Selected Works in Probability and Statistics

Selected Works of Terry Speed

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Editor

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Springer

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To Terry — teacher, colleague, and friend


NO
WAR


THEY TAKE THE PRIZES
THE Police Commissioner, Mr Porter, presented the prizes at the University High School's jubilee speech night in Melbourne Town Hall last night.
Pictured with their prizes are two matriculation students, Suzanne Cory, 18, of East Kew, and Terry Speed, 17, of Elwood. Suzanne came first in six subjects.
Mr Porter said that parents should not set their children "an impossible task" by forcing them into a career to which they were not suited.
At other speech nights last night—
Malvern Gramm
Dux, Geoffrey C
17, also won a
prize for dram
Special
awarded t
Grammar
LABOR ATTACKS

Preface to the Series

Springer's Selected Works in Probability and Statistics series offers scientists and scholars the opportunity of assembling and commenting upon major classical works in statistics, and honors the work of distinguished scholars in probability and statistics. Each volume contains the original papers, original commentary by experts on the subject's papers, and relevant biographies and bibliographies.

Springer is committed to maintaining the volumes in the series with free access of SpringerLink, as well as to the distribution of print volumes. The full text of the volumes is available on SpringerLink with the exception of a small number of articles for which links to their original publisher is included instead. These publishers have graciously agreed to make the articles freely available on their websites. The goal is maximum dissemination of this material.

The subjects of the volumes have been selected by an editorial board consisting of Anirban DasGupta, Peter Hall, Jim Pitman, Michael Sörensen, and Jon Wellner.

Preface

The purpose of this volume is to provide an overview of Terry Speed's contributions to statistics and beyond. Each of the fifteen chapters concerns a particular area of research and consists of a commentary by a subject-matter expert and selection of representative papers. Note that, due to space constraints, not all articles discussed in the commentaries are reprinted in this volume. The reader is referred to the book website for access to these papers (http://www.stat.berkeley.edu/~sandrine/Pubs/SelectedWorksTerrySpeed/). The chapters, organized more or less chronologically in terms of Terry's career, encompass a wide variety of mathematical and statistical domains, along with their application to biology and medicine. Accordingly, earlier chapters tend to be more theoretical, covering some algebra and probability theory, while later chapters concern more recent work in genetics and genomics. The chapters also span continents and generations, as they present research done over four decades, while crisscrossing the globe.

The commentaries provide insight into Terry's contributions to a particular area of research, by summarizing his work and describing its historical and scientific context, motivation, and impact. I've enjoyed reading the personal anecdotes, which remind us that one cannot always dissociate the scholar from the person and show how relationships beginning as professional collaborations can turn into long-lasting friendships. In addition to shedding light on Terry's scientific achievements, the commentaries reveal endearing aspects of his personality, such as his intellectual curiosity, energy, humor, and generosity. The title of Bin Yu's piece, "the $n \to \infty$ dimensions of Terry", says it all and captures Terry as an avid and tireless scholar and explorer.

Due to space constraints, this volume is only the tip of the iceberg, as it is clearly impossible to give a complete account of Terry's work. And it is certain that additional significant contributions are forthcoming — Terry's thirst for knowledge has not abated, and neither has his dynamic pace. For "coming attractions", one will have to wait for another such volume ...

Berkeley, CA
June 2011

Sandrine Dudoit

Acknowledgements for the Series

This series of selected works is possible only because of the efforts and cooperation of many people, societies, and publishers. The series editors originated the series and directed its development. The volume editors spent a great deal of time organizing the volumes and compiling the previously published material. The contributors provided comments on the significance of the papers. The societies and publishers who own the copyright to the original material made the volumes possible and affordable by their generous cooperation:

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University of North Carolina Press

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First and foremost, I would like to thank the fifteen contributors for their insightful and inspiring commentaries. This project would not have been possible without their hard work, support, generosity, and enthusiasm. In particular, they were remarkably gracious and efficient while working under pressure to meet tight publication deadlines. I have thoroughly enjoyed interacting with each of them and have learned much about Terry's career and life while reading their commentaries.

I am grateful to the reviewers for immediately and kindly agreeing to contribute to this volume and for their thoughtful reports.

I have greatly appreciated Jim Pitman's guidance and support throughout this project, regarding both the general editing process and technical aspects of bibliography management.

Matthew Watkins' work on Terry's bibliography was very valuable.

I am also thankful for Darlene Goldstein's encouragement and help during the final stages of this project.

Last but not least, editing this volume was an extraordinary opportunity to work with Terry again and "delve into his many lives". I have learned immensely from him, whether in a Berkeley classroom, sipping a milkshake on Bancroft Avenue, attending a performance at the San Francisco Opera, or trying to keep up with him on a morning jog in the mountains overlooking Lago Maggiore. His sharp intellect, vast culture, humanism, energy, enthusiasm, and humor never cease to inspire me. For this, I am most thankful.

Berkeley, CA *Sandrine Dudoit*
June 2011

Contents

Biographical Sketch of Terry Speed

Terence Paul (Terry) Speed was born on March 14th, 1943, in Victor Harbor, South Australia. He grew up in Melbourne, attending Westgarth Central School and University High School. In his final year of high school, he decided that he wanted to pursue a career in medical research, influenced by the award that year (1960) of the Nobel Prize in Medicine to Sir Frank Macfarlane Burnet, the Director of Melbourne's Walter and Eliza Hall Institute (WEHI) of Medical Research. In 1961, Terry enrolled in a joint Medicine and Science degree at the University of Melbourne. By the end of the first term, his lack of enthusiasm for laboratory work prompted him to change his enrollment to Science alone, majoring in mathematics, while maintaining a strong interest in genetics. He graduated in 1964 with an honours degree in mathematics and statistics. In his final year, he edited the magazine *Matrix* of the mathematics students society and also attended lectures on algebra at Monash University, located in an outer suburb of Melbourne. At the end of that year, he married Freda Elizabeth (Sally) Pollard, whom he had met at a party at the home of Carl Moppert, then a Senior Lecturer in the Department of Mathematics at the University of Melbourne.

Although an attempt to join the PhD program in the Department of Statistics at the University of California, Berkeley (UCB) fell through, Terry was awarded an Australian Commonwealth Postgraduate Research Scholarship in the Department of Statistics at the University of Melbourne. He began his graduate studies in 1965, under the supervision of Professor Evan J. Williams. Rather than pursuing research in the area of his supervisor (Fisherian statistics), Terry developed an interest in probability theory, along the lines of Michel Loève's work at Berkeley. He did not however complete his doctoral degree at that point. In mid-1965, he took a job as a tutor in the Department of Mathematics at nearby Monash University and enrolled for a part-time PhD in mathematics under the supervision of Professor Peter D. Finch. With hindsight, it is interesting to note that several elements that were to feature prominently in Terry's later life had already manifested themselves: interests in medical and genetic research, the Walter and Eliza Hall Institute, and probability and statistics as practiced in the Department of Statistics at the University of California, Berkeley.

At Monash, Finch had eclectic interests in probability theory and mathematics and encouraged Terry to examine probability and measure theory on the class of lattices generalizing Boolean algebras that corresponded to the intuitionistic logic of L. E. J. Brouwer. This led to Terry's 1969 PhD thesis entitled *Some topics in the theory of distributive lattices*. In addition to working on his doctoral research, Terry taught introductory probability and statistics to large classes for four years running, and developed and presented undergraduate lecture courses on information theory (introductory and advanced), measure theory, projective geometry, and lattice theory. He also lectured on the theory of games to students in the Department of Mathematics at the new La Trobe University, located in another outer suburb of Melbourne. On top of this, he completed a Diploma of Education at Monash, reasoning that, if all else failed, he would be happy as a secondary school mathematics teacher and that it would be wise to be qualified.

While waiting for the examiners' reports on his thesis, Terry met Professor Joe Gani, then Director of the Manchester-Sheffield School of Probability and Statistics in the United Kingdom. Gani encouraged him to consider a lecturing position in the School. The presence in Manchester of Professor K. R. Parthasarathy — who carried out research on probability theory over algebraic structures such as locally compact abelian and Lie groups — proved to be the clincher. So off to Sheffield he and Sally went! Sheffield was an exciting place at that time, with excellent staff and lots of visitors. Equally important was its accessibility to other centers of probability and statistics such as Manchester and London. Initially, Terry travelled to Manchester weekly to attend Partha's seminar and went down to London to attend seminars at Imperial College, meetings of the Royal Statistical Society, and the like. There was lots of train travel. However, Terry's career in Partha-style probability theory did not take off and, in due course, he found himself collaborating with Elja Arjas on the topic of random walks, an experience that was both satisfying and productive. A later visitor to Manchester, Professor Debrabata Basu, re-kindled his interest in Berkeley-style statistics and led to a new obsession: sufficiency.

Terry returned to Australia to head the small group of statisticians in the Department of Mathematics at the University of Western Australia (UWA). He started at UWA as Associate Professor in 1974, became Professor in 1975, and spent a very happy and productive period there, culminating in being appointed Head of Department in 1982. From late 1977 until early 1979, he had his first sabbatical, spending time at the University of Copenhagen, Princeton University, Rothamsted Experimental Station, and UC Berkeley, all with Sally, and on his own at the Indian Statistical Institute in Calcutta.

In 1982, Terry was invited to apply for the position of Chief, Division of Mathematics and Statistics, at Australia's Commonwealth Scientific and Industrial Research Organization (CSIRO). He took up that appointment in 1983 and had a very hectic first year, being based in Canberra, but travelling to visit members of his division in every state capital and several other centers around Australia.

In 1984, he spent two enjoyable months visiting the Department of Statistics at the University of California, Berkeley, in a way, fulfilling his unrealized dream from 1964. While there, he was encouraged to apply for a permanent position, and three

years later, in fall 1987, joined UCB as a tenured professor. On the basis of his administrative experience with CSIRO, he was appointed Department Chair 1989–94, after which he took a second sabbatical, encouraged by Sally to explore job opportunities back in Australia. Nothing happened on this front for two years, but in 1996, a former classmate from University High School, Professor Suzanne Cory, by then Director of the Walter and Eliza Hall Institute of Medical Research, invited him to start up bioinformatics at WEHI. Sally said "yes!" and so he half accepted. From August 1997 to July 2009, Terry split his time evenly between UCB and WEHI or, as he used to say, spent half his time in Berkeley, half in Melbourne, and the other half in the air in between.

Following yet more encouragement from Sally, Terry officially retired from teaching at UC Berkeley at the end of the US academic year 2008–9 and took on a full-time appointment at WEHI. At the time of writing, he still has four PhD students, three postdoctoral fellows, and a number of continuing collaborations at Berkeley. He visits there for short periods every 1–2 months and remains as active as ever.

To quote from one of Terry's recent e-mails: "Life has been hectic!"

Terry has (co-)authored over 300 refereed articles, in journals such as *Nature* and *The Annals of Statistics*, and on a wide variety of subjects, ranging from distributive lattices and ring theory in algebra, to pre-processing of high-throughput microarray and sequencing data in genomics. He contributes a regular column, *Terence's Stuff*, to the *Institute of Mathematical Statistics Bulletin*, with his unique and provocative opinions on the current state of statistical practice and education. His book *Stat Labs: Mathematical Statistics Through Applications* provides a glimpse into his teaching philosophy, which integrates the theory of statistics with its practice through case studies. As illustrated by his dizzying travel schedule, he is a much sought-after speaker worldwide. He has delivered prestigious lectures such as the 2001 Wald Memorial Lectures and 2006 Fisher Lecture, at the Joint Statistical Meetings, and the 2007 Hotelling Lectures, at the University of North Carolina, Chapel Hill.

Terry is an active and dedicated member of the main statistical and biological professional societies, journal editorial boards, and grant and peer review committees. He is also highly-solicited as a consultant and scientific advisory board member in industry. He is a Fellow of the Institute of Mathematical Statistics (1984), the American Statistical Association (1989), the American Association for the Advancement of Science (1990), and the Australian Academy of Science (2001). He has received various honors, including the 2002 Pitman Medal (Statistical Society of Australia), the 2003 Moyal Medal (Macquarie University), an Australian Government Centenary Medal (2003), the 2004 American Statistical Association Outstanding Statistical Application Award (for the paper Irizarry et al. (2003), *Biostatistics*, 4(2):249–264), as well as an Achievement Award for excellence in health and medical research (2007) and an Australia Fellowship (2009) from Australia's National Health and Medical Research Council (NHMRC).

In addition to his invaluable contributions to research, Terry is an extraordinary teacher, who has trained and influenced generations of students at Berkeley, in Australia, in the United Kingdom, and beyond. According to the Mathematics Genealogy Project (http://genealogy.math.ndsu.nodak.edu/id.php?id=30979), he has advised or co-advised 60 PhD students and has over 120 "descendants". He is a most inspiring and generous mentor. His contagious enthusiasm and intellectual curiosity have made him one of the most popular advisors in the UC Berkeley Department of Statistics and a great resource for students in other departments.

Berkeley, CA
June 2011

Sandrine Dudoit
with contributions from Terry Speed

PhD Students of Terry Speed

Name	Institution	Year
Michael Evans	Monash University	1973
Philip Pegg	University of Sheffield	1973
James (Jim) Pitman	University of Sheffield	1974
John Whitehead	University of Sheffield	1975
Anne Houtman	Princeton University	1980
Harri Kiiveri	University of Western Australia	1982
Matthew Knuiman	University of Western Australia	1983
Jens Breckling	University of Western Australia	1987
Bin Yu	University of California, Berkeley	1990
Sang Ho Lee	University of California, Berkeley	1991
Trang Nguyen	University of California, Berkeley	1991
Rudy Guerra, Jr.	University of California, Berkeley	1992
Darlene Goldstein	University of California, Berkeley	1993
Ferdinand Verweyen	University of California, Berkeley	1993
Mary Sara McPeek	University of California, Berkeley	1993
Steven Rein	University of California, Berkeley	1993
Ann Kalinowski	University of California, Berkeley	1995
David Nelson	University of California, Berkeley	1995
Hongyu Zhao	University of California, Berkeley	1995
Gregory Alexander	The American University	1996
Mark Grote	University of California, Berkeley	1996
Karl Broman	University of California, Berkeley	1997
Barathi Sethuraman	University of California, Berkeley	1997
William Forrest, III	University of California, Berkeley	1998
Lei Li	University of California, Berkeley	1998
Sandrine Dudoit	University of California, Berkeley	1999
Ru-Fang Yeh	University of California, Berkeley	1999
Shiying Ling	University of California, Berkeley	2000
Simon Cawley	University of California, Berkeley	2000

Name	Institution	Year
Alexandre Bureau	University of California, Berkeley	2001
Nicola Armstrong	University of California, Berkeley	2001
Yevgeniya (Jane) Fridlyand	University of California, Berkeley	2001
Fletcher (Hank) Ibser	University of California, Berkeley	2002
Von Bing Yap	University of California, Berkeley	2002
Yee Hwa (Jean) Yang	University of California, Berkeley	2002
Yongchao Ge	University of California, Berkeley	2003
Pratyaksha Wirapati	University of Melbourne	2003
Jacqueline Wicks	Australian National University	2003
Xiaoyue Zhao	University of California, Berkeley	2004
Natalie Thorne	University of Melbourne	2004
Benjamin Bolstad	University of California, Berkeley	2005
Yu Chuan Tai	University of California, Berkeley	2005
Ingileif (Inga) Hallgrímsdóttir	University of California, Berkeley	2005
Frédéric Schütz	University of Melbourne	2005
Ingrid Lönnstedt	Uppsala Universitet	2005
Tracey Wilkinson	University of Melbourne	2005
Richard Bourgon	University of California, Berkeley	2006
Jing Yi	University of California, Berkeley	2006
Yun Zhou	University of California, Berkeley	2006
Hui Tang	University of California, Berkeley	2007
Mark Robinson	University of Melbourne	2008
Tobias Sargeant	University of Melbourne	2008
Gregory Hather	University of California, Berkeley	2008
Margaret Taub	University of California, Berkeley	2009
Xin Wang	University of California, Berkeley	2009
Nancy Wang	University of California, Berkeley	2009
Frances Tong	University of California, Berkeley	2010
Oleg Mayba	University of California, Berkeley	2011
Alex Gout	University of Melbourne	2011

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Chapter 1
Algebra

Brian A. Davey

It gives me great pleasure to present this brief commentary on some of T. P. Speed's papers on algebra. It may come as a surprise to many of Speed's colleagues to know that his 1968 PhD thesis was entitled *Some Topics in the Theory of Distributive Lattices*. Moreover, of his first 15 papers only one was in probability theory with the remainder in algebra. Nevertheless, this fruitful excursion into algebra has its roots in the foundations of probability theory. In the introduction to his PhD thesis, Speed writes:

> In July 1965, the author began to look at the lattices associated with intuitionistic logic which are called variously – relatively pseudo-complemented, brouwerian or implicative lattices. This was under the direction of Professor P. D. Finch and aimed towards defining probability measures over these lattices. It was hoped that a probability theory could be developed for the intuitionistic viewpoint similar to the Kolmogorov one for classical logic.

Speed never returned to the search for an intuitionistic probability theory for, as he says later in the introduction to his thesis, he became "*sold on distributive lattices*". In the summer of 1968–1969, between my third and honours years, I spent three months on a Monash University Graduate Assistantship during which I read Speed's PhD thesis. By the end of that summer I was also *sold on distributive lattices* and have been ever since [2].

Between 1969 and 1974, Speed published 17 papers on a range of algebraic topics: distributive lattices, including their topological representation (9), Baer rings (3), Stone lattices (2), semigroups (2), and ℓ-groups (1). In the commentary below, I will discuss five of these papers. Only one of these papers, the first discussed, comes from Speed's thesis.

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Distributive lattices in general

Most of Speed's work on distributive lattices revolves around the role of particular sorts of prime ideals, with an emphasis on minimal prime ideals. In this section, we will look at two of the seven papers that fall into this category, namely, *On rings of sets* [10] and *On rings of sets. II. Zero-sets* [16].

In the first of these papers, Speed provides a unified approach to a number of representations of distributive lattices as rings of sets, that is, as lattices of subsets of some set in which the operations are set-theoretic union and intersection. Each of these characterisations was originally given in terms of the existence of enough elements of a special form, and their proofs looked quite different. Given cardinals $\mathfrak{m}$ and $\mathfrak{n}$, a lattice L is called $(\mathfrak{m},\mathfrak{n})$-complete if it is closed under the operations of least upper bound and greatest lower bound of sets of at most $\mathfrak{m}$ and $\mathfrak{n}$ elements, respectively. An $(\mathfrak{m},\mathfrak{n})$-complete lattice of sets is an $(\mathfrak{m},\mathfrak{n})$-*ring of sets* if $\mathfrak{m}$-ary least upper bounds and $\mathfrak{n}$-ary greatest lower bounds are given by set union and intersection, respectively. For example, the open sets of a topological space form an $(\mathfrak{m},2)$-ring of sets for every cardinal $\mathfrak{m}$. Speed introduces $\mathfrak{n}$-prime $\mathfrak{m}$-ideals and employs them to give natural necessary and sufficient conditions for an $(\mathfrak{m},\mathfrak{n})$-complete lattice to be isomorphic to an $(\mathfrak{m},\mathfrak{n})$-ring of sets. As Speed remarks in the introduction to the paper, *It is interesting to note that the elementary methods used in representing distributive lattices carry over completely and yield all these results, although this is hardly obvious when one considers special elements of the lattice*.

In *On rings of sets. II. Zero-sets* [16], Speed turns his attention to an important example of $(2,\omega)$-rings of sets, the *lattice* $\mathbf{Z}(X)$ *of zero-sets* of continuous real-valued functions on a topological space X. The paper, which is deeper and somewhat more technical than the first, includes lattice-theoretic characterisations of $\mathbf{Z}(X)$ in two important cases, when X is compact (Theorem 4.1) and when X is an arbitrary topological space (Theorem 5.9). In both cases, the characterisations involve minimal prime ideals. Along the way he proves a result (Theorem 3.1) that very nicely generalises Urysohn's Lemma for normal topological spaces and the fact that, in a completely regular space, disjoint zero-sets can be separated by a continuous function.

Distributive lattices—Priestley duality

About the same time that Speed was writing his PhD thesis at Monash University, H. A. Priestley was writing her DPhil at the University of Oxford. Speed was amongst the first to realise the importance of the new duality for bounded distributive lattices that Priestley established in her thesis (see Priestley [8, 9] and Davey and Priestley [2]).

In *On the order of prime ideals* [13], Speed addresses the question, raised by Chen and Grätzer [1], of characterising *representable* ordered sets, that is, ordered sets that arise as the ordered set of prime ideals of a bounded distributive

lattice. By using Birkhoff's duality between finite distributive lattices and finite ordered sets, he shows that an ordered set is representable if and only if it is the inverse limit of an inverse system of finite ordered sets. Speed observes that, when combined with deep results of Hochster [5], this tells us that an ordered set is isomorphic to the ordered set of prime ideals of a commutative ring with unit if and only if it is isomorphic to an inverse limit of finite ordered sets. This cross fertilisation in Speed's work between commutative rings with unit and bounded distributive lattices will arise again in Section 1.

Soon after writing Speed [13], Speed became aware of Priestley's results. He quickly realised that, since an inverse limit of finite sets is endowed with a natural compact topology, his characterisation of representable ordered sets could be lifted to a characterisation of compact totally order-disconnected spaces, the ordered topological spaces that arise in Priestley duality (and are now referred to simply as *Priestley spaces*). In *Profinite posets* [12], he proved that an ordered topological space is a Priestley space if and only if it is isomorphic, both order theoretically and topologically, to an inverse limit of finite discretely topologised ordered sets.

Baer rings

Speed's PhD thesis was strongly influenced by the seminal paper *Minimal prime ideals in commutative semigroups* [6]. He took ideas from Kist's paper and reinterpreted them in the context of distributive lattices. Speed saw that there was some informal connection between the commutative Baer rings introduced and studied in Kist [6] and Stone lattices, a class of distributive lattices introduced by Grätzer and Schmidt [4]. A commutative ring R is a *Baer ring* if, for every element $a \in R$, the annihilator $\operatorname{ann}(a) := \{ x \in R \mid xa = 0 \}$ is a principal ideal generated by a (necessarily unique) idempotent a^*. A bounded distributive lattice L is a *Stone lattice* if, for every element $a \in L$, the annihilator $\operatorname{ann}(a) := \{ x \in L \mid x \wedge a = 0 \}$ is a principal ideal generated by an element a^*, and in addition the equation $a^* \vee a^{**} = 1$ is satisfied. While quite different looking, the requirements that a^* be an idempotent, in the ring case, and the identity $a^* \vee a^{**} = 1$, in the lattice case, guarantee that the elements a^* form a Boolean algebra and correspond precisely to the direct product factorisations of the ring or lattice.

While the proofs will typically be quite different, it is often true that a result about Baer rings will translate to a corresponding result about Stone lattices and vice versa. For example:

(i) Grätzer [3] proved that Stone lattices form an equational class; Speed and Evans [17] proved that Baer rings also form an equational class. (In both cases, * is added as an additional unary operation.)
(ii) Grätzer and Schmidt [4] proved that, in a Stone lattice, each prime ideal contains a unique minimal prime ideal; Kist [6] proved that precisely the same condition holds in a Baer ring.

In separate papers on Stone lattices [11] and Baer rings [14], Speed proves that there are broad classes of distributive lattices and rings, respectively, within which Stone lattices and Baer rings are characterised by the property that each prime ideal contains a unique minimal prime ideal.

In his third and final paper on Baer rings [15], Speed considers the question of embedding a commutative semiprime ring R into a Baer ring B. Two such embeddings had already been given: the first by Kist [6] and the second by Mewborn [7]. In both cases, the Baer ring B was constructed as a ring of global sections of a sheaf over a Boolean space. Speed shows that, in fact, there is a hierarchy of Baer extensions of R, the smallest being Kist's and the largest Mewborn's. Moreover, he is able to replace the sheaf-theoretic construction with a purely algebraic one similar in nature to one that had been used previously in the theory of lattice-ordered groups. The underlying lattice of a lattice-ordered group is distributive, so again we see Speed's fruitful use of the interplay between rings and distributive lattices.

References

[1] C. C. Chen and G. Grätzer. Stone lattices. II. Structure theorems. *Canad. J. Math.*, 21:895–903, 1969.
[2] B. A. Davey and H. A. Priestley. *Introduction to Lattices and Order*. Cambridge University Press, New York, second edition, 2002.
[3] G. Grätzer. Stone algebras form an equational class. Remarks on lattice theory. III. *J. Aust. Math. Soc.*, 9:308–309, 1969.
[4] G. Grätzer and E. T. Schmidt. On a problem of M. H. Stone. *Acta Math. Acad. Sci. Hungar.*, 8:455–460, 1957.
[5] M. Hochster. Prime ideal structure in commutative rings. *Trans. Amer. Math. Soc.*, 142:43–60, 1969.
[6] J. Kist. Minimal prime ideals in commutative semigroups. *Proc. Lond. Math. Soc. (3)*, 13:31–50, 1963.
[7] A. C. Mewborn. Regular rings and Baer rings. *Math. Z.*, 121:211–219, 1971.
[8] H. A. Priestley. Representation of distributive lattices by means of ordered stone spaces. *Bull. Lond. Math. Soc.*, 2:186–190, 1970.
[9] H. A. Priestley. Ordered topological spaces and the representation of distributive lattices. *Proc. Lond. Math. Soc. (3)*, 24:507–530, 1972.
[10] T. P. Speed. On rings of sets. *J. Aust. Math. Soc.*, 8:723–730, 1968.
[11] T. P. Speed. On Stone lattices. *J. Aust. Math. Soc.*, 9:297–307, 1969.
[12] T. P. Speed. Profinite posets. *Bull. Aust. Math. Soc.*, 6:177–183, 1972.
[13] T. P. Speed. On the order of prime ideals. *Algebra Univers.*, 2:85–87, 1972.
[14] T. P. Speed. A note on commutative Baer rings. *J. Aust. Math. Soc.*, 14:257–263, 1972.
[15] T. P. Speed. A note on commutative Baer rings. III. *J. Aust. Math. Soc.*, 15: 15–21, 1973.

[16] T. P. Speed. On rings of sets. II. Zero-sets. *J. Aust. Math. Soc.*, 16:185–199, 1973. Collection of articles dedicated to the memory of Hanna Neumann, II.
[17] T. P. Speed and M. W. Evans. A note on commutative Baer rings. *J. Aust. Math. Soc.*, 13:1–6, 1972.