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Giuseppe Buttazzo
Aldo Frediani *Editors*

Variational Analysis and Aerospace Engineering: Mathematical Challenges for Aerospace Design

Contributions from a Workshop held at the School of Mathematics in Erice, Italy

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VOLUME 66

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Variational Analysis and Aerospace Engineering: Mathematical Challenges for Aerospace Design

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ISSN 1931-6828 Springer Optimization and Its Applications
ISBN 978-1-4614-2434-5 e-ISBN 978-1-4614-2435-2
DOI 10.1007/978-1-4614-2435-2
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2012933756

Mathematics Subject Classification (2010): 49J15, 49J20, 35Q93, 47N10, 65K10, 65N30, 65N38, 74R20, 76M10, 76N25

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

This book is dedicated to
Professor Franco Giannessi
on the occasion of his 75th birthday.

Preface

The new challenges in Aerospace Sciences and Engineering are not limited to partial improvement of the systems available today, but the ambition is to design innovative machines with a jump forward of efficiency to reduce fuel consumption and noxious emissions or, in synthesis, to fly cleaner and quieter. Mathematics is fundamental in this respect. The series of the workshops held at the "*Ettore Majorana Foundation and Centre for Scientific Culture*" of Erice continues bringing together mathematicians and aerospace engineers coming from both Academia and Industry. Erice is a place where the dialog is easy and fruitful and young research fellows can interact and discuss in a pleasant and sophisticated scientific atmosphere.

The present volume collects most of the papers presented at the workshop "*Variational Analysis and Aerospace Engineering II*" held on 8–16 September 2010; some papers, dealing with new challenges in Aeronautics, were added in order to present a set of new problems requiring an extensive application of mathematical tools.

The editors wish to continue this series and are confident, as written in the volume published on 2009, "to capture the interest of people, ..., particularly, young researchers working on new frontiers of mathematical application to engineering".

This volume is dedicated to Franco Giannessi, eminent professor of Mathematics at the University of Pisa and Director of the School of Mathematics "G. Stampacchia" of the Erice Centre, on the occasion of his 75th birthday. Franco continues to be a guide to the new generations of scientists.

Pisa, Italy

Giuseppe Buttazzo
Aldo Frediani

Acknowledgements

This volume collects contributions presented in the workshop "*Variational Analysis and Aerospace Engineering II*" held at the School of Mathematics of the E. Majorana Centre in Erice on 8–16 September 2010.

The workshop has been possible thanks to the contributions of the following organizations:

- University of Pisa (Italy),
- Department of Aerospace Engineering "L. Lazzarino", University of Pisa (Italy),
- Department of Mathematics "L. Tonelli", University of Pisa (Italy),
- Technical University of Technology, Delft (Holland),
- IDS Company, Pisa (Italy),
- GNAMPA (Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni).

We gratefully acknowledge the E. Majorana Centre and Foundation for Scientific Culture and the precious help by Franco Giannessi and Vittorio Cipolla.

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PrandtlPlane Propelled with Liquid Hydrogen: A Preliminary Study

Nicola Beccasio, Marco Tesconi, and Aldo Frediani

1 Introduction

In 1998, the world's airlines transported more than 1600 million of passengers and 29 million tons of freight, generating an annual turnover of US$ 307 billion. These numbers have raised up and, in the future, are supposed to improve with a predicted growth of revenue passengers-km of 4–5% every year.

Today transport aviation is supposed to produce about 2.5% of the total CO_2 emissions, and, considering its growth, this is a problem for the future of aviation industry [13].

The document "*A Vision for 2020*", published by the Advisory Council for Aeronautics Research in Europe, October 2002, summarizes the main guidelines to make Aircraft and Air Transport System responding to society's needs, despite a threefold increase in air transporting. The main points of interest are Quality and Affordability, Safety, and Environment; Environment is the starting point of this paper. The main exhausts of aircraft engines are Carbon Nitride CO_2, Water vapour, Nitride Oxides NO_x, non-combusted fuels, and particulates. CO_2 has a long-term permanence in the atmosphere (about 100 years), and the impact on climate is independent of the height of flight. It can be reduced by reducing the engine specific consumption and improving the efficiency of the kerosene propelled aircraft; the other possibility is to

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G. Buttazzo, A. Frediani (eds.), *Variational Analysis and Aerospace Engineering: Mathematical Challenges for Aerospace Design*, Springer Optimization and Its Applications 66,
DOI 10.1007/978-1-4614-2435-2_1,

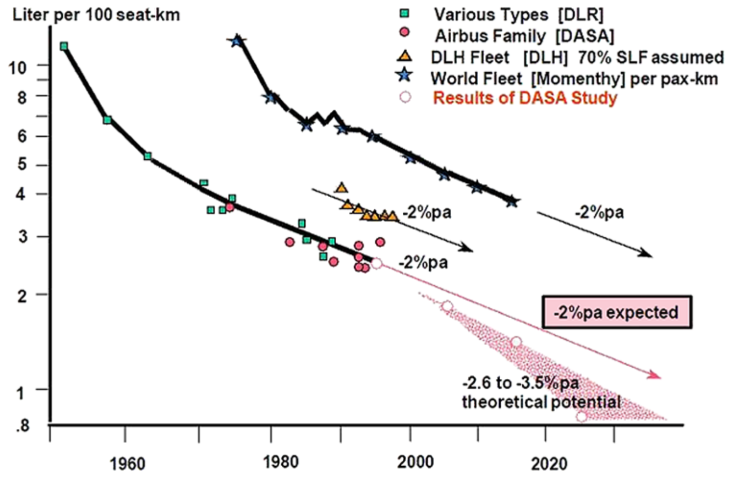

Fig. 1 Efficiency trend of aircraft engines

eliminate the kerosene by introducing new fuels. The water vapour effect vanishes soon at low altitudes but is persistent at high altitudes and produces some greenhouse effect due to the contrails formation [1]. The result of the engine combustion is NO, and, then, NO_2 is produced by oxidation of NO into the atmosphere; these products are dangerous for human health. Other noxious products are UHC and CO deriving from incomplete combustion, which generally occurs at low regimes of the engines. Particulate is made of small flying particles of carbon and hydrogen; it is not toxic but produces smog. The efficiency of civil transport aircraft has improved constantly in the past. The increase of efficiency has invested new materials, new engines and on-board systems. In 2000 the material used for the main structures were the following: Aluminum alloys 65%, composites 15%, Titanium 5% and steel 15%; in 2020 the following is forecast: 65% composites, 15% Aluminum, 5% Titanium and 15% steel. The fuel consumed per 100 seat-km have been reduced constantly as shown in Fig. 1 [6].

The new requirements on environment are a reduction in perceived noise to one half of the current average levels, noise nuisance outside the airport boundary eliminated by quieter aircraft, CO_2 emissions per passenger kilometer cutted by 50% (i.e. a cut of 50% in specific fuel consumption) and NO_x cutted by 80%.

To achieve these goals in the medium term, it is necessary to change the way to design airplanes, introducing innovative configurations; in the long-term perspective, the fuel needs to be changed in addition. In this paper, both aircraft configuration and a new fuel are indicated; the new configuration is the PrandtlPlane, and the candidate fuel is Hydrogen.

The PrandtlPlane is potentially the most efficient aircraft configuration and, also, permits an easy integration of different types of engines, including the hydrogen ones. When passing from kerosene to Hydrogen, the general architecture of the aeronautical engines does not change apart from the combustion chambers; because the

Table 1 Comparative characteristics of some fuels

	H_2	CH_4	$CH_{1.93}$
Molecular mass	2.016	16.04	≈168
Heat of combustion [kJ/g]	120	50	42.08.00
Density [g/cm^3]	0.071	0.29375	≈0.811
Boiling point, 1 atm [K]	20.27	112	440–539
Freezing point [K]	14.04	91	233
Specific heat capacity [J/g K]	0.422917	3.05	0.109722
Heat of vaporization [J/g]	446	510	360

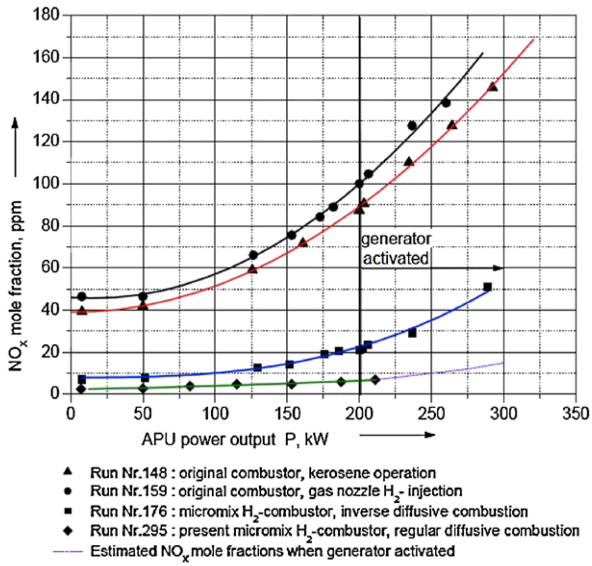

Fig. 2 NO_x production with different combustors

flame speed of Hydrogen is eight times higher than for the kerosene, the chambers are smaller and shorter, with the consequences of a lower permanence of fuel into the chambers (and a lower probability of NO_x formation) and lower refrigeration temperature. A comparison between the characteristics of Hydrogen and Kerosene, taken from the literature, are presented in Table 1.

Liquid Hydrogen reduces significantly NO_x emissions (from 30% to 50% compared to a kerosene propelled aircraft). Recent progresses in the combustor design (Micromix) have produced a further reduction of NO_x; a comparison of the kerosene and hydrogen NO_x emissions vs. power of an APU unit is presented in Fig. 2, which shows that a reduction of about 85% compared with a standard combustor of kerosene is obtained.

Hydrogen engines eliminate totally CO_2, reduce the fuel mass by a factor of 2.8 but improve the water vapour by a factor 2.6 compared to kerosene for completing the same mission (Fig. 3).

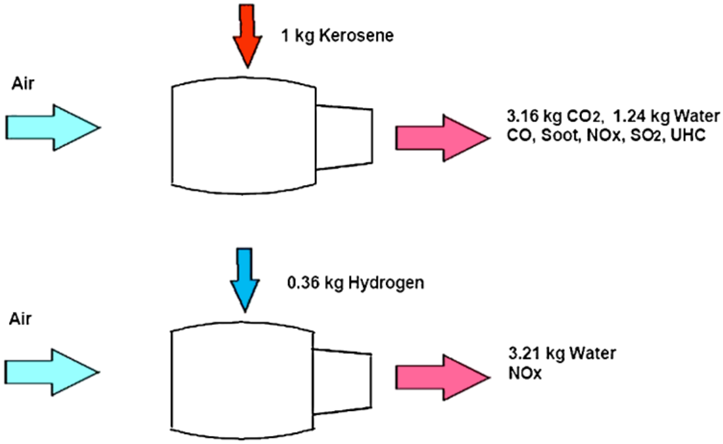

Fig. 3 Emissions of kerosene and Hydrogen propelled engines

Fig. 4 Example of LH_2 propelled aircraft [18]

The noxious emissions and water vapour productions indicated before are related to a conventional cruise altitude, but the conclusions could be different when changing it. So, if the generation of pollution (including water vapour) is the objective function to be minimized, the cruise altitude becomes a main parameter, and, in addition, in order to define a compromise with aerodynamic efficiency, the second main parameter is the aspect ratio. There are also undesired properties connected to liquid Hydrogen as, for example, low energy required for ignition, hide range of flammability, high volatility, high flame speed, etc.; these aspects are well known and discussed in the literature [6, 9, 12, 14–16]. In addition, because of its low density, about $4\times$ fuel volume is required in comparison to conventional fuel; consequently, very large pressurized fuel tanks are needed, and new concepts in Aerodynamics and overall architecture become the main challenges. The liquid hydrogen configurations proposed so far have, fundamentally, conventional layout apart from the positioning of the LH_2 tanks into the fuselage; in Fig. 4, a model of LH_2 aircraft developed in the CryoPlane Project [6] is illustrated.

Table 2 Technical specifications of the PrandtlPlane

Design payload	250 pax.
Range	2500 nm
Flight crew	2
Cabin crew	8
Standard pax weight	75 kg
Luggage	20 kg
Fuel	Liquid Hydrogen
Unusuable fuel	5% of fuel weight
Propulsion	2 engines
Cruise speed	0.75 Mach
Approach speed	<150 kts
Max take off field length	7000 ft
Max landing field length	6000 ft
Airport altitude	0 m
Landing performance	$WLA = 0.95WTO$
N° of external tanks	2
Tank differential pressure	1.4 bar

The most serious disadvantage of this solution are tanks positioned over the heads of passengers, and safety could be dramatically reduced.

In this paper a new PrandtlPlane configuration is proposed with the aim of proving that the Hydrogen tanks could be fully separated from the passenger cabin contrary to a conventional aircraft.

2 Preliminary Design

The application of liquid Hydrogen propulsion to a PrandtlPlane configuration [7] is proposed in the case of a 250-seat medium-size aircraft, designed for continental flights (max 2500 nm) and for a reduced cruise speed (0.75 Mach). The complete list of design requirements is reported in Table 2.

As said before, a cut to pollution and, at the same time, an efficient air transport system inside continental routes are the main scopes of the project. Wing aspect ratios ($AR = b^2/S$, where S is the sum of front and rear wing surfaces) and cruise altitudes (h_{cruise}) are assumed as variables in convenient ranges of variation; the aspect ratio varies in the interval 4–11, and the cruise altitude between 27.000 and 35.000 ft.

The present optimization procedure of the lifting system is affected by more approximations than the ones introduced in [8], due to the very preliminary design using Hydrogen as a fuel and, also, the lack of previous experience; thus, forward and rear wing reference surfaces are fixed as 45% and 55% of total one, respectively. The design process is summarized in Fig. 5.

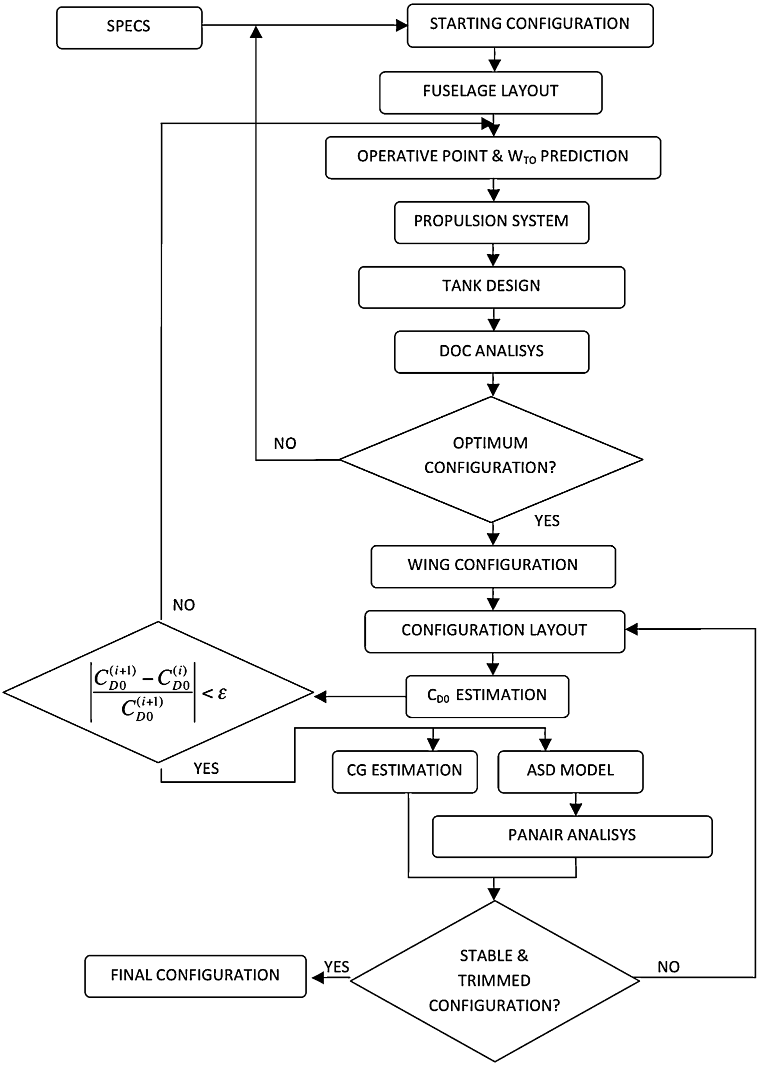

Fig. 5 Design procedure of the aircraft [2]

Starting from an initial layout, the efficiency during cruise is optimized meanwhile the aircraft is trimmed and stable in flight.

The fuselage layout is designed in order to minimize the wetted area; in this analysis the fuselage is assigned and not subjected to any modification during the design process. The passenger deck is the same of the other PrandtlPlane configurations, where about 250 passengers in 2 classes or 300 passengers in one single class are embarked; a typical solution is depicted in Fig. 6, in the case of two classes.

The cargo deck allocates two containers abreast (Fig. 7) for a total of 22 LD1 containers. A central axial support connects the passenger beam to the bottom fuse-

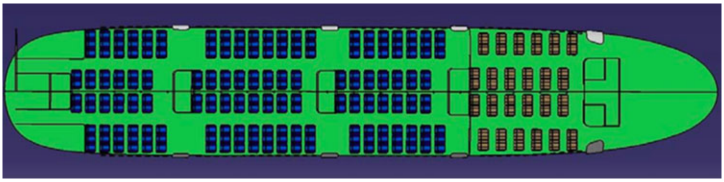

Fig. 6 General overview of the seats' disposition

Fig. 7 General fuselage cross section (business class, 250-pax. version)

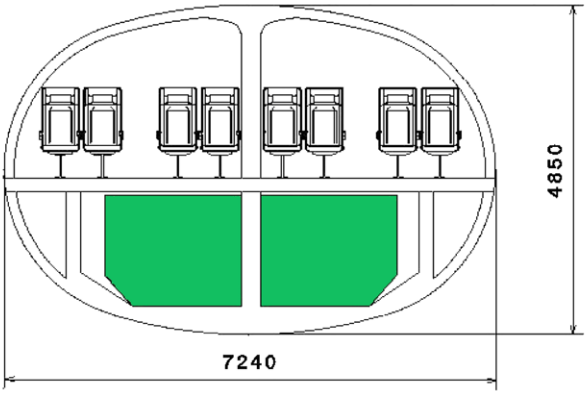

Fig. 8 General fuselage layout

lage in order to save weight and improve the stiffness; a truss loaded in traction (due to pressurization) is mounted between the top and bottom central fuselage.

As said before, due to the low density of hydrogen, we must provide a four times fuel volume of an equivalent aircraft propelled with Kerosene; for this reason, a part of the fuselage after the rear bulkhead could be used to allocate a fraction of total fuel. The presence of this tank could be avoided by reducing the range to about 1800 nm; in the case of a long route where the rear cabin tank is necessary, the relative positions of the Centre of gravity and the neutral point can be changed by a proper translation of the lifting system along the fuselage direction. The main part of fuel is set on two pressurized cylindrical tanks at the wing tips, insulated with Polymethacrylamide and all contained into an exterior aluminium case. In this paper, we do not discuss about the fuel tank detail design, but only their external layout is defined; the tank weight is predicted by means of statistic formulas.

In the center fuselage, two sponsons contain the main undercarriage (Fig. 8); previous experience have shown that the sponsons are not critical as far as transonic Aerodynamics is concerned; even more so, when the cruise speed has been reduced.

Once the external fuselage layout is fixed, preliminary wing layout and engine integration are necessary to determine the operative point. In particular, consid-

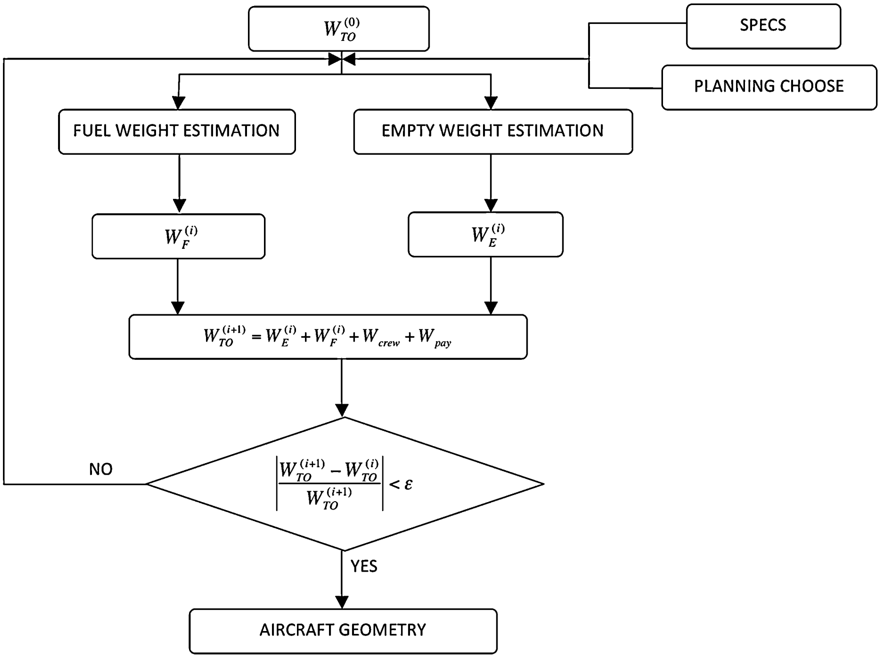

Fig. 9 Logic diagram used to predict take-off weight

ering FAR25 Regulations and the technical specifications in terms of Max Take Off/Landing field length, the minimum value of thrust required is selected together with the maximum wing load.

To predict the takeoff weight, the standard procedure indicated in the logic scheme in Fig. 9 is adopted.

Fuel weight is first estimated by fuel fraction method, where a typical mission is divided into steps, and fuel burnt is evaluated step by step using predictions taken from the literature and modified with proper corrective coefficients.

Semi-empirical expressions from NASA CR-151970 [3], adapted to take the specific configuration into account, are used to evaluate the empty weight of the single components of the aircraft.

Once Thrust/Weight (T/W) and Takeoff weight (W_{TO}) are calculated, the necessary thrust is given, and the engine features can be established. Many engine integrations are possible so avoiding to mix the presence of passengers in the cabin with hydrogen pipelines; the engines could be positioned at the rear fuselage, under the front wing or under the rear wing, depending on the layout of the hydrogen pipelines connecting the engines and the tanks. The configuration with the engines positioned on the lateral fuselage is typical of the propulsion with Kerosene, but, in the case of hydrogen propulsion, it is not allowed for safety reasons with the fuselage tank, and, in the case of tip tanks only, the pipeline layout along the vertical and rear wings is too long and unsafe.

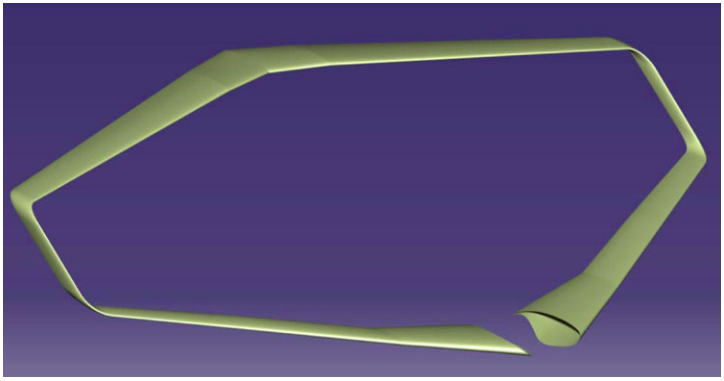

Fig. 10 The box wing system of the present PrandtlPlane

Table 3 Airfoils

1	SC 20714
2	SC 20714
3	GRUMMAR K2
4	GRUMMAR K2
5	GRUMMAR K2
6	SC 20412
7	GRUMMAR K2

The solution with two engines under the front wing is convenient when hydrogen is contained in the tip tanks only. The solution with the engines under the rear wings is always valid and safe, both with tip tanks and rear fuselage tank [5].

In analogy with others aircraft, a reference engine is fixed, and the final choice is obtained using the so called Engine Scale Factor method; the Engine Scale Factor method used here is based on ESF factor, defined as

$$ESF = \frac{T_{des}}{T_{ref}}, \quad (1.1)$$

where T_{ref} is the reference engine static thrust, and T_{des} is the actual engine static thrust. Once ESF is obtained, the reference engine can be scaled to be adapted to the configuration studied; this method is valid in the range $0.8 < ESF < 1.2$. After having fixed the wing load, W/S, we can define the wing shape; a typical PrandtlPlane wing system is illustrated in Fig. 10.

Airfoils are fixed in analogy with others 250-pax (Table 3). PrandtlPlane configurations; a typical aerodynamic solution is shown in Fig. 11: hereinafter, sweep angle, wing system condition and other wing geometrical characteristics will be fixed to make the aircraft stable and trimmed longitudinally.

The dihedral angles are set according to other PrandtlPlane configurations, The optimization procedure allows us to obtain the following quantities (functions of cruise altitude and Aspect Ratio): Maximum Take Off Weight, Empty Operating

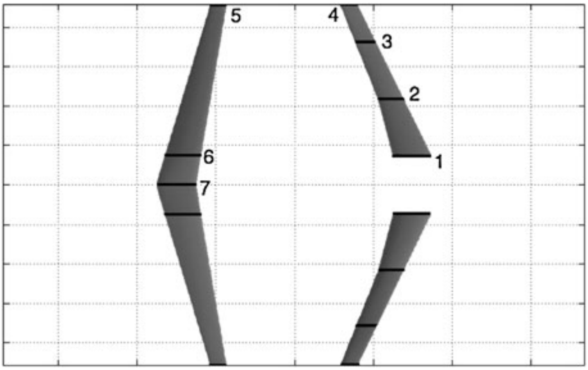

Fig. 11 Airfoil definition

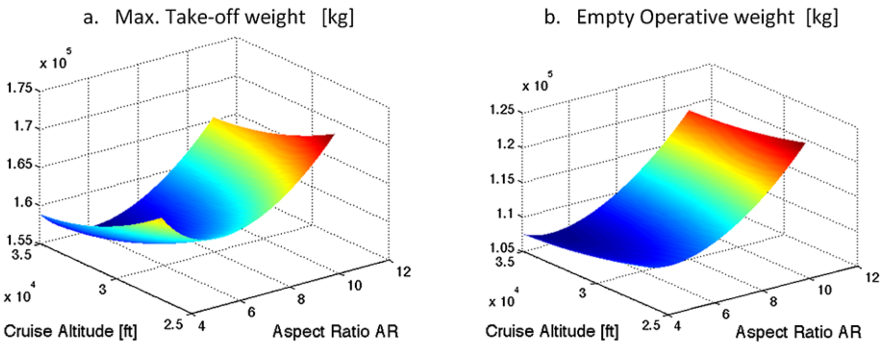

Fig. 12 Results a and b

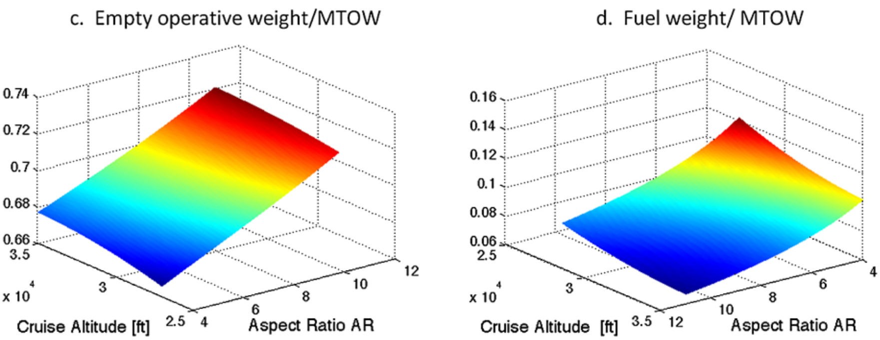

Fig. 13 Results c and d

Weight, Empty Operating Rate/Maximum Take Off Weight, Fuel Weight/Maximum Take Off Weight, Wing Span, ESF, Wing Surface, Fuel Weight, Landing Weight/ Maximum Take Off Weight, Wing Tank Weight including fuel inside. The results obtained are summarized in Figs. 12, 13, 14, 15 and 16.

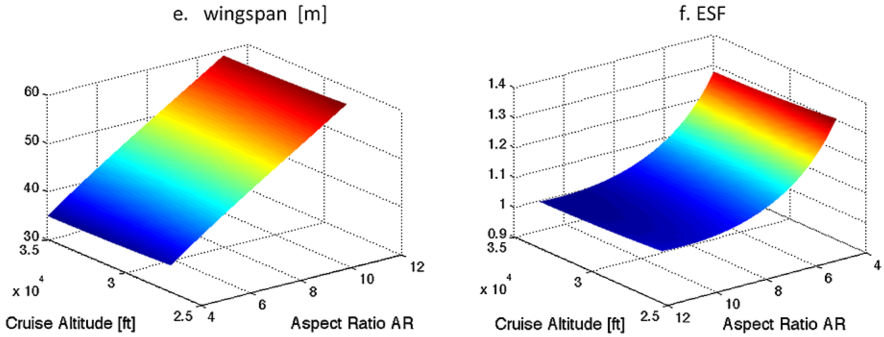

Fig. 14 Results e and f

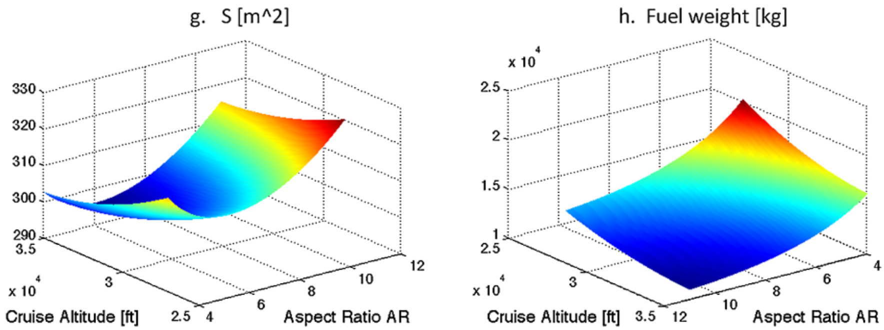

Fig. 15 Results g and h

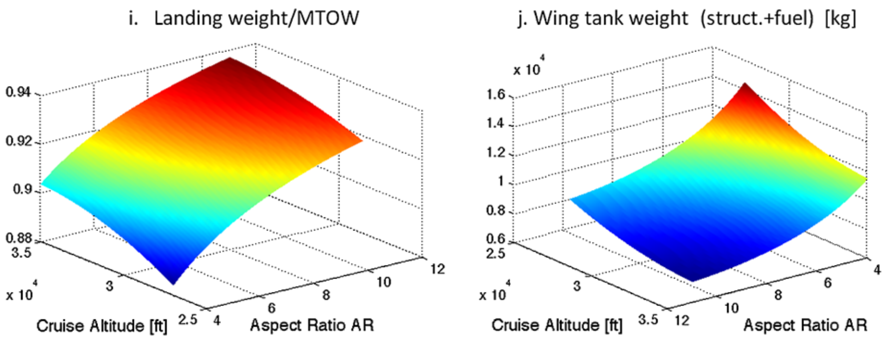

Fig. 16 Results i and j

Figure 12a shows how the maximum take-off weight depends on both cruise altitude and aspect ratio. The results confirm that fuel consumption is reduced when flying at high altitudes and also that, given a cruise altitude, maximum take off weight is a parabolic function of the Aspect Ratio: a low AR corresponds to a low efficiency, and, as AR increases, the efficiency will raise with the same law of the

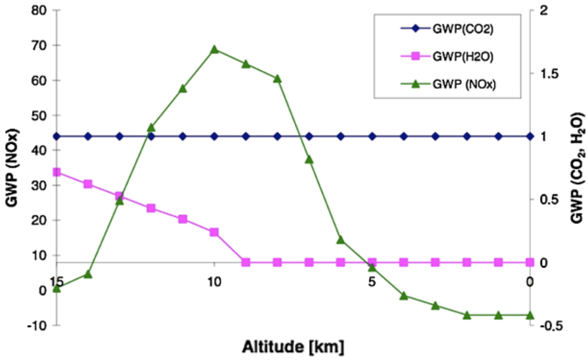

Fig. 17 GWP for CO_2, NO_x and water vapour emissions of a typical aircraft

empty operative weight (Fig. 12b). It is worth noting that operative weight is very high (about 68%–70% of take-off weight) because of the low density of liquid hydrogen; this conclusion is confirmed by Fig. 12d, where the fuel weight is small (about 7%–9% of take-off weight). If only aircraft performances are considered, the optimum cruise altitude is the maximum one, but, when noxious emissions and environmental impact are considered, the optimum altitude is different. In this respect, we make reference to some results in the literature to establish the cruise flight altitude as a compromise between a high aerodynamic efficiency and a low environmental impact.

3 The Choice of Flight Altitude

The cruise flight altitude of minimum fuel consumption is the maximum one (35.000 ft), but it corresponds to an unacceptable level of noxious emissions. The influence of the emissions on the global warming is conventionally considered through the so-called Global Warming Parameter (GWP). Figure 17 [16] shows the effect on heart heating of the main emissions in terms of GWP for a typical Kerosene propelled aircraft. The effects of CO_2 are nearly independent of flight altitude, the contrary for NO_x, and no warming effect due to contrails is produced under about 9-km altitude. Under 4000 m, NO_x produces a refrigeration, and the maximum warming effect occurs at 10000 m due the interactions of NO_x with other gases (e.g. Ozone, Methane, etc.); the global effect is the superposition of the partial ones. It appears that, should CO_2 be eliminated by an Hydrogen propelled aircraft, we could obtain a good compromise between performances and pollution at altitudes lower than 9000 m, with no effect of contrails.

Figures 18 and 19 show the GWP concentrations at the altitudes of interest in the case of kerosene and hydrogen propelled aircraft, respectively [16].

Figures 20 and 21 show the effects of Hearth warming in terms of change of fuel burnt and MTOW for the two engines. This results that a kerosene engine has no benefit to reduce the cruise altitude; when passing, for example, from FL390 to FL310 the fuel burnt increases by 20%, the MTOW increases by 2% and the GWD

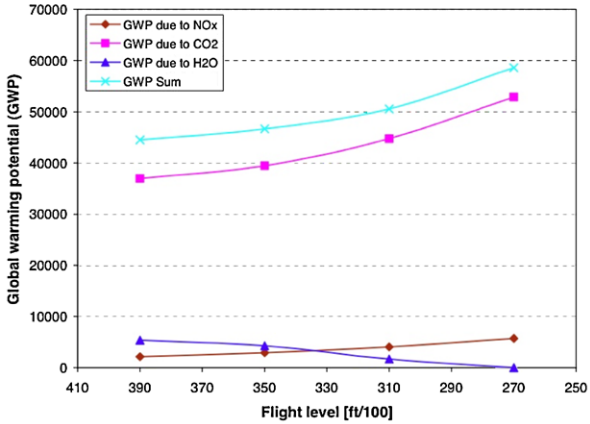

Fig. 18 GWP for a typical Kerosene propelled aircraft

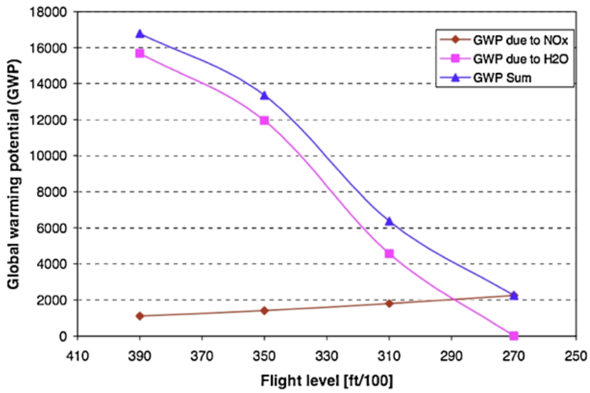

Fig. 19 GWP for a Hydrogen propelled aircraft

by 10%; under the same conditions, a hydrogen propelled aircraft experiences an increase of MTOW of 2%, a specific consumption of 10% and a reduction of GWD (or NO_x) of 60%.

In the present analysis, the attention is focused on the reduction of noxious emissions, and, thus, we assume a flight altitude of 29000 ft (8.840 m). After that the cruise level has been stated, it is possible to reduce all figures from Figs. 12 to 16 to the simple curves shown in the figures from Figs. 22 to 26.

Now the aircraft configuration is obtained by means of the optimization procedure presented before; the starting configuration is defined with the following characteristics: 50-m maximum wing span, maximum value of $AR = 8$ and best value of $AR = 6.5$ (with these values, the take-off weight is minimized, and reasonable tank weights and dimensions result).

When the optimization procedure converges, the aerodynamic drag is minimized (the relevant parameter is the friction drag coefficient CD_0). The results of the optimization procedure are presented in the next section. Given the aerodynamic configuration, the range of positions of the Centre of Gravity is defined consequently in

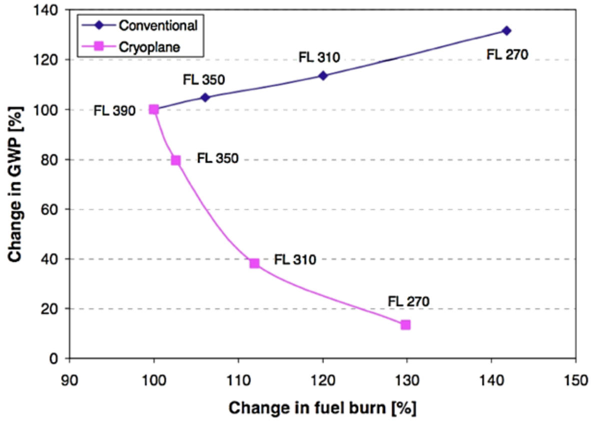

Fig. 20 GWP vs change in fuel burn

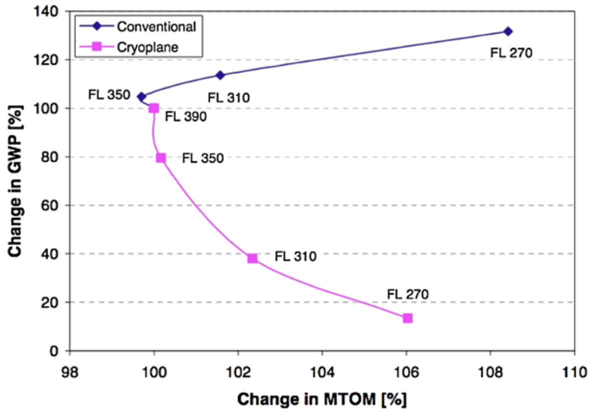

Fig. 21 GWP vs change in fuel burn

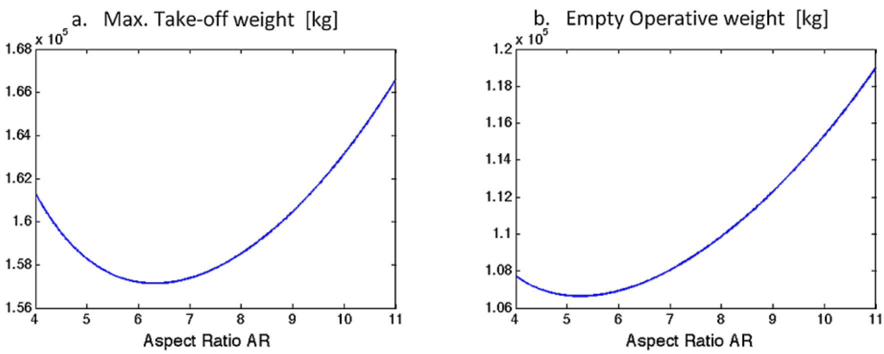

Fig. 22 The effect of the aspect ratio on the aircraft performances, a and b

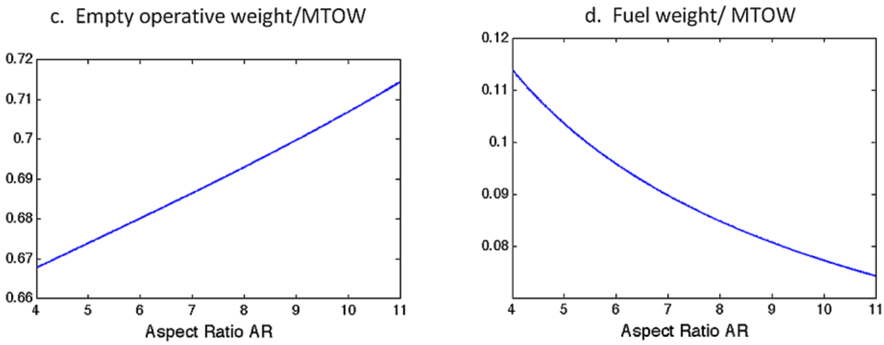

Fig. 23 The effect of the aspect ratio on the aircraft performances, c and d

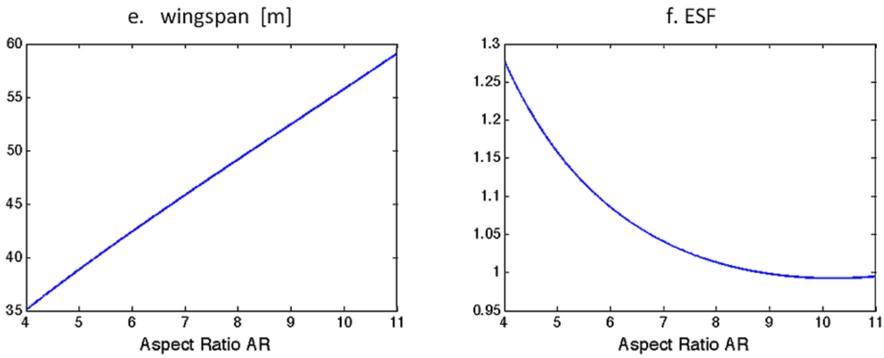

Fig. 24 The effect of the aspect ratio on the aircraft performances, e and f

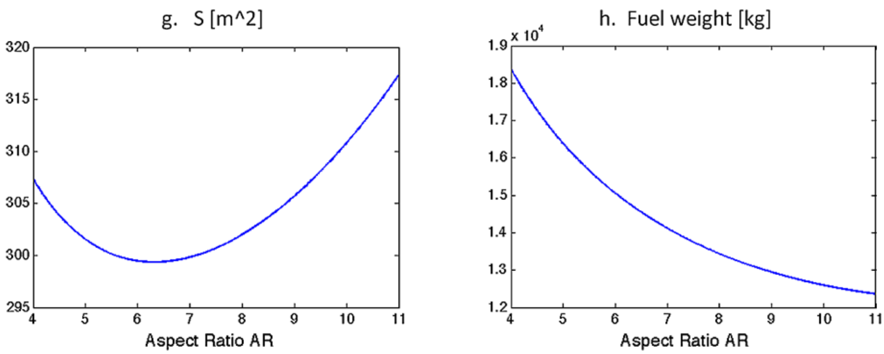

Fig. 25 The effect of the aspect ratio on the aircraft performances, g and h

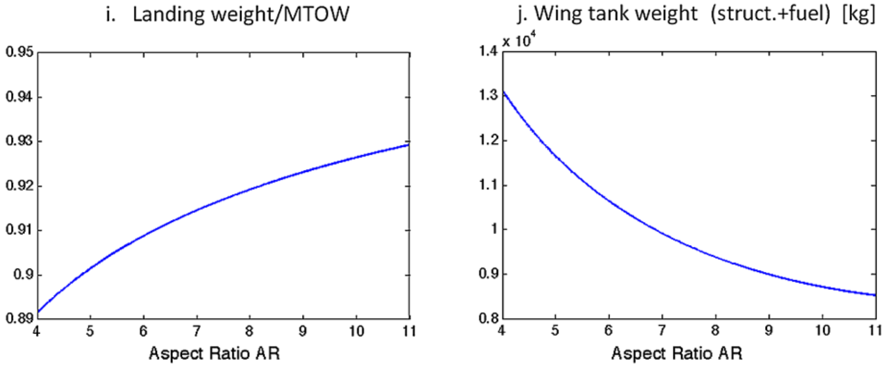

Fig. 26 The effect of the aspect ratio on the aircraft performances, i and j

such a way to obtain the static stability of flight in all the flight envelope; different operative conditions are considered, namely:

- full loaded aircraft (design condition);
- empty wing tanks;
- empty fuselage tank;
- empty tanks (wing and fuselage);
- no freight;
- no passengers, luggage and fuel;
- no passengers, luggage, fuel and freight.

Excursions of CG have been also examined in order to avoid any possible failure for "off design" conditions. Finally, the aerodynamic shape is constructed using [4], and the aerodynamic analysis is performed.

4 Final Configuration of the Aircraft

An artistic view of the final baseline configuration is illustrated in Fig. 27 in the case of a range of 2500 nm, with three tanks (at wing tips and inside rear fuselage).

All main characteristics of the final configuration have been calculated starting the formulas reported in the iterative loops illustrated in Figs. 5 and 9, using the commercial software Matlab. In Fig. 28 below it is possible to find out the iterative cycles necessary to have final values regarding the main aircraft weights.

In Tables 4–9 all the main data of the aircraft are summarized.

The aerodynamic chord is the average of the mean aerodynamic chords of the two horizontal wings; the results are relevant to 250 passengers with their luggage without any additional cargo. The empty weight has been obtained with standard statistical methods; corrections have been introduced to take the influence of the large fuel tanks, plants, etc., into account.

The total length of the fuselage becomes 51.64 m, resulting from the passenger accommodation and the presence of the rear fuselage Hydrogen tank. The wing span

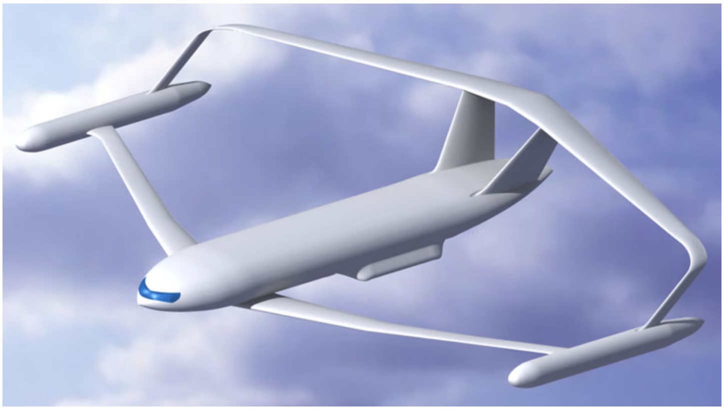

Fig. 27 Configuration resulting from a preliminary optimization

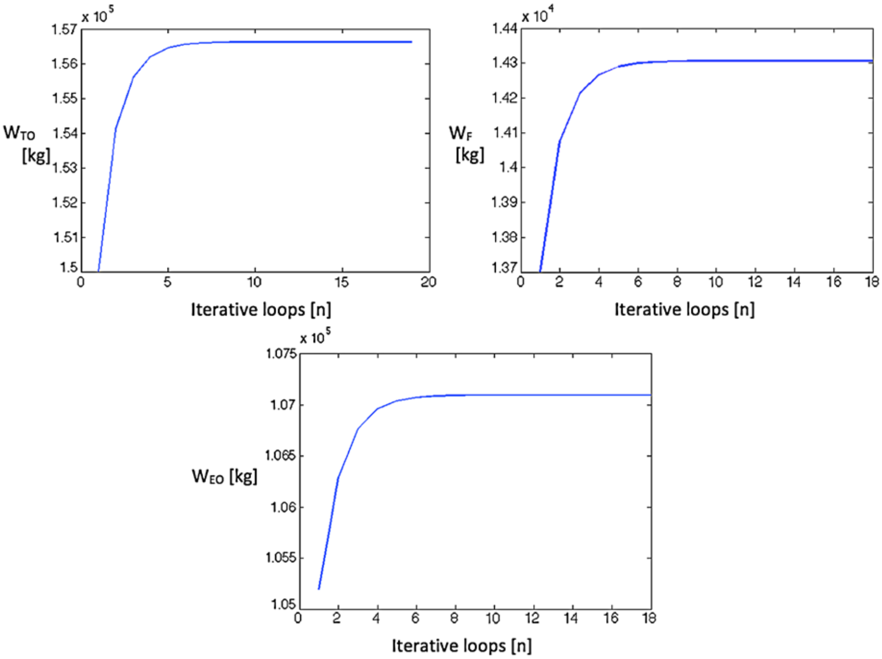

Fig. 28 Configuration resulting from a preliminary optimization

results from the optimization process; the final span of the optimized wing is 44 m, including the rounds to connect horizontal wings to the vertical ones (for a total of 3.24 m).

The preliminary weight estimation of this paper was conducted by using statistical formulas (see [10, 11, 17]), properly modified. For example, according to [8],

Table 4 Main LH2 PrandtlPlane characteristics

Max take-off weight [kg]	W_{TO}	156627
Empty operative weight [kg]	W_{EO}	107098
Fuel weight [kg]	W_{FUEL}	14306
Payload weight [kg]	W_{PAY}	23750
Thrust/max. take-off weight	T/W	0.32
Wing load [kg/m^2]	W/S	525
Cruise efficiency	$(L/D)_{CR}$	14.26
Maximum efficiency	$(L/D)_{MAX}$	16.47
Total fuselage length [m]	L_{TOT}	51.64
Wing span [m]	b	44
Reference surface [m^2]	S	298.4
Main aerodynamic chord [m]	MAC	3.6

Table 5 Forward wing geometry

Wing surface [m^2]	S_{FOR}	134.27
Wing span [m]	b	44.04
Main aerodynamic chord [m]	MAC_{FOR}	3.56
Aspect ratio	AR_{FOR}	11.8
Taper ratio	$(CT/CR)_{FOR}$	0.45
Relative average thickness	$(t/c)_{FOR}$	0.12
Sweep angle 25% [deg]	$\Lambda 25_{FOR}$	35
Sweep angle LE [deg]	ΛLE_{FOR}	35.94
Sweep angle TE [deg]	ΛTE_{FOR}	32.12
Root choord [m]	Cr_{FOR}	5.08
Kink choord [m]	Ck_{FOR}	3.37
Tip choord [m]	Ct_{FOR}	2.28
Dihedral angle [deg], Bay n°1	$\Gamma 1_{FOR}$	0
Dihedral angle [deg], Bay n°2	$\Gamma 2_{FOR}$	5

the empty weight of a PrandtlPlane wing system is nearly the same of that of a conventional aircraft; thus, we start from the statistical evaluation of the wing weight of a conventional aircraft to be improved in order to take the position of the tip tanks, heat exchangers and the tip local reinforcements into account and to be reduced because the present wings are free of the engines.

Figures 29 and 30 show some aerodynamic results obtained from a BEM analysis using Panair, even though, in these figures, vertical tail is single and not double as the final configuration (the reason is that only longitudinal stability has been studied). The figures give just an indication that the aerodynamic behaviour of the aircraft is not critical.

The aircraft is stable during trimmed cruise flight, as shown in Fig. 31, where the main longitudinal aerodynamic derivatives are reported.