Transportation and Traffic Theory 2009: Golden Jubilee

William H. K. Lam · S. C. Wong · Hong K. Lo
Editors

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Papers selected for presentation at ISTTT18,
a peer reviewed series since 1959

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Preface

It is our great privilege and honor to present the proceedings of the *18th International Symposium on Transportation and Traffic Theory* (ISTTT), held at The Hong Kong Polytechnic University in Hong Kong, China on 16-18 July 2009. The 18th ISTTT is jointly organized by the Hong Kong Society for Transportation Studies and Department of Civil and Structural Engineering of The Hong Kong Polytechnic University.

The ISTTT series is the main gathering for the world's transportation and traffic theorists, and those who are interested in contributing to or gaining a deep understanding of traffic and transportation phenomena in order to better plan, design and manage the transportation system. Although it embraces a wide range of topics, from traffic flow theories and demand modeling to road safety and logistics and supply chain modeling, the ISTTT is hallmarked by its intellectual innovation, research and development excellence in the treatment of real-world transportation and traffic problems. The ISTTT prides itself in the extremely high quality of its proceedings. Previous ISTTT conferences were held in Warren, Michigan (1959), London (1963), New York (1965), Karlsruhe (1968), Berkeley, California (1971), Sydney (1974), Kyoto (1977), Toronto (1981), Delft (1984), Cambridge, Massachusetts (1987), Yokohama (1990), Berkeley, California (1993), Lyon (1996), Jerusalem (1999), Adelaide (2002), College Park, Maryland (2005), and London (2007).

This 18th ISTTT celebrates the 50th Anniversary of this premier conference series. The first Symposium, organized by Professor Robert Herman, was held on 7-8 December 1959. A total of 15 papers were presented in the 1st Symposium. The scope of this Symposium series has since broadened, from the *Symposium on the Theory of Traffic Flow* to the *International Symposium on Transportation and Traffic Theory*. The ISTTT has also grown in size, but is still limited to around 35 papers. The rationale is, as was since the 1st Symposium, to allow ample time for presentation and informal discussion. Indeed, this time, in celebrating the 50th Anniversary of this tradition, we have arranged roundtable discussions to further enhance the interactions among researchers, scientists, and practitioners. We hope to have an opportunity to reminisce advances made in the past, and to outline important, uncharted territories. In reviewing the outline of the 1st Symposium, we were awed by the foresights of the researchers then, addressing research topics such as traffic control, distribution of traffic on a network and that of households and workplaces, clustering tendency of vehicular traffic, modeling traffic via stochastic processes, and simulation of bottlenecks, etc. These topics appear as fresh today as they were posed 50 years ago, despite much progress having been made. Transportation and traffic theories renew themselves as technology advances, as human activities are re-organized, and as scarce resources become gradually depleted, etc, representing our best effort at the time to understand and hence manage the needs and consequences of connecting activities, goods, and people. From this vantage point, we are confident that the best years for ISTTT are yet to come in the future.

It is timely to organize the 18th ISTTT in Hong Kong, as the thrust of transportation infrastructure development has emerged strongly in Asia. Indeed, many parts of Asia are currently undertaking extensive transportation infrastructure programs. Hong Kong, for example, will initiate 10 major infrastructure projects with an investment of about HK$250 billion (roughly US$32 billion), in which 6 are transportation projects, including 4 railway and 2 highway infrastructure projects. These transportation infrastructures will bring about an economic benefit of more than HK$100 billion (roughly US$12 billion) annually, amounting to some 7% of the GDP of Hong Kong in 2006. This points to the imminent demand of high caliber transportation and traffic planners and engineers for the planning, design, management and operation of the transportation systems. The 18th ISTTT offers an excellent platform to elevate the role of transportation and traffic theories for transportation infrastructure planning and operations.

Special thanks are given to Members of the International Advisory Committee (IAC) and the local organizing committee for reviewing the extended abstracts and then full paper submissions, especially under the short review time requested of them. Our particular appreciation is extended to the referees who have contributed their considerable time and effort to the two-tier review process. With their dedicated support, each paper submission received at least three reviews, typically four to five, sometimes up to six reviews. Given the extremely high selectivity, we have tried our very best to ensure that each paper submission received a sufficient number of reviews to evaluate its merit. In reality, the tight constraint on the number of papers to be accepted for this Symposium have forced us to decline a number of very high quality submissions, which would be acceptable for publications in quality transportation journals. All in all, out of 230 extended abstract submissions, we have finally selected 35 papers to be included in this volume. We sincerely hope that this volume will serve as a vehicle to stimulate novel research initiatives in transportation and traffic theories.

As this volume was heading towards press, the news of Ryuichi Kitamura's untimely death on 19 February 2009 struck us with sadness and a profound sense of loss. A professor at Kyoto University, Ryuichi had given his unstinting support to ISTTT as a Member of the IAC. All of us who had the good fortune of having met him were often touched by his cheerfulness, kindness, and generosity. His positive attitude and warmth endured even when he was suffering from a long illness. We shall sorely miss his scholarship and friendship.

This commemorative Symposium volume is dedicated to researchers, scientists, and practitioners who have spent their career advancing the state-of-the-art in transportation and traffic theories. We celebrate their accomplishments and honor the memory of those who are not with us today.

Finally, we express our gratitude to the organizations whose financial contribution has made it possible for us to host the 18th ISTTT in Hong Kong.

William H. K. Lam, S.C. Wong and Hong K. Lo

March 2009

Remembering Ryuichi Kitamura

On February 19, 2009, many of us were saddened to hear of Ryuichi Kitamura's passing. His close friends and colleagues had known for some time of his struggle with cancer, but had drawn hope from his appearances at conferences and occasional correspondence. He taught us a lot about how to be graceful in the face of adversity, as he remained his wonderful positive self until the end, and continued to write and conduct far-reaching research with his students and collaborators. He had hoped to attend the 18th ISTTT meeting in Hong Kong, but unfortunately it was not meant to be.

Ryuichi returned to Kyoto University, his alma mater, in 1993, as a Professor of Urban Management in the Faculty of Engineering, after a distinguished 15-year career on the faculty of the Department of Civil and Environmental Engineering at the University of California at Davis, where he was instrumental in founding the Institute for Transportation Studies (ITS). He received his BS in Civil Engineering and MS in Transportation in 1972 and 1974, respectively, from Kyoto University, and a PhD in Civil Engineering from the University of Michigan at Ann Arbor in 1978.

Through his research, teaching and professional service, Ryuichi played a key role in advancing the state of the art as well as state of practice in travel demand modeling and the dynamic analysis of transportation systems through micro-simulation of household travel and activity behaviour. He realized early on that travel demand does not occur for its own sake, but is part of a broader set of activities undertaken by individuals and households in fulfilling their various needs. He became a major force in the US and internationally in promoting greater behavioural realism in transportation models, leading the way towards comprehensive activity-based models of travel demand.

Ryuichi challenged conventional wisdom and brought a fresh perspective to nearly all topics on which he worked. He was a scholar, with a probing inquisitive mind, who subtly but firmly made you look at problems from a different angle. He was a keen observer of social trends, and among the first to recognize how they might impact travel and transportation. Telecommuting, changing gender roles, increased environmental awareness and shifting preferences are examples of phenomena he sought to understand and quantify in terms of transportation implications. Ryuichi was one of few researchers who had the methodological firepower to analyze these kinds of trends rigorously, ranging from novel survey methods to advanced econometric and psychometric techniques.

His quest for models based on sound behavioural theories was promoted through several disciples who studied under him at both Davis and Kyoto, and who went on to become influential scholars and practitioners in their own right. He helped shape the field through his service as chair of the Traveller Behaviour and Values committee of the Transportation Research Board in the mid to late 1980's, a time of major advances in both research and practice. He served as President of the International Association of Travel Behaviour Research (IATBR) in 1992-94, and hosted its triennial meeting in Kyoto in 2006. In that same year, the IATBR recognized his contributions by awarding him the Lifetime Achievement Award.

Ryuichi was a member of the International Advisory Committee of the ISTTT, and had discussed with us his desire to host it in Kyoto at some future date. It is unfortunate he did not live to see this wish fulfilled. We will miss his thoughtful interventions and good humour, the depth of insight and breadth of perspective he brought to the ISTTT, and his warm, pleasant and congenial personality. I will miss a dear friend, who contributed so much to making what we do the exciting privilege it truly is.

Hani S. Mahmassani

March 2009

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Chapter 1

A Game Theoretic Approach to the Determination of Hyperpaths in Transportation Networks

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Abstract In transit assignment, the common lines problem leads to the notion of a hyperpath, which is a set of paths that when used according to the "take whichever attractive line arrives next" strategy minimises the expected travel time. Similarly, the game theoretic approach to risk-averse traffic assignment leads to the generation of a set of paths which minimises expected travel time when a pessimistic assumption is made about on-trip events. The equivalence between the hyperpath of transit assignment and the set of paths generated by a multi-agent, zero sum game is shown in this paper. In particular, game theory is used to show that the path split probabilities proposed by Spiess and Florian (1989) are optimal for the risk-averse traveller who needs to make an on-the-spot decision between alternative routes. An alternative two-agent (single demon), zero-sum game is considered. The results of the multiple- and two-agent games are compared on a small example network, showing that the single demon game can lead to denser hyperpaths.

1. Introduction

It is well known that route choice in transit networks differs from that in traffic networks. This is because passengers often face uncertainties at each boarding point along their paths. Unless a timetable is published and the trains arrive perfectly to schedule, passengers do not know how long they have to wait at boarding points, although they might know from experience how long they expect to wait.

The concept of the hyperpath, namely the set of paths any one of which may be the fastest depending on line arrival times, emanates from this problem (Spiess and Florian 1989; Nguyen and Pallottino 1988) and is associated with the concept of common lines. When arriving at a bus stop or a train station it is often the case that there are a number of attractive paths and the choice of which to take often depends on which line happens to arrive next. Under the "take whichever attractive line arrives next" rule, and assuming either that lines arrive randomly with given frequencies or that passengers arrive randomly (or both), it is possible to determine which paths (and therefore which lines) minimise the expected travel time for the trip and are therefore attractive. The lines that are attractive at a given stop or station hence constitute the common lines referred to earlier.

This paper contributes to the on-going discussion of appropriate path choice models. Nökel and Wekeck (2007) review the assumptions made in frequency-based assignment models and point out shortcomings of existing models with regards to the role of available information to passengers at platforms. Kato et al. (2007) compared the appropriateness of various econometric and deterministic as well as stochastic equilibrium assignment models by applying these to Tokyo's rail and underground data. They found surprisingly little difference in the model fit, which might be due to the high reliability of Tokyo's network. Kato et al. (2007) ignored the common lines problem, which might in any case not be appropriate in networks where passengers do not feel the need to develop strategies to counter uncertainty in waiting times.

This paper returns to the basic Spiess and Florian model but replaces the link service frequency by a link-specific penalty which may be interpreted as the maximum link delay expected by the passenger, in order to highlight the link to game theory. It will be shown that a risk-averse transit user who follows the path choice probabilities as proposed in Spiess and Florian (1989) minimises his maximum expected travel time to his destination. Therefore, the Spiess and Florian algorithm provides optimal risk-averse node split probabilities which can also be applied to road networks, as proposed by Bell (2008) in the hyperstar algorithm.

The reminder of this paper is organised as follows. Section 2 reviews the main idea of the Spiess and Florian hyperpath as well as criticism it has attracted recently in Nökel and Weckek (2007) that lead to the formulation of more complex choice models. In Section 3 the idea of constructing path sets through game theory as proposed by several authors is reviewed. In Section 4 the notation is introduced and in Section 5 it is shown that the node split probabilities in Spiess and Florian can also be derived through game theory. Section 6 then shows the equivalence of the Spiess and Florian linear programming formulation and a zero-sum, multi-agent game. Section 7 presents an alternative two-agent (single demon), zero-sum game that has been discussed in the existing literature, in particular for the hazardous material transportation problem (see Bell 2007). Section 8 illustrates the results numerically, and Section 9 concludes the paper.

2. Transit Choice Models for Different Assumptions

If passengers only have information about the service frequency, the common lines problem arises whereby, as stated in Spiess and Florian (1989), passengers board the first line to arrive among a set of attractive lines at a boarding node i, designated by A_i^+. This means that the waiting times at nodes and the node split probabilities can be calculated as in equations (1) and (2) where f_a are the link service frequencies (the links are line specific) and α is a parameter indicating the line regularity with $\alpha = 1$ describing exponential headway distribution.

$$w(A_i^+) = \frac{\alpha}{\sum_{a \in A_i^+} f_a}, \tag{1}$$

$$p_a(A_i^+) = \frac{f_a}{\sum_{a \in A_i^+} f_a}. \tag{2}$$

A non-trivial task is to find the optimal strategy, namely the set of paths which are used when expected journey time is minimised. Spiess and Florian presented the problem as a linear program where the objective is to find the strategy that minimises the sum of the expected total travel time including waiting time for flow from all origins to a specified destination. The resulting strategy defines a set of potentially optimal paths, referred to by Nguyen and Pallottino (1988) as a hyper-path.

The Spiess and Florian approach is, however, only correct if a number of assumptions about the transit service are made (see, in particular Billi et al. 2004; Gentile et al. 2005 and Nökel and Wekeck 2007). The latter paper points out three practical considerations that lead to more complex route choice models. Firstly, the service regularity might vary significantly between subsequent arrivals leading to choice sets more biased to a particular line. Equation (1) makes the strong assumption that the intervals between all services of all attractive lines serving the stop are equal on average, i.e. if two lines with the same frequency are part of the choice set, the expected waiting time is simply halved. Secondly, passengers often have more information than the service frequency alone. In many cities, countdown clocks have been installed telling passengers when to expect the next arrival of a number of lines. Thirdly, the structure of the choice set depends on the layout of the platforms or the fare system. If transit stops at the same station are not close together passengers may have to make a choice of line before the arrival of any transit line that could be in their set of attractive lines. Similarly, complex fare structures often lead to a much reduced attractive set of lines the passengers might take.

To cope with some of these assumptions, Gentile et al. (2005) developed a more general formulation that considers different regularity assumptions on the headways. Billi et al. (2004) considered the dynamically changing choice of the best line when passengers can observe elapsed waiting time on all lines and can assume that departures are regular. Nökel and Wekeck (2007) summarise these cases and look at further cases, such as the difference in choice probabilities when passengers can observe only one line (instead of all lines departing from the transit stop).

3. Path Choice and Game Theory

The idea that a split between multiple paths can minimise the maximum expected travel cost in the presence of scenario uncertainty has also been applied in other contexts. In particular, "risk-averse assignment" leads to the creation of a number of paths. In this case, a set of paths are created to reduce the exposure to potential loss or delay. To minimise the maximum travel cost, a zero-sum game is played between a rational traveller and a "demon", to emphasise that the game can be understood as a search for worst case scenarios. By employing a mixed routing strategy, the traveller can then minimise his maximum exposure to cost.

This game notion was first introduced by Bell (2000) as a two player game, noting that such a pessimistic game might be used as a benchmark to assess network reliability and to find critical links in the network. The game notion has then been applied and extended in several ways: Cassir and Bell (2000) introduce a game between one network tester and *n* heterogeneous players. Cassir et al. (2003) test the approach on a larger network and introduce a "tree spoiler" to find the reliability of particular origins or destinations. Bell (2007) applies the game approach to the transportation of hazardous materials and shows some interesting properties of the game, for example that the overall expected cost of the solution is equal to the route cost plus the maximum exposure on any link. Nagae and Akamatsu (2007) introduce entropy terms to the objective function to uniquely define link choice and link attack probabilities, leading to a logit function for the path choice as well as for the attack probabilities. This allows one to compare various degrees of risk aversion as well as degrees of sensitivity to path cost (although there are theoretical objections to using the logit model for path choice). Bell et al. (2008) focus on the logit formulation of the attack probabilities.

Szeto et al. (2007) note that this line of work so far has only considered the failure of a single link. They introduce therefore multiple spoilers in the network, each of which can independently fail one link each. The game is formulated as a non-linear complementary problem and solved by minimising the resulting gap function. Different scenarios are tested and in particular it is shown that multiple solutions exist.

The games described above have been envisaged to be primarily applicable for road traffic scenarios. The following will investigate the connection between "risk averse path sets" created through a game analogy and the Spiess and Florian hyperpath created to describe the behaviour of public transport passengers who aim to minimise their expected travel times. It will be shown that the Spiess and Florian hyperpath is equivalent to a particular type of a zero-sum game.

4. Notation

Define the following sets and variables:

A Set of all links

I Set of all nodes

H Set of links contained in the hyperpath from origin r to destination s

g_k Expected travel time on path from node i to destination s when link k is attacked

A^+_i Set of links emanating from node i

A^-_i Set of links leading into node i

λ_i Expected cost to reach destination s from node i (node potential)

w_i Exposure to maximum delay at node i

f_a Service frequency on link a

d_a Penalty on link a ($= 1 / f_a$)

q_a Probability penalty d_a occurs on link a

p_a Probability link a is used

c_a Undelayed travel time on link a

u_a Undelayed travel time on link a plus expected travel time from the downstream node of link a to destination s

A hyperpath is in the following defined as a set of paths any of which has a non-zero probability of being taken by the traveller. It follows therefore that $p_a > 0$ $\forall a \in H$. Further, it should be noted that throughout the paper congestion effects are not considered. This means that it is assumed that the service frequency f_a (and hence also d_a) as well as the link travel times c_a are assumed to be independent of p_a.

5. Risk-averse Path Choice at a Node

Spiess and Florian (1989) proposed the path split probabilities in equation (2), where the choice of a line is equivalent to the frequency of the line divided by the sum of the frequencies of all the attractive lines emanating from the same node. The following proof shows that the same path split probabilities are also following from a game theoretic formulation. This means that under two different sets of assumptions the same path split probabilities can be derived. The traveller in Spiess in Florian aims to minimise expected travel times under the assumption that the service has exponential headways. Contrary to this, the traveller considered in the game described in this paper considers a link-specific maximum delay.

Proposition 1 The risk-averse traveller, who fears a maximum delay of d_a on any one link emanating from a node should use the split probabilities as in equation (2) for all attractive links, independent of the travel time on any downstream link c_a.

Proof Following the above definition on one link maximum delay might occur which can be expressed as a game between the traveller with arc choice probabilities p_a and a demon with arc attack probabilities q_a. For the choice of n links from a node this can be written as a pay-off matrix as in Table 1. The rows indicate the links in the set of optimal paths and the columns the attack probabilities. In choosing a link, the link specific cost u_a, which consists of the arc cost c_a plus the node cost of the tailnode, j, where $a = (i, j)$, is considered.

$$u_a = c_a + \lambda_j \, . \tag{3}$$

Attacked links suffer an additional (maximum) delay of d_a. Writing this as a pay-off table means that only in the main diagonal the link specific maximum delay is added to the costs. It is supposed here that there are n links out of node i, of which k are used.

Table 1. Game pay-off matrix at a node

	q_1	q_2	…	q_k	…	q_n
p_1	$u_1 + d_1$	u_1	…	u_1	…	u_1
p_2	u_2	$u_2 + d_2$	…	u_2	…	u_2
…	…	…	…	…	…	…
p_k	u_k	u_k	…	$u_k + d_k$	…	u_k
…	…	…	…	…	…	…
p_n	u_n	u_n	…	u_n	…	$u_n + d_n$

The traveller employs a mixed strategy in order to reduce the probability of encountering a large link delay, and the demon might choose a strategy that maximizes the expected cost. Wlog assume that the traveller only includes the first k links emanating from the node in his or her choice set, i.e. $p_a > 0\ \forall a = 1,\ldots k$ and $p_a = 0\ \forall a = k+1,\ldots n$. Note that Proposition 3 in the following shows that the set of arcs used by the demon is a subset of set of arcs used by the traveller, i.e. $q_a \geq 0\ \forall a = 1,\ldots k$ and $q_a = 0\ \ \forall a = k+1,\ldots n$. The costs encountered by the traveller are $\sum_{a \in A_i^+} u_a p_a + q_a p_a d_a$ and the corresponding optimisation problem for this zero-sum game can hence be written as:

$$\mathrm{Min}_{\boldsymbol{p}}\ \mathrm{Max}_{\boldsymbol{q}} \sum_{a \in A_i^+} u_a p_a + q_a p_a d_a \, , \ \forall i \in \mathrm{I}. \tag{4}$$

The traveller's problem is hence to choose a (mixed) strategy **p** that minimizes his or her expected (or rather feared) cost of travel λ_i from origin i to destination s. This can be written as follows:

Min λ_i so that

$$\begin{array}{llllll} p_1(u_1 + d_1) & + p_2 u_2 & + \ldots & + p_k u_k & = g_1 & \leq \lambda_i \\ p_1 u_1 & + p_2(u_2 + d_2) & + \ldots & + p_k u_k & = g_2 & \leq \lambda_i \\ \ldots & + \ldots & + \ldots & + \ldots & = \ldots & \leq \lambda_i \\ p_1 u_1 & + p_2 u_2 & + \ldots & + p_k(u_k + d_k) & = g_k & \leq \lambda_i \end{array} \tag{5}$$

$$p_1 + p_2 + \ldots + p_k = 1, \tag{6}$$

$$p_i > 0\ \forall i = 1,\ldots, k. \tag{7}$$

Following the expected value principle at the saddle point the costs of all used strategies will be equal, i.e. $g_1 = g_2 = \ldots g_l = \ldots = g_k = \lambda_i$ where λ_i is the expected game value of the saddle point.

Wlog solving the above set of linear equations for p_1 leads to

$$p_a = p_1\,(d_1/d_a)\ \forall a=2,\ldots,k, \tag{8}$$

and with (6) this leads to

$$p_1 + p_1\,(d_1/d_2) + p_1\,(d_1/d_3) + \ldots + p_1\,(d_1/d_k) = 1, \tag{9}$$

Solving for p_1 leads to

$$p_1 = \frac{\frac{1}{d_1}}{\sum_{a=1,..,k}\frac{1}{d_a}}, \tag{10}$$

with $f_a = 1/d_a$ this is therefore equivalent to the path split proposed in equation (2). Q.E.D.

In particular it should be noted that the path split probabilities are independent of any delay occurrence probability, q_a, at the node as well as of any link costs c_a. Further note that by inserting equation (2) in any of the equations (5) one can derive the expected game value as in equation (11) which is also equivalent to the expected travel time in Spiess and Florian.

$$\lambda_i = (u_1 + \frac{1}{f_1})\frac{f_1}{\sum_{a=1}^{k} f_a} + u_2\frac{f_2}{\sum_{a=1}^{k} f_a} + \ldots + u_k\frac{f_k}{\sum_{a=1}^{k} f_a} = \frac{1+\sum_{a=1}^{k} f_a u_a}{\sum_{a=1}^{k} f_a}. \tag{11}$$

In the same way as for the path split probabilities, at the Nash equilibrium solution for all non-dominated attack strategies the following set of equations must hold:

Max λ_i so that

$$\begin{array}{llllll} q_1(u_1+d_1) & +\,q_2u_1 & +\ldots & +\,q_ku_1 & = g_1 & \geq \lambda_i \\ q_1u_1 & +\,q_2(u_2+d_2) & +\ldots & +\,q_ku_2 & = g_2 & \geq \lambda_i \\ \ldots & +\ldots & +\ldots & +\ldots & = \ldots & \geq \lambda_i \\ q_1u_k & +\,q_2u_k & +\ldots & +\,q_k(u_l+d_l) & = g_k & \geq \lambda_i \end{array} \tag{12}$$

$$q_1 + q_2 + \ldots + q_k = 1, \tag{13}$$

$$q_i \geq 0\ \forall i = 1,\ldots, k, \tag{14}$$

with equation (13) and applying the saddle point theorem it follows:

$$u_a(q_1 + q_2 + \ldots + q_l) + d_a q_a = u_a + d_a q_a = \lambda_i,\ \forall a = 1,\ldots l, \tag{15}$$

with λ_i as determined in equation (11) or solving equation (10) with equation (16) it follows therefore that:

$$q_a = f_a(\lambda_i - u_a) = f_a\left(\frac{1+\sum_{a'=1}^{k} f_{a'}u_{a'}}{\sum_{a'=1}^{k} f_{a'}} - u_a\right) = \frac{f_a + f_a\sum_{a'=1}^{k} f_{a'}(u_{a'} - u_a)}{\sum_{a'=1}^{k} f_{a'}}. \tag{16}$$

Hence with equation (16) the optimal attack probabilities at each node can be identified, giving a useful measure of the importance of the link to the expected travel time to the destination. The higher q_a the more the traveller fears any delay on this link. $q_a = 0$ means that even if the link is attacked the traveller has sufficient rerouting options so that a failure does not increase the expected travel time.

Proposition 2 Any arc $a = (i, j)$ with an undelayed travel time that reduces the node cost λ_i is part of the optimal hyperpath. In other words any arc that potentially reduces the node costs will be considered independent of its potential delay.
Proof For simplicity of notation, subscript i is omitted in the following from λ and λ'. Let λ' denote the node cost if arcs 1,…,k, $k+1$ are included in the traveller's strategy and λ denotes the node cost if only arcs 1,…, k are considered. With equation (9) it follows that:

$$\begin{aligned} \lambda' < \lambda &\Leftrightarrow \frac{1+\sum_{a=1}^{k} f_a u_a + f_{k+1}u_{k+1}}{\sum_{a=1}^{k} f_a + f_{k+1}} < \frac{1+\sum_{a=1}^{k} f_a u_a}{\sum_{a=1}^{k} f_a} \\ &\Leftrightarrow \sum_{a=1}^{k} f_a\left(1+\sum_{a=1}^{k} f_a u_a + f_{k+1}u_{k+1}\right) < \left(1+\sum_{a=1}^{k} f_a u_a\right)\left(\sum_{a=1}^{k} f_a + f_{k+1}\right) \\ &\Leftrightarrow f_{k+1}u_{k+1} < \frac{1+\sum_{a=1}^{k} f_a u_a}{\sum_{a=1}^{k} f_a} f_{k+1} = \lambda f_{k+1} \\ &\Leftrightarrow u_{k+1} < \lambda \end{aligned} \quad (17)$$

Hence if $u_{k+1} < \lambda$ then λ can be reduced by the inclusion of link $k+1$ in the hyperpath. Q.E.D.

Note that the following complementary Proposition 2a also follows from the proof of Proposition 2. Alternatively the proof below leads to the same conclusion.

Proposition 2a An arc $a = (i, j)$ is not included in H if it does not reduce the expected travel time. In other words if $u_a \geq u_{a'} + d_{a'} q_{a'} = \lambda_i$ for any arc $a' \in H$ with a, $a' \in A_i^+$ it follows that $p_a = 0$.
Proof Let us assume that arc 1 and arc 2 are part of the hyperpath but that $u_1 = u_2 + d_2 q_2$. The demon strategy will be to maximise the saddle point value of the game, hence to maximise the set of equations (10). It follows

$$q_1(u_2 + q_2 d_2 + d_1) + q_2(u_2 + q_2 d_2) + q_3(u_2 + q_2 d_2) + \ldots + q_n(u_2 + q_2 d_2) = g_1, \quad (18)$$
$$q_1(u_2) + q_2 u_2 + q_3 u_2 + \ldots + q_n u_2 = g_2. \quad (19)$$

with a_1, $a_2 \in H$ it follows that $g_1 = g_2$. With d_1, d_2, $u_2 > 0$ it follows further that independent of any demon strategy the expected cost of using arc 1 is always at least as high as using arc 2 and therefore $p_1 = 0$, so $a_1 \notin H$ which is a contradiction. Q.E.D.

The following proposition shows that the demon will focus the resources on any links that are part of the hyperpath.

Proposition 3 For $a, a' \in A^{+}_{i}$ and $a \notin H$ but $a' \in H$, $q_a = 0$.
Proof The demon problem is to $\text{Max}_{\mathbf{q}} \sum_{a\in A_i^+} u_a p_a + q_a p_a d_a$ for any set of arc flows **p**. From the definition of hyperpaths it follows that $p_{a'} > 0$ and that $p_a=0$. Since the traveller only considers one incident to occur at this node $\sum_{a\in A_i^+} q_a$ is limited to 1 which means that from $q_a = 0 + \delta$ with $\delta > 0$ it follows that $q_{a'} \leq 1 - \delta$. This clearly cannot be optimal as increasing $q_{a'}$ by δ and reducing q_a by δ to $q_a = 0$ leads to an increase in the objective function by $\delta d_{a'} p_{a'}$. Q.E.D.

Note that in equation (3) the cost u_a of any arc $a = (i, s)$ equals c_a. Therefore for any penultimate node s-1 the optimal node cost at the saddle point equation (11) can be uniquely determined as λ_{s-1}. This is only depending on the fixed costs d_a and c_a of the links emanating from s-1 that are part of the attractive path set. In particular note that the node costs are independent of any upstream path choice and attack probabilities once the attractive path set has been identified. Similarly note that from equation (4) it follows that:

$$\begin{aligned}\lambda_i &= \sum_{a\in A_i^+} ((c_a+\lambda_j)p_a + q_a p_a d_a) \\ &= \sum_{a\in A_i^+} (c_a+q_a d_a)p_a + \sum_{a=(i,j)\in A_i^+} p_a \lambda_j\end{aligned} . \quad (20)$$

This means that the cost from node i to the destination node s is the sum of the average cost for all links emanating from node i (1st term) plus the weighted cost from the downstream nodes j to the destinations.

This observation means that a Dijkstra-type algorithm can be applied to determine the optimal hyperpath in which each link is only visited once in increasing cost order from the destination. Whenever a new arc is found which decreases the node costs equation (9), the path split probabilities equation (8) and the attack probabilities equation (14) are updated until all arcs have been visited. The following section provides the linear programming formulation of the path choice problem with the arc split probabilities established in this section. Nguyen and Pallottino (1988) show that for this problem indeed a Dijkstra-type algorithm can find the solution.

6. Spiess and Florian (1989) Path Choice and Equivalence to Zero-sum Game Formulation

Following Spiess and Florian (1989), the optimal hyperpath from origin r to destination s under the assumption of path split probabilities as in equations (2) and (10) is identified by solving the following linear program:

$$Min_{\mathbf{p},\mathbf{w}} \sum_{a\in A} c_a p_a + \sum_{i\in I} w_i , \quad (21)$$

subject to

$$\sum_{a\in A_i^+} p_a - \sum_{a\in A_i^-} p_a = g_i \,,\ i\in I, \tag{22}$$

$$p_a d_a \le w_i \,,\ a \in A_i^+ , i \in I \,, \tag{23}$$

$$p_a \ge 0 \,,\ a \in A \,, \tag{24}$$

which may be solved by any LP solver if flow constraints equation (22) can be explicitly set in the LP solver. The objective function equation (21) is the travel time expected pessimistically. Correspondingly, the expected node delay is that which would be expected by a pessimistic passenger. If link a is the only link out of node i included in the hyperpath, then the passenger is fully exposed to maximum delay on that link and $w_i = d_a$. To mitigate this exposure, the passenger requires alternative ways out of node i.

Constraint equation (22) enforces flow conservation. Note that $g_i = 1$ if $i = r$, $g_i = -1$ if $i = s$, and $g_i = 0$ otherwise. Constraint equation (23) ensures that the usage of link a is inversely proportional to its maximum delay, if link a is used ($p_a > 0$). If link a is not part of the hyperpath and hence not used ($p_a = 0$), the traveller exposes himself to a larger w_i as equation (23) has to be fulfilled for all arcs emanating from i unless the node can be avoided altogether by the traveller. Constraint equation (23) further enforces $\mathbf{w} \ge 0$ as $\mathbf{p} \ge 0$ and $\mathbf{d} \ge 0$.

The equivalence between the above problem and a non-cooperative zero sum game can be demonstrated by the method of Lagrange multipliers. The Lagrangian function for the above problem is

$$L(\mathbf{p},\mathbf{w},\boldsymbol{\lambda},\mathbf{q}) = \sum_{a\in A} c_a p_a + \sum_{i\in I} w_i - \sum_{i\in I}\sum_{a\in A_i^+} q_a (w_i - p_a d_a) + \sum_{i\in I} \lambda_i (g_i - \sum_{a\in A_i^+} p_a + \sum_{a\in A_i^-} p_a) \,, \tag{25}$$

where $\mathbf{q} \ge 0$ and $\boldsymbol{\lambda}$ are sets of Lagrange multipliers. In Ahuja et al. (1993) the dual variable of the link choice probabilities $\mathbf{p}$ are further proven to be the node costs, or also referred to as node potential, which is in accordance with the usage of $\boldsymbol{\lambda}$ in previous Section 3. By means of the Lagrangian function, a constrained optimisation problem is transformed into an optimisation problem constrained only by non-negativity conditions. In this case, the L(**p**, **w**, **λ**, **q**) is minimised with respect to $\mathbf{p} \ge 0$ and **w** and maximised with respect to the Lagrange multipliers, subject to non-negativity conditions on **p** and **q**. The solution is given by the saddle point.

Maximisation of the third term in equation (25), $-\sum_{i\in I}\sum_{a\in A_i^+} q_a (w_i - p_a d_a)$, with respect to **q** enforces the non-negativity constraint provided $q_a \ge 0$. If the term in brackets is positive, $q_a = 0$ is the solution. On the other hand, should the term in brackets try to go negative, $q_a > 0$ but minimisation with respect to **p** restores the term in brackets to 0. At the saddle point a value for q_a is found that is large enough to enforce the constraint.

The maximisation of the fourth term in equation (25), $\sum_{i\in I} \lambda_i (g_i - \sum_{a\in A_i^+} p_a + \sum_{a\in A_i^-} p_a)$, with respect to **λ** enforces the equality constraint. If the term in brackets is positive, $\lambda_i > 0$, but minimisation with respect to **p** returns the term in brackets to 0. Conversely, if the term in brackets is negative, λ_i