. This is a topic of practical importance because many of the mathematical models in quantitative finance cannot be treated analytically, and therefore must be solved numerically. Frequently this requires intensive computation on large grids of computers. In some respects, the development of numerical methods has kept pace with the development of computing hardware; however, many complex and high-dimensional problems are beyond the scope of even the most powerful contemporary computer clusters. Therefore, new numerical algorithms are required, which are fast, accurate and efficient for such problems. A wide range of topics and applications are presented in this volume. These offer both academic and practitioner appeal, reflecting the broad scope of the conference.

The 2011 conference was held under the joint auspices of the Institute for Numerical Computation and Analysis, Dublin, and the Kemmy Business School, University of Limerick. It is a pleasure to thank all members of the various committees who helped with the onerous burdens placed on them by the local organisers. The vital and generous support of the sponsors is also acknowledged with much gratitude. The dedicated work of all reviewers in the pre-conference review process and the post-conference proceedings review process is greatly appreciated. Finally, it was the participants who made this conference a lively, friendly and technically stimulating event. Particular thanks are extended to the

keynote speakers who encouraged and facilitated the fascinating discussions and debates that emerged. It is to be hoped that participants will return to future conferences in the series.

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About the Editors

Mark Cummins is a Lecturer in Finance at the Dublin City University Business School. He holds a PhD in Quantitative Finance, with specialism in the application of integral transforms and the fast Fourier transform (FFT) for derivatives valuation and risk management. Mark has previous industry experience working as a Quantitative Analyst within the Global Risk function for BP Oil International Ltd., London. Mark has a keen interest in a broad range of energy modelling, derivatives, risk management and trading topics. He also has a growing interest in the area of sustainable energy finance, with particular focus on the carbon markets. Linked to Mark's industry experience, he holds a further interest in the area of model risk and model validation.

Finbarr Murphy is a Lecturer in Quantitative Finance at the University of Limerick, Ireland. Finbarr's key teaching and research interests lie in the field of credit risk and derivatives and more recently, in carbon finance. His research is focused on the application of generalised Lévy Processes and their application in the pricing and risk management of derivative products. Finbarr is also interested in the application of econometric techniques in finance. Prior to taking up his position in UL, Finbarr was a Vice President of Convertible Bond Trading with Merrill Lynch London.

John J.H. Miller is Director of INCA, the Institute for Numerical Computation and Analysis, in Dublin, Ireland. He is also a Fellow Emeritus of Trinity College, Dublin, where he was a member of the Mathematics Department. He received his Sc.D. from the University of Dublin and his Ph.D. in numerical analysis from the Massachusetts Institute of Technology. He completed his undergraduate degrees at Trinity College Dublin.

Contents

On Weak Predictor–Corrector Schemes for Jump-Diffusion Processes in Finance

Nicola Bruti-Liberati[†] **and Eckhard Platen**

Abstract Event-driven uncertainties such as corporate defaults, operational failures, or central bank announcements are important elements in the modeling of financial quantities. Therefore, stochastic differential equations (SDEs) of jump-diffusion type are often used in finance. We consider in this paper weak discrete time approximations of jump-diffusion SDEs which are appropriate for problems such as derivative pricing and the evaluation of risk measures. We present regular and jump-adapted predictor–corrector schemes with first and second order of weak convergence. The regular schemes are constructed on regular time discretizations that do not include jump times, while the jump-adapted schemes are based on time discretizations that include all jump times. A numerical analysis of the accuracy of these schemes when applied to the jump-diffusion Merton model is provided.

1 Introduction

Several empirical studies indicate that the dynamics of financial quantities exhibit jumps, see [2, 7, 16, 17]. Announcements by central banks, for instance, create jumps in the evolution of interest rates. Moreover, events such as corporate defaults and operational failures have a strong impact on financial quantities. These events cannot be properly modeled by pure diffusion processes. Therefore, several financial models are specified in terms of jump diffusions via their corresponding stochastic differential equations (SDEs), see [3, 8, 10, 20, 23].

[†]Died tragically in a traffic accident on his way to work.

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The class of jump-diffusion SDEs that admits explicit solutions is rather limited. Therefore, it is important to develop discrete time approximations. An important application of these methods arises in the pricing and hedging of interest rate derivatives under the LIBOR market model. Since the arbitrage-free dynamics of the LIBOR rates are specified by nonlinear multidimensional SDEs, Monte Carlo simulation with discrete time approximations is the typical technique used for pricing and hedging. Recently, LIBOR market models with jumps have appeared in the literature, see [10, 28]. Here, efficient schemes for SDEs with jumps are needed.

Discrete time approximations of SDEs can be divided into the classes of strong and weak schemes. In the current paper, we study weak schemes which provide an approximation of the probability measure and are suitable for problems such as derivative pricing, the evaluation of moments, risk measures, and expected utilities. Strong schemes, instead, provide pathwise approximations which are appropriate for scenario simulation, filtering, and hedge simulation, see [19, 27].

A discrete time approximation Y^{Δ} converges weakly with order β to X at time T, if for each $g \in \mathscr{C}_P^{2(\beta+1)}(\mathbb{R}^d,\mathbb{R})$ there exist a positive constant C, independent of Δ, and a positive and finite number $\Delta_0 > 0$, such that

$$\varepsilon_w(\Delta) := |E(g(X_T)) - E(g(Y_T^{\Delta}))| \leq C\Delta^{\beta}, \tag{1}$$

for each $\Delta \in (0,\Delta_0)$. Here, we denote by $\mathscr{C}_P^{2(\beta+1)}(\mathbb{R}^d,\mathbb{R})$ the space of $2(\beta+1)$ continuously differentiable functions which, together with their partial derivatives of order up to $2(\beta+1)$, have polynomial growth. This means that for any $g \in \mathscr{C}_P^{2(\beta+1)}(\mathbb{R}^d,\mathbb{R})$ there exist constants $K > 0$ and $r \in \{1,2,\ldots\}$, depending on g, such that

$$|\partial_y^j g(y)| \leq K(1+|y|^{2r}),$$

for all $y \in \mathbb{R}^d$ and any partial derivative $\partial_y^j g(y)$ of order $j \leq 2(\beta+1)$.

In the case of pure diffusion SDEs, there is a substantial body of research on discrete time approximations, see [19]. The literature on weak approximations of jump-diffusion SDEs, instead, is rather limited, see [11–13, 21, 22, 24]. In this paper, we propose several new weak predictor–corrector schemes for jump-diffusion SDEs with first and second order of weak convergence.

For pure diffusion SDEs arising in applications to LIBOR market models, specific weak predictor–corrector schemes have been proposed and analyzed in [15, 18]. These authors show that for the numerical approximation of the nonlinear dynamics of discrete forward rates, predictor–corrector schemes outperform the simpler Euler scheme and allow the use of a single time step within reasonable accuracy. The weak predictor–corrector schemes proposed in the current paper can be applied to pricing and hedging of complex interest rate derivatives under LIBOR market models with jumps.

The paper is organized as follows. Sect. 2 introduces the class of jump-diffusion SDEs under consideration. In Sect. 3, we propose several weak predictor–corrector schemes for SDEs with jumps. These are divided into regular predictor–corrector schemes and jump-adapted predictor–corrector schemes. Finally, we present in Sect. 4 a numerical study of these schemes applied to the jump-diffusion Merton model.

2 Model Dynamics

The continuous uncertainty is modeled with an $\underline{\mathscr{A}}$-adapted m-dimensional standard Wiener process denoted by $W = \{W_t = (W_t^1, \dots, W_t^m)^\top, t \in [0,T]\}$, while the event-driven uncertainty is represented by an $\underline{\mathscr{A}}$-adapted r-dimensional compound Poisson process denoted by $J = \{J_t = (J_t^1, \dots, J_t^r)^\top, t \in [0,T]\}$. Each component J_t^k, for $k \in \{1,2,\dots,r\}$, of the r-dimensional compound Poisson process $J = \{J_t = (J_t^1, \dots, J_t^r)^\top, t \in [0,T]\}$ is defined by

$$J_t^k = \sum_{i=1}^{N_t^k} \xi_i^k,$$

where $N^1, \dots, N^r$ are r independent standard Poisson processes with constant intensities $\lambda^1, \dots, \lambda^r$, respectively. Let us note that each component of the compound Poisson process J^k generates a sequence of pairs $\{(\tau_i^k, \xi_i^k), i \in \{1,2,\dots, N_T^k\}\}$ of jump times and marks. We will denote with $F^k(\cdot)$ the distribution function of the marks ξ_i^k, for $i \in \{1,2,\dots,N_T^k\}$, generated by the kth Poisson process N^k.

We consider the dynamics of the underlying d-dimensional factors specified with the jump-diffusion SDE

$$\mathrm{d}X_t = a(t,X_t)\mathrm{d}t + b(t,X_t)\mathrm{d}W_t + c(t,X_{t-})\,\mathrm{d}J_t, \tag{2}$$

for $t \in [0,T]$, with $X_0 \in \mathbb{R}^d$. Here $a(t,x)$ is a d-dimensional vector of real-valued functions on $[0,T] \times \mathbb{R}^d$, while $b(t,x)$ and $c(t,x)$ are a $d \times m$-matrix of real-valued functions on $[0,T] \times \mathbb{R}^d$ and a $d \times r$-matrix of real-valued functions on $[0,T] \times \mathbb{R}^d$, respectively. Moreover, we denote by $Z_{t-} = \lim_{s \Uparrow t} Z_s$ the almost sure left-hand limit of $Z = \{Z_s, s \in [0,T]\}$ at time t. Let us note that in the following we adopt a superscript to denote vector components, which means, for instance, $a = (a^1, \dots, a^d)^\top$. Moreover, we write b^i and c^i to denote the ith column of matrixes b and c, respectively.

We assume that the coefficient functions a, b, and c satisfy the usual linear growth and Lipschitz conditions sufficient for the existence and uniqueness of a strong solution of Eq. (2), see [25]. Moreover, when we will indicate the orders of weak convergence of the approximations to be presented in Sect. 3 we will assume that smoothness and integrability conditions similar to those required in [19] for pure diffusion SDEs are satisfied. The specific conditions along with a proof of the convergence theorem will be given in forthcoming work.

If we choose multiplicative coefficients in the one-dimensional case with one Wiener and one Poisson process, $d = m = r = 1$, then we obtain the SDE

$$\mathrm{d}X_t = X_{t-}\left(\mu \mathrm{d}t + \sigma \mathrm{d}W_t + \mathrm{d}J_t\right), \tag{3}$$

which describes the *jump-diffusion Merton model*, see [23]. For this linear SDE, we have the explicit solution

$$X_t = X_0\, \mathrm{e}^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \prod_{i=1}^{N_t} (1 + \xi_i), \tag{4}$$

which we will use in Sect. 4 for a numerical study. In [23] $(1+\xi_i) = \mathrm{e}^{\zeta_i}$ is the ith outcome of a log-normal random variable with $\zeta_i \sim \mathcal{N}(\rho, \varsigma)$. If instead $(1+\xi_i)$ is drawn from a log-Laplace random variable we recover the *Kou model*, see [20]. Moreover, a simple degenerate case arises when $(1+\xi_i)$ is a positive constant.

Other important examples of jump-diffusion dynamics of the form Eq. (2) arise in LIBOR market models. [28], for instance, consider a LIBOR market model with jumps for pricing short-term interest rate derivatives. Given a set of equidistant tenor dates $T_1, \ldots, T_{d+1}$, with $T_{i+1} - T_i = \delta$ for $i \in \{1, \ldots, d\}$, the components of the vector $X_t = (X_t^1, \ldots, X_t^d)^\top$ represent discrete forward rates at time t maturing at tenor dates $T_1, \ldots, T_d$, respectively. Moreover, they consider one driving Wiener process, $m = 1$, and two driving Poisson processes, $r = 2$. The diffusion coefficient is specified as $b(t,x) = \sigma x$, with σ a d-dimensional vector of positive numbers, and the jump coefficient $c(t,x) = \beta x$, where β is a $d \times 2$-matrix with $\beta^{i,1} > 0$ and $\beta^{i,2} < 0$, for $i \in \{1, \ldots, d\}$. In this way the first jump process generates upward jumps, while the second jump process creates downward jumps. Moreover, the marks are set to $\xi_i = 1$ so that the two driving jump processes are standard Poisson processes. A no-arbitrage restriction on the evolution of forward rates under the T_{d+1}-forward measure, see [3] and [10], imposes a particular form on the nonlinear drift coefficient $a(t,x)$ whose ith component is given by

$$a^i(t,x) = -\Bigg\{ \sum_{j=i+1}^{d} \frac{\delta x^j}{1+\delta x^j} \sigma^j + \lambda^1 \prod_{j=i+1}^{d} \left(1 + \beta^{j,1} \frac{\delta x^j}{1+\delta x^j}\right) + \lambda^2 \prod_{j=i+1}^{d} \left(1 + \beta^{j,2} \frac{\delta x^j}{1+\delta x^j}\right) \Bigg\}. \tag{5}$$

A complex nonlinear drift coefficient, as that in Eq. (5), is a typical feature of LIBOR market models. Therefore, it makes the application of numerical techniques essential in the pricing of complex interest rate derivatives.

To recover some empirical features observed in the market, it is sometimes important to consider a jump behavior more general than that driving the SDE, Eq. (2). By considering jump-diffusion SDEs driven by a Poisson random measure it is possible to introduce, for instance, state-dependent intensities. The numerical schemes to be presented can be naturally extended to the case with Poisson random measures.

3 Weak Predictor–Corrector Schemes

In this section, we present several discrete time weak approximations of the jump-diffusion SDE, Eq. (2). First, we consider regular schemes based on regular time discretizations which do not include jump times of the Poisson processes. Then we present jump-adapted schemes constructed on time discretizations which include all jump times.

3.1 Regular Weak Predictor–Corrector Schemes

We consider an equidistant time discretization $0 = t_0 < t_1 < \cdots < t_{\bar{n}} = T$, with $t_n = n\Delta$ and step size $\Delta = \frac{T}{\bar{n}}$, for $n \in \{0, 1, \ldots, \bar{n}\}$ and $\bar{n} \in \{1, 2, \ldots\}$. We denote a corresponding discrete time approximation of the solution X of the SDE, Eq. (2), by $Y^\Delta = \{Y_n^\Delta,\ n \in \{0, 1, \ldots, \bar{n}\}\}$.

Before introducing advanced predictor–corrector schemes, we present the *Euler scheme* which is given by

$$Y_{n+1} = Y_n + a\Delta + \sum_{j=1}^{m} b^j \Delta W_n^j + \sum_{k=1}^{r} c^k \hat{\xi}_n^k \Delta p_n^k, \tag{6}$$

for $n \in \{0, 1, \ldots, \bar{n}-1\}$, with initial value $Y_0 = X_0$. For ease of notation, we omit here and in the following the dependence on time and state variables in the coefficients of the scheme, this means we simply write a for $a(t_n, Y_n)$, etc.

In Eq. (6) we denote by $\Delta W_n^j = W_{t_{n+1}}^j - W_{t_n}^j \sim \mathcal{N}(0, \Delta)$ the nth increment of the jth Wiener process W^j and by $\Delta p_n^k = N_{t_{n+1}}^k - N_{t_n}^k \sim \mathrm{Poiss}(\lambda^k \Delta)$ the nth increment of the kth Poisson process N^k with intensity λ^k. Moreover, $\hat{\xi}_n^k$ is the nth independent outcome of a random variable with given probability distribution function $F^k(\cdot)$. The Euler scheme achieves, in general, weak order of convergence $\beta = 1$.

It is possible to replace the Gaussian and Poisson random variables ΔW_n^j and Δp_n^k with simpler multipoint distributed random variables that satisfy certain moment-matching conditions. For instance, if we use in Eq. (6) the two-point distributed random variables $\Delta \widehat{W}_n^j$ and $\Delta \hat{p}_n^k$, where

$$P(\Delta \widehat{W}_n^j = \pm\sqrt{\Delta}) = \frac{1}{2}, \tag{7}$$

for $j \in \{1, \ldots, m\}$, and

$$P\left(\Delta \hat{p}_n^k = \frac{1}{2}(1 + 2\lambda^k \Delta \pm \sqrt{1 + 4\lambda^k \Delta})\right) = \frac{1 + 4\lambda^k \Delta \mp \sqrt{1 + 4\lambda^k \Delta}}{2\,(1 + 4\lambda^k \Delta)}, \tag{8}$$

for $k \in \{1, \ldots, r\}$, then we obtain the *simplified Euler scheme* which still achieves weak order of convergence $\beta = 1$. Let us note that this scheme can be implemented in a highly efficient manner by resorting to random bit generators and hardware accelerators, as shown for pure diffusion SDEs in [5, 6].

As indicated in [14] for pure diffusion SDEs and in [12, 13] for jump-diffusion SDEs, explicit schemes have narrower regions of numerical stability than corresponding implicit schemes. For this reason, implicit schemes for diffusion and jump-diffusion SDEs have been proposed. Despite their better numerical stability properties, implicit schemes carry, in general, an additional computational burden since they usually require the solution of an algebraic equation at each time step. Therefore, in choosing between an explicit and an implicit scheme one faces a trade-off between computational efficiency and numerical stability.

Predictor–corrector schemes are designed to retain the numerical stability properties of similar implicit schemes, while avoiding the additional computational effort required for solving an algebraic equation in each time step. This is achieved with the following procedure implemented at each time step: at first, an explicit scheme is generated, the so-called predictor, and afterward a de facto implicit scheme is used as corrector. The corrector is made explicit by using the predicted value $\bar{Y}_{n+1}$, instead of Y_{n+1}. The orders of weak convergence of the predictor–corrector schemes to be presented can be obtained by applying the Wagner–Platen expansion for jump-diffusion SDEs, see [26]. We refer to [4, 27] for the weak convergence of explicit and implicit approximations for SDEs with jumps.

The *weak order one predictor–corrector scheme* has corrector

$$Y_{n+1} = Y_n + \frac{1}{2}\left\{a(t_{n+1}, \bar{Y}_{n+1}) + a\right\}\Delta + \sum_{j=1}^{m} b^j \Delta W_n^j + \sum_{k=1}^{r} c^k\, \hat{\xi}_n^k\, \Delta p_n^k, \tag{9}$$

and predictor

$$\bar{Y}_{n+1} = Y_n + a\Delta + \sum_{j=1}^{m} b^j \Delta W_n^j + \sum_{k=1}^{r} c^k, \quad \hat{\xi}_n^k \Delta p_n^k. \tag{10}$$

The predictor–corrector scheme, Eqs. (9)–(10), achieves first order of weak convergence. Also in this case, we can use the two-point distributed random variables Eqs. (7) and (8) without affecting the order of weak convergence of the scheme. Let us note that the difference $Z_{n+1} := \bar{Y}_{n+1} - Y_{n+1}$ between the predicted and the corrected value provides an indication of the local error. This can be used to implement more advanced schemes with step-size control based on Z_{n+1}.

A more general *family of weak order one predictor–corrector schemes* is given by the corrector

$$\begin{aligned} Y_{n+1} = {} & Y_n + \left\{\theta\, \bar{a}(t_{n+1}, \bar{Y}_{n+1}) + (1-\theta)\, \bar{a}\right\}\Delta \\ & + \sum_{j=1}^{m} \left\{\eta\, b^j(t_{n+1}, \bar{Y}_{n+1}) + (1-\eta)\, b^j\right\} \Delta W_n^j + \sum_{k=1}^{r} c^k\, \hat{\xi}_n^k\, \Delta p_n^k, \end{aligned} \tag{11}$$

for $\theta, \eta \in [0,1]$, where

$$\bar{a} = a - \eta \sum_{j=1}^{m} \sum_{i=1}^{d} b^{i,j} \frac{\partial b^j}{\partial x^i}, \tag{12}$$

and the predictor is as in Eq. (10). Here, one can tune the degree of implicitness in the drift coefficient and in the diffusion coefficient by changing the parameters $\theta, \eta \in [0,1]$, respectively. Note that when the degree of implicitness η is different from zero, it is important to use bounded random variables as $\Delta\widehat{W}_n^j$ and $\Delta\hat{p}_n^k$ in an implicit scheme. These prevent the effect of possible divisions by zero in the algorithm, see [19]. For a predictor–corrector method, this can be computationally advantageous, but it is no longer required. One can still use the Gaussian and Poisson random variables, ΔW_n^j and Δp_n^k, in the above scheme as in Eq. (11).

By using the Wagner–Platen expansion for jump-diffusion SDEs, it is possible to derive higher-order regular weak predictor–corrector schemes. However, these schemes are quite complex as they involve the generation of multiple stochastic integrals with respect to time, Wiener processes, and Poisson processes.

3.2 Jump-Adapted Weak Predictor–Corrector Schemes

As introduced in [26], let us consider a *jump-adapted time discretization* $0 = t_0 < t_1 < \cdots < t_M = T$ constructed as follows. First, as in Sect. 3.1, we choose an equidistant time discretization $0 = \bar{t}_0 < \bar{t}_1 < \cdots < \bar{t}_{\bar{n}} = T$, with $\bar{t}_n = n\Delta$, for $n \in \{1, \ldots, \bar{n}\}$, and step size $\Delta = \frac{T}{\bar{n}}$. Then we simulate all jump times τ_i^k, for $i \in \{1, 2, \ldots, N_T^k\}$ and $k \in \{1, \ldots, r\}$, generated by the r Poisson processes, and superimpose these on the equidistant time discretization. The resulting jump-adapted time discretization includes all jump times τ_i^k of the r Poisson processes and all equidistant time points $\bar{t}_1, \ldots, \bar{t}_n$. Its maximum step size is then guaranteed to be not greater than $\Delta = \frac{T}{\bar{n}}$. Note that the number $M+1$ of points in the jump-adapted time discretization is random and, thus, changes in each simulation. It equals the total number of jumps τ_i^k of the r Poisson processes plus $\bar{n}+1$. Therefore, the average number of grid points and, thus, of operations of jump-adapted schemes is for large intensity almost proportional to the total intensity $\bar{\lambda} = \sum_{k=1}^{r} \lambda^k$, which is defined as the sum of the intensities of the r Poisson processes.

From now on for convenience, we use the notation $Y_{t_n} = Y_n$ and denote by $Y_{t_{n+1}-} = \lim_{s \Uparrow t_{n+1}} Y_s$ the almost sure left-hand limit of Y at time t_{n+1}.

Within a jump-adapted time discretization, by construction jumps arise only at discretization times and we can separate the diffusion part of the dynamics from the jump part. Therefore, the *jump-adapted Euler scheme* is given by

$$Y_{t_{n+1}-} = Y_{t_n} + a\Delta_{t_n} + \sum_{j=1}^{m} b^j \Delta W_{t_n}^i, \tag{13}$$

and

$$Y_{t_{n+1}} = Y_{t_{n+1}-} + \sum_{k=1}^{r} c^k(t_{n+1}-, Y_{t_{n+1}-})\Delta J^k_{t_{n+1}}, \qquad (14)$$

for $n \in \{0, \ldots, M-1\}$, where $\Delta_{t_n} = t_{n+1} - t_n$ and $\Delta W^j_{t_n} = W^j_{t_{n+1}} - W^j_{t_n} \sim \mathcal{N}(0, \Delta_{t_n})$. Here, $\Delta J^k_{t_{n+1}}$ equals $\xi^k_{N^k_{t_{n+1}}}$ if t_{n+1} is a jump time of the kth Poisson process or zero otherwise. The solution X follows a diffusion process between discretization points and is approximated by Eq. (13). If we encounter a jump time as discretization time, then the jump impact is simulated by Eq. (14). The jump-adapted Euler scheme has first order of weak convergence. By replacing the Gaussian random variable $\Delta W^j_{t_n}$ in Eq. (13) with the two-point random variable

$$P(\Delta \widehat{W}^j_{t_n} = \pm\sqrt{\Delta_{t_n}}) = \frac{1}{2}, \qquad (15)$$

for $j \in \{1, \ldots, m\}$, we obtain the *jump-adapted simplified Euler scheme* which still achieves first order of weak convergence.

The *jump-adapted weak order one predictor–corrector scheme* is given by the corrector

$$Y_{t_{n+1}-} = Y_{t_n} + \frac{1}{2}\left\{a(t_{n+1}-, \bar{Y}_{t_{n+1}-}) + a\right\}\Delta + \sum_{j=1}^{m} b^j \Delta W^j_{t_n}, \qquad (16)$$

the predictor

$$\bar{Y}_{t_{n+1}-} = Y_{t_n} + a\Delta_{t_n} + \sum_{j=1}^{m} b^j \Delta W^j_{t_n}, \qquad (17)$$

and Eq. (14). This scheme achieves the same first order of weak convergence of the jump-adapted Euler scheme. Thanks to the quasi-implicitness in the drift it has, in general, better numerical stability properties. Also in this case, it is possible to replace the Gaussian random variables in Eqs. (16) and (17) with the two-point random variables in Eq. (15).

A more general *family of jump-adapted weak order one predictor–corrector schemes* is given by the corrector

$$\begin{aligned} Y_{t_{n+1}-} = {} & Y_{t_n} + \left\{\theta \bar{a}(t_{n+1}-, \bar{Y}_{t_{n+1}-}) + (1-\theta)\bar{a}\right\}\Delta \\ & + \sum_{j=1}^{m} \left\{\eta\, b^j(t_{n+1}-, \bar{Y}_{t_{n+1}-}) + (1-\eta)\, b^j\right\}\Delta W^j_n, \end{aligned} \qquad (18)$$

for $\theta, \eta \in [0,1]$. Here $\bar{a}$ is defined as in Eq. (12) and the predictor as in Eq. (17) again together with a relation as in Eq. (14). This scheme achieves in general first order of weak convergence. Also in this case one can use the two-point random variables as in Eq. (15).

Within the class of jump-adapted schemes, we can derive higher-order weak predictor–corrector schemes which do not involve multiple stochastic integrals with respect to the Poisson processes. By using a second order weak implicit scheme as corrector and a second order weak explicit scheme as predictor, we obtain the *jump-adapted weak order two predictor–corrector scheme*. It is given by the corrector

$$Y_{t_{n+1}-} = Y_{t_n} + \frac{1}{2}\Big\{a(t_{n+1}-, \bar{Y}_{t_{n+1}-}) + a\Big\}\Delta_{t_n} + \Psi_{t_n}, \tag{19}$$

with

$$\Psi_{t_n} = \sum_{j=1}^{m}\left\{b^j + \frac{1}{2}L^0 b^j \Delta_{t_n}\right\}\Delta W_{t_n}^j + \frac{1}{2}\sum_{j_1,j_2=1}^{m} L^{j_1} b^{j_2}\left(\Delta W_{t_n}^{j_1}\Delta W_{t_n}^{j_2} + V_{t_n}^{j_1,j_2}\right), \tag{20}$$

the predictor

$$\bar{Y}_{t_{n+1}-} = Y_{t_n} + a\Delta_{t_n} + \Psi_{t_n} + \frac{1}{2}L^0 a\,(\Delta_{t_n})^2 + \frac{1}{2}\sum_{j=1}^{m} L^j a \Delta W_{t_n}^j \Delta_{t_n}, \tag{21}$$

and a relation as in Eq. (14). The differential operator L^0 is defined by

$$L^0 := \frac{\partial}{\partial t} + \sum_{i=1}^{d} a^i(t,x)\frac{\partial}{\partial x^i} + \frac{1}{2}\sum_{i,k=1}^{d}\sum_{j=1}^{m} b^{i,j}(t,x)b^{k,j}(t,x)\frac{\partial^2}{\partial x^i \partial x^j}, \tag{22}$$

and the operator L^j by

$$L^j := \sum_{i=1}^{d} b^{i,j}(t,x)\frac{\partial}{\partial x^i}, \tag{23}$$

for $j \in \{1,\ldots,m\}$. The random variables $V_{t_n}^{j_1,j_2}$ are two-point distributed with

$$P(V_{t_n}^{j_1,j_2} = \pm\sqrt{\Delta_{t_n}}) = \frac{1}{2}, \tag{24}$$

for $j_2 \in \{1,\ldots,j_1-1\}$, where

$$V_{t_n}^{j_1,j_1} = -\Delta_{t_n}, \tag{25}$$

and

$$V_{t_n}^{j_1,j_2} = -V_{t_n}^{j_2,j_1} \tag{26}$$

for $j_2 \in \{j_1+1,\ldots,m\}$ and $j_1 \in \{1,\ldots,m\}$. The Gaussian random variable $\Delta W_{t_n}^k$ can be replaced by the three-point random variable $\Delta\widetilde{W}_{t_n}^k$ defined by

$$P(\Delta\widetilde{W}_{t_n}^k = \pm\sqrt{3\Delta_{t_n}}) = \frac{1}{6}, \qquad P(\Delta\widetilde{W}_{t_n}^k = 0) = \frac{2}{3}, \tag{27}$$

for $k \in \{1,\ldots,m\}$.