

International Cryogenics Monograph Series

Franco Pavese

Gianfranco Molinar Min Beciet

# Modern Gas- Based Temperature and Pressure Measurements

*2nd Edition*



Springer

# Modern Gas-Based Temperature and Pressure Measurements

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## INTERNATIONAL CRYOGENICS MONOGRAPH SERIES

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# Modern Gas-Based Temperature and Pressure Measurements

Second Edition

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# Preface

Since the beginning of the preparation of this book, which is the second edition of a previous book printed in 1992, we have been strongly convinced that temperature and pressure measurements should not be separated, particularly in different applications at low temperatures. This approach has been followed in the preparation of this second edition because advanced applications and modern experimental investigations in science and technology need the combination of various professional experiences, and this is particularly true for the thermodynamic quantities as temperature and pressure.

Although the book is divided in two parts (Part I by Franco Pavese and Part II by Gianfranco Molinar Min Beciet), plus the new Chap. 11 common to the two, we always tried to correlate low temperatures with low-medium pressures as much as possible.

This second edition book has been substantially revised in respect of the first edition, by considering new measurement methods, new systems and devices of the last 20 years. This reflects as well new achievements of metrology in general as the treatment of uncertainty, that is now more stable and well defined, and attention was posed as well to the effects of the BIPM-CIPM 1999 Mutual Recognition Arrangement (MRA) that have produced since 1999 many key comparison results and approval of many calibration and measurement capabilities.

We are pleased to acknowledge our debt to our research group colleagues at the Istituto di Metrologia Gustavo Colonnetti of CNR (IMGC-CNR), merged in 2006 within Istituto Nazionale di Ricerca Metrologica (INRIM) and to many persons at international level to which we have gladly cooperated. In particular, Franco Pavese acknowledges the competent help kindly obtained from INRIM colleague Peter P.M. Steur for the revision of Chaps. 3–5.

However, the persons that we really want to thank are our families; they have been always supporting us and they were able to create around us the “perfect atmosphere” in order to be relaxed and able of working always with great pleasure.

The first edition of our book was dedicated to our wives and sons (Ghita, Carlo and Matteo Pavese; Dida and Daniele Molinar Min Beciet).

This second edition is particularly dedicated to our—present—grand children's (Nicolai, Leonardo, Luca and Viola Pavese; Matteo Molinar Min Beciet) as they are the future of our dreams.

Torino (Italy)

Franco Pavese  
Gianfranco Molinar Min Beciet

# Contents

## Part I Temperature Measurements in the Range from 0.1 to 300 K

<b>1</b>	<b>The Concept of Temperature</b>	3
1.1	Definitions of Temperature	4
1.1.1	The Phenomenological Approach	4
1.1.2	The Axiomatic Approach	10
1.1.3	The Microscopic Approach	13
1.1.4	Negative and Nonequilibrium Thermodynamic Temperatures and for (Fast)-Moving Bodies	16
1.2	Temperature Scales	17
1.2.1	Thermodynamic Temperature	20
1.2.2	Empirical Temperature	24
1.2.3	Official Temperature Scales	29
1.2.4	“Mise en Pratique” for the Definition of the Kelvin	31
1.2.5	Difference Between the Realized Thermodynamic Temperature and the $T_{90}$	38
<b>2</b>	<b>Gas-Based Reference Points for Thermometry</b>	45
2.1	Thermodynamic States Versus Standard Reference Materials	46
2.1.1	Substances Versus Standard Reference Materials	46
2.1.2	Thermodynamic States and Phase Diagrams	48
2.1.3	Reference Points: Triple, Boiling, and Critical Points	50
2.2	Physical Chemistry Associated with Gas Triple Points	50
2.2.1	Triple Point of a Pure Substance	50
2.2.2	Triple Point of an Impure Substance. Cryoscopy	54
2.3	The Realization of Temperature Fixed Points Using Gas Triple Points	76
2.3.1	Basic Problems in the Design of Cells for Accurate Realization of Triple Points	77
2.3.2	Chemical-Physical Problems in the Realization of Triple Points	85
2.3.3	Thermal Errors in Sealed Cells and Related Problems in Measurement.	89



2.4	Modern Design of Temperature Fixed Points	
	Based on the Triple Point of Gases . . . . .	94
2.4.1	The Technique of Sealed Cells for the Realization of a Temperature Reference . . . . .	94
2.4.2	Further Checks Necessary to Achieve a Total Uncertainty of $\approx 50 \mu\text{K}$ in Triple Point Best Realizations . . . . .	101
2.4.3	Performance of Sealed Cell Models . . . . .	109
2.5	Fixed Points Using Other Phase Transitions . . . . .	123
2.5.1	Liquid-To-Liquid Transitions . . . . .	123
2.5.2	Solid-To-Solid Transitions . . . . .	129
2.6	The ITS-90 Between 13.80 and 273.16 K and Scale	
	Approximations Using Sealed Fixed Points . . . . .	131
2.6.1	Realization of the ITS-90 in the Laboratories . . . . .	132
2.6.2	Approximating the ITS-90 . . . . .	135
2.7	Gaseous Standard Reference Materials, and Sealed Cells . . . . .	139
<b>3</b>	<b>Gas Thermometry Between 0.5 and 273.16 K</b> . . . . .	<b>147</b>
3.1	Constant-Volume Gas Thermometry . . . . .	148
3.1.1	Influence of Physical Parameters . . . . .	150
3.1.2	Influence of Technical Parameters . . . . .	160
3.1.3	Gas Thermometers with a Built-in Pressure-Measuring Device . . . . .	172
3.2	Interpolating Constant-Volume Gas Thermometers . . . . .	176
3.2.1	ICVGT Types with Stipulation of the Virial Function . . . . .	179
3.3	Constant-Volume Gas-Thermometer Realizations . . . . .	183
3.3.1	CVGTs with Reference at Temperatures Lower than 273.16 K . . . . .	183
3.3.2	CVGTs with Reference Temperature at 273.16 K . . . . .	191
3.3.3	CVGTs Using a Cryogenic Pressure Measurement . . . . .	193
3.3.4	Realizations of Interpolating Gas Thermometers . . . . .	195
3.3.5	Gas Thermometry in the ITS-90 . . . . .	198
3.4	Dielectric-Constant, Refractive-Index, and Acoustic Gas Thermometers . . . . .	200
3.4.1	Dielectric-Constant Gas Thermometry . . . . .	201
3.4.2	Refractive-Index Gas Thermometry . . . . .	205
3.4.3	Acoustic Gas Thermometry . . . . .	207
3.4.4	The Boltzmann Project . . . . .	214
<b>4</b>	<b>Vapor-Pressure Thermometry</b> . . . . .	<b>221</b>
4.1	Influence of Physical Parameters . . . . .	224
4.1.1	Purity of the Substance . . . . .	224
4.1.2	Effect of the Amount of Thermometric Substance . . . . .	232
4.2	Influence of Technical Parameters . . . . .	234
4.2.1	Thermometer with Gauge for Pressure Measurement at Room Temperature . . . . .	234

4.2.2	Use of Sealed Cells for Vapor-Pressure Measurements . . . . .	239
4.2.3	Thermometer with Gauge for Pressure Measurement at Low Temperature . . . . .	240
4.2.4	Liquid–Vapor Versus Solid–Vapor Equilibria . . . . .	241
4.3	Realization of Vapor-Pressure Temperature Scales . . . . .	242
4.3.1	Equations for Vapor Pressure . . . . .	243
4.3.2	Helium Vapor Pressure . . . . .	246
<b>5</b>	<b>Thermometry Based on the Melting Line of <math>^3\text{He}</math></b> . . . . .	<b>261</b>
5.1	The $^3\text{He}$ Melting-Line Thermometer . . . . .	263
5.1.1	Melting-Line Experiments Below $T_{\min}$ . . . . .	266
5.1.2	Melting-Line Experiments Above $T_{\min}$ . . . . .	267
5.1.3	Pressure and Temperature Measurement . . . . .	267
5.1.4	The $^3\text{He}$ Melting-Line Scale . . . . .	268
<b>6</b>	<b>Cryostats for Thermometry and Gas-Based Temperature Control</b> . . .	<b>269</b>
6.1	Cryostats . . . . .	269
6.1.1	Refrigerator-Based Cryostats . . . . .	270
6.1.2	Liquid-Refrigerant Cryostats . . . . .	274
6.2	Temperature Control in Thermometry and Gas-Based Temperature Controls . . . . .	278
6.2.1	Control of the Cryogen Bath Temperature . . . . .	279
6.2.2	Vapor-Flowrate Control . . . . .	280
6.2.3	Passive Thermostats . . . . .	280
6.2.4	Self-Regulating Passive Shields . . . . .	280
6.2.5	Gas-Filled Heat Switches . . . . .	282
<b>Part II Pressure Measurements in the Range from <math>10^2</math> to <math>10^8</math> Pa</b>		
<b>7</b>	<b>Primary Standards for Pressure Measurements</b> . . . . .	<b>291</b>
7.1	Liquid-Column Manometers for Pressure Measurements . . . . .	298
7.1.1	Operating Principles and General Precautions for Absolute Pressure Measurements Below 0.3 MPa . . . . .	299
7.1.2	Basic Apparatus in Metrological Laboratories: Different Methods for Height Measurements . . . . .	301
7.1.3	Calculations Applicable with Mercury-Column Manometers: Pressure Measurement Uncertainty . . . . .	316
7.1.4	Liquid Columns for Different Applications . . . . .	327
7.2	Gas-Operated Pressure Balances . . . . .	330
7.2.1	Basic Description . . . . .	331
7.2.2	Absolute Pressure Measurements to 5 MPa . . . . .	336
7.2.3	Gauge Pressure Measurements up to 100 MPa . . . . .	351
7.2.4	Uncertainty of Absolute and Gauge Pressure Measurements with Pressure Balances . . . . .	368

7.2.5	Results of Comparison Measurements in Gas Media with the Use of Pressure Balances . . . . .	374
7.2.6	Comparisons of Liquid-Column Manometers and Pressure Balances . . . . .	379
7.2.7	What Future for Primary Pressure Standards? . . . . .	389
7.3	Summary of Typical Uncertainty Levels for Liquid-Column Manometers and Pressure Balances . . . . .	391
<b>8</b>	<b>Pressure Transducers for Gaseous Media . . . . .</b>	<b>393</b>
8.1	Transfer Standards for Absolute Pressure Measurements Below 120 kPa . . . . .	395
8.1.1	Transfer Standards for Atmospheric Pressure Measurements . . . . .	398
8.1.2	Transfer Standards for Gas Pressure Measurements from 100 Pa to 10 kPa . . . . .	403
8.2	Transfer Standards for Pressure Measurements up to 100 MPa . . . . .	406
8.3	Transducers Used for Differential Pressure Measurements and as Null Detectors . . . . .	412
8.3.1	Differential Pressure Transducers for High Line Pressures . . . . .	412
8.3.2	Differential Pressure Transducers Used as Null Detectors . . . . .	416
8.4	Transducers for Pressure Measurements Extended to Cryogenic Environments . . . . .	418
8.4.1	Strain-Gauge Pressure Transducers for Cryogenic Applications . . . . .	422
8.4.2	Resonant- and Capacitance-Type Pressure Transducers for Cryogenic Applications . . . . .	426
8.5	Glossary: Terms Used in the Metrological Characterization of a Pressure Transducer . . . . .	429
8.6	Typical Procedure for the Static Calibration of a Pressure Transducer in Gaseous Media . . . . .	430
<b>9</b>	<b>Gas-Based Pressure Fixed Points . . . . .</b>	<b>433</b>
9.1	The Pressure Scale from $10^2$ to $10^8$ Pa Based on Pressure Fixed Points . . . . .	435
9.1.1	Triple Points . . . . .	436
9.1.2	Other Pressure Fixed Points . . . . .	437
9.2	Experience in the Use of Fixed Points as Pressure Transfer Standards . . . . .	438
9.3	Summary of Possible Fixed Points to be Used as Transfer Standards for Gas Pressure Measurements from 100 Pa to Approximately 100 MPa . . . . .	443
9.4	Appendix: Properties of Substances not Gaseous at Room Temperature of Interest for Realization of Pressure Fixed Point . . . . .	444

<b>10 The Thermomolecular Pressure Difference Effect</b> .....	447
10.1 Introduction .....	447
10.2 The Calculation of the Thermomolecular Pressure Difference .....	448
10.3 Experimental Measurements of the Thermomolecular Pressure Difference Effect .....	456
10.4 General Considerations on Experimental Measurements and Theoretical Calculations of the Thermomolecular Pressure Difference .....	461
<b>11 The Mutual Recognition Arrangement and Its Implementation in     Temperature and Pressure</b> .....	463
11.1 Introduction .....	463
11.2 The Key Comparisons .....	464
11.2.1 Key Comparisons Related to Gas Pressure Measurements in the Range from 100 Pa to 100 MPa .....	465
11.2.2 Key Comparisons Related to Temperature Measurements Extending from 273.16 K Downward .....	479
11.3 The Calibration and Measurement Capabilities .....	481
11.3.1 Calibration and Measurement Capabilities for Gas Pressure Measurements in the Range from 100 Pa to 100 MPa .....	481
11.3.2 Calibration and Measurement Capabilities (CMC) for Cryogenic Temperature Measurements .....	491
11.4 The BIPM Key Comparison Database (KCDB) .....	491
<b>Appendix A The International Temperature Scale of 1990</b> .....	495
<b>Appendix B List of Temperature and Pressure Fixed Points     (Table B.1, B.2, and B.3)</b> .....	511
<b>Appendix C Reference Data on Gases</b> .....	515
<b>Appendix D Vapor Pressure Equations</b> .....	541
<b>Appendix E Reference Data for Liquid-Column Manometers     (Tables E.1, E.2, E.3, E.4)</b> .....	549
<b>Appendix F Reference Data for Pressure Balances</b> .....	555
<b>Appendix G</b> .....	561
<b>Appendix H General Terminology in Measurements</b> .....	575
<b>References</b> .....	581
<b>Index</b> .....	627

# Introduction

The use of substances that are gaseous at room temperature for temperature measurements and for standards realization is traditional in the cryogenic field. Vapor-pressure thermometry and gas thermometry have been since a long time ago the commonest nonelectrical methods for temperature measurements in physics and chemistry. Correspondingly, specific primary and secondary standards have been developed to measure with improved accuracy pressure in gaseous media.

Most of the studies on thermophysical properties of these substances were carried out in the first half of the twentieth century, but work, though with less momentum, progressed especially at NIST (formerly NBS) in the USA and in Russia also in the second half. As far as its use in metrology is concerned, the adoption of the IPTS-68 in 1968 stimulated a new activity both intensive and extensive, as some of these properties form the basis of low-temperature thermometry, which led to the adoption of the new International Temperature Scale which came into effect on 1 January 1990 (ITS-90). Now, after more than 20 years that this scale is adopted, revisions have started. However, with the shift in 2006 of the meaning of “temperature scale” in the definition of the unit kelvin, caused by the introduction of the concept of “*mise en pratique*” of the kelvin, adjustments are now possible without having to promulgate a new ITS-xx.

This activity resulted in a sizeable upgrading of the accuracy in the determination of the gas properties relevant to temperature standards and on standards traceability to the thermodynamic temperature. New techniques were also developed, which greatly improved the reliability of fixed point realization; the possibility of simplifying the use of the existing standards and of adopting new gas-based standards was explored and it is now extensively exploited.

This book is intended to collect up-to-date information on the latest developments in thermometry and manometry that involve the use of gaseous substances and that are likely to be valid methods also in the future. At present, this information is dispersed in a large number of papers published in international journals and most of it is probably available only to a limited number of specialists. While other books on thermometry deal, in a comparable number of pages, with the *whole* range of temperatures and techniques, the part of the present work devoted to thermometry intends, in the first place, to introduce selected methods, leaving the general description of thermometry

to textbooks. Secondly, being limited to low-temperature and gas-based techniques, the present book intends to supply the reader with information about the very tools for their implementation. Instead of the usual “Problems”, a synopsis of “Solutions” to problems of thermometry implementation is therefore added at the end of each chapter.

As regards to manometry and pressure measurements in general, this book fills a gap in the international literature, as no other recent book provides a comprehensive survey of methods for pressure measurements in gaseous media used in the medium-to-low pressure range closely connected with thermometry.

Although the two parts of the book on temperature and on pressure measurements both give special attentions to future-oriented techniques, their approach to deal with the subject is very different. Part I deals with thermometric techniques for which, apart few recent exceptions, no commercial devices are available: individual users must directly implement these types of thermometers. Consequently, most of the information collected is intended to help them to select the best design, from both standpoints of simplicity and accuracy, and to be self-sufficient to supply all data necessary for their implementation. On the contrary, for most of the pressure-measuring techniques dealt with in Part II, commercially apparatuses, particularly in the case of modern pressure balances and pressure transducers are available. Accordingly, users can find the basic description of such instruments and all the data necessary for appropriate criteria of selection, in view especially of their use at the best possible accuracy for thermometry and manometry applications.

The methods and the instruments dealt with, which allow medium-to-high temperature and pressure accuracy to be achieved, are *not* intended only for applications which need the top measurement accuracy of interest for standard laboratories (though the error analysis is always pushed to this level), but they can be used in a broader range of applications. However, the book does not include methods or instruments intrinsically limited to low accuracy. This second edition book was revised according to different advances made in the last 20 years in metrology, particularly to give evidence of the important role that the CIPM-MRA have assumed since its starting in 1999.

In Part I, basic concepts of temperature and temperature scale are first introduced together with a short review of the different temperature definitions, so that the reader may be made aware of the difficulties involved in defining temperature, especially when it becomes lower and lower.

The use of well-specified thermodynamic states of condensed gases as temperature fixed points (within the temperature range of 2.2–220 K) are then illustrated, and the most effective method for their realization, the sealed-cell method, is fully described, also concerning the further improvement obtained from year 2000 on thermal issues and on the effect of isotopic composition of the substances used.

In the subsequent chapters, thermometric methods exploiting a pressure-temperature relationship are described. For the gaseous state and for the range 1–300 K, the different types of gas thermometry are discussed, with special emphasis being given to the constant-volume type, not only as an absolute thermometer, but also as an interpolating thermometer (as required by the ITS-90, but in a broader

temperature range) and as a simple and practical self-contained device. Also, the more recent progress in acoustic thermometry is extensively illustrated.

In connection with condensed gases, vapor-pressure thermometry is described for the helium-isotopes in the range 0.3–5 K and for its implementation with other gases in the range up to 300 K; special attention is given to simplified realizations using sealed devices.

The  $^3\text{He}$  melting-curve thermometry is then introduced as the official, accurate temperature-measuring means below the present range of the ITS-90.

The last chapter of Part I offers a survey of the hardware specifically required for the implementation of these thermometry's and it considers in particular the modern use of closed-cycle refrigerators above 4 K and uses of gases in temperature control.

In Part II, gas pressure measurements are considered in the range from 100 Pa to 100 MPa, in connection with the former applications to thermometry. Modern primary standards for accurate pressure measurements of gaseous media are first reviewed with a detailed and comprehensive description of their best use.

Liquid-column manometers are described for absolute, gauge, and differential pressure measurements in the range from few pascal to less than 0.3 MPa. Subsequently, modern gas-operated pressure balances are extensively discussed for absolute pressure measurements up to about 5 MPa, relative pressure measurements up to 100 MPa and differential pressure measurements.

Liquid manometers and pressure balances will be particularly described analyzing each physical quantity affecting pressure measurement uncertainty.

A survey is made of pressure transducers, particularly of those used for differential measurements and others that can directly be employed in a cryogenic environment. Problems involved in the assessment of their metrological characteristics, mostly stability with time and thermal cycling, are discussed connected with their use as transfer standards. Typical procedures to be used for a correct data acquisition and calibration of significant parameters of pressure transducers are given.

The gas-based fixed points (triple points, critical points,) available in the considered pressure range are reviewed from the standpoint of their use as transfer standards for interlaboratory comparisons.

Physical quantities and phenomena that affect pressure measurements are thoroughly discussed, as they must be taken into account to obtain top accuracy when using primary standards. In this context, special attention is devoted to a specific and controversial problem of cryogenic measurements: the thermomolecular pressure-difference effect.

The last chapter of this book, common to both Part I and II, is new and deals with the CIPM-MRA, putting into evidence the effort that NMIs made to realize pressure and temperature key comparisons, that are shortly reviewed, and the preparation of calibration and measurements capabilities (CMC) available to users in many application fields. Full text of CIPM-MRA is given in Appendix G.

Appendix A introduces the commented text of the International Temperature Scale of 1990 (ITS-90) below 273.15 K, while the modifications contained in the Technical Annex to the *mise en pratique* are reported and commented in the text. Its implementation, which always requires gas-based thermometry below 0 °C, is deeply discussed in Part I.

Reference data are extensively supplied too in Appendices B, C, D, E, F, G and H. They include: a comprehensive list of temperature (with values in ITS-90) and pressure fixed points; relevant thermophysical data and advices for their specific use in thermometric and manometry fields, given in the form of data sheets for each of 15 substances commonly used in manometry and thermometry; tables for the main manometry and pressure balances corrections (according to ITS-90) with specific examples of uncertainty evaluation; the text of the MRA; general terminology in measurements.

Finally, an extensive Bibliography is provided covering all the subjects dealt with, and including a “Further Readings” section for the main topics.



# List of Acronyms

AGT	Acoustic gas thermometer
ASMW	Amt für Standardisierung, Messwesen und Warenprüfung (after 1990 merged in PTB) (G)
BCR	Bureau Communautaire de Référence of the European Union (no more existing)
BIPM	Bureau International des Poids et Mesures
CCM	Comité Consultatif pour la Masse et les Grandeurs apparentées
CCT	Comité Consultatif de Thermométrie
CGPM	Conférence Générale des Poids et Mesures
CIAME	Commission Interministerielle des appareils électriques et électroniques de mesure (F)
CIPM	Comité International des Poids et Mesures
CSIRO	Commonwealth Scientific Industrial Research Organization (AU)
CVGT	Constant volume gas thermometer
DCGT	Dielectric constant gas thermometer
GUM	Guide to the expression of uncertainty in measurement (BIPM)
EU	European Union
ICVGT	Interpolating constant volume gas thermometer
IEC	International Electrotechnical Commission
IEN	Istituto Elettrotecnico Nazionale “G. Ferraris” (since 2006 merged with IMGC in INRIM)
IMGC	Istituto di Metrologia “G. Colonnetti” of the Italian National Research Council (since 2006 merged with IEN into INRIM)
INRIM	Istituto Nazionale di Ricerca Metrologica
INTiBS	Instytut Niskich Temperatur i Badan Strukturalnych (PL)
IPRT	Industrial platinum resistance thermometer
IPTS-48	International Practical Temperature Scale of 1948
IPTS-68	International Practical Temperature Scale of 1968
ISA	Instrument Society of America
ISO	International Standards Organization
ITS-90	International Temperature Scale of 1990
IUPAC	International Union of Pure and Applied Chemistry

IUPAP	International Union of Pure and Applied Physics
KCDB	Key Comparison Data Base
KOL	Kamerling Onnes Laboratorium (NL)
KRISI	Korean Research Institute of Standards and Science (ROC)
LCM	Laboratoire Commun de Métrologie LNE/CNAM (F) (formerly Institut National de Métrologie INM/BNM)
LNE	Laboratoire National d'Essais (F)
MRA	Mutual Recognition Agreement (BIPM)
n.b.p.	Normal boiling point (nbp as subscript)
NIM	National Institute of Metrology (CN)
NIST	National Institute of Standards and Technology (formerly NBS) (USA)
NMI	National Metrology Institute
NMIJ	National Institute of Metrology of Japan (AIST, J)
NML	National Measurement Laboratory (also DAP, Division of Applied Physics, of CSIRO), now National Measurement Institute (NMI) (AU)
NPL	National Physical Laboratory (UK)
NPL-I	National Physical Laboratory (India)
NRC	National Research Council (CA)
NRLM/AIST	National Research Laboratory of Metrology (formerly NRLM) (J)
OIML	Organization Internationale de Métrologie Légale
PLTS-2000	Provisional Low Temperature Scale of year 2000
PTB	Physikalische-Technische Bundesanstalt (D)
RIGT	Refractive index gas thermometer
RG	Residual gas analyzer
RM	Reference Material
SLAP	Standard Light Antarctic Precipitation (Water standard)
SPRT	Standard platinum resistance thermometer
SRM	Standard Reference Material
s.s.t.	Solid-to-solid transition (sst as subscript)
$T_{58}$	Former $^4\text{He}$ vapor-pressure Scale
$T_{62}$	Former $^3\text{He}$ vapor-pressure Scale
$T_{68}, t_{68}$	Kelvin and Celsius temperature in IPTS-68
$T_{76}$	Kelvin temperature in EPT-76
$T_{90}, t_{90}$	Kelvin and Celsius temperature in ITS-90
TC	Thermocouple thermometry
t.p.	Triple point (tp as subscript)
ULT-2000	Ultra-Low Temperature Scale of 2000
VIM	International Vocabulary of Basic and General Terms in Metrology
VNIIFTRI (also PRMI)	State Institute for Physicotechnical and Radiotechnical Measurements (Russia, formerly USSR)
VSL	Van Swinden Laboratory (NL)
V-SMOW	Vienna Standard Mean Ocean Water
WGx	Working Group x of CCT

# List of Symbols

Symbols are in order of appearance and by chapter. Some symbols differ from chapter to chapter.

In general,  $u$  is used here to indicate the standard uncertainty (also known as the  $1\sigma$  or  $s$  uncertainty) and  $U = ku$  the expanded uncertainty ( $k \approx 2$ ). The symbol  $u$  is equivalent to  $u_c$  in the GUM notation.

## Chapter 1

$Q$	Amount of heat (energy) (J, joule)
$\dot{Q}$	Heat rate (or “heating”), such as $\delta Q = c \times \delta T$ (W, watt)
$c, c_p, c_v$	Specific heats
$q$	Heat flow direction, defined as $q = -k \times \text{grad } \theta$
$\lambda$	Thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ )
$k$	Boltzmann constant = $1.380\,6488(13) \times 10^{-23}$ ( $\text{J K}^{-1}$ ) (CODATA 2010)
$\theta$	Empirical temperature $\theta = kT$
$T$	Thermodynamic temperature
$T_{emp} (T_{90})$	Empirical International temperature: International Scale temperature
$\Theta, \Theta^*$	Thompson (Lord Kelvin) temperatures
$\tau$	Norm of a (statistical) distribution, kinetic temperature
$t$	Celsius temperature ( $^{\circ}\text{C}$ , degree Celsius)
$t$	Celsius-like temperature
$t$	Time (s, second)
$p$	(Static) pressure (Pa, pascal)
$V$	Volume ( $\text{m}^3$ , cubic meter)
$v$	Velocity ( $\text{m s}^{-1}$ )
$W$	(External) mechanical work (J)
$S$	Entropy ( $\text{J K}^{-1}$ )
$\eta$	Efficiency of a cycle
$\mu$	Carnot function ( $\text{K}^{-1}$ )

$\mu$	Integrating factor of the Pfaffian
$J$	Mechanical equivalent of heat
$m_B$	Mass of substance B (kg, kilogram)
$M_{r,B}$	Relative molecular mass
$\phi$	Phase of a substance
$U$	Internal energy of a system (J)
$E$	(Kinetic) energy (J)
$N$	Number of molecules
$\alpha$	Cubic (thermal) expansion coefficient ( $K^{-1}$ )
$P$	Physical quantity
$n_B$	Amount of substance B, defined as $n_B/L$ (mol)
$L$	Avogadro constant (also indicated $N_A$ ) = $6.022\,141\,29(27) \times 10^{23} \text{ mol}^{-1}$ (CODATA, 2010)
$R$	Molar gas constant = $8.314\,4621(75) \text{ J K}^{-1} \text{ mol}^{-1}$ (CODATA, 2010)
$\chi$	Magnetic susceptibility, defined as $\chi = \mu/\mu_0 - 1$ , where $\mu_0 = 4\pi \cdot 10^{-7} \text{ H m}^{-1}$ exactly (henry per meter)

## Chapter 2

$T_r, p_r$	Room temperature, room pressure
$T_{tp}, p_{tp}$	Triple point temperature, triple point pressure (of an impure substance)
$T_{LP}$	Liquidus-point temperature at the triple point
$T_c, p_c$	Critical temperature, critical pressure
$\rho$	Density ( $\text{kg m}^3$ )
$T_{tp}^*$	Triple point temperature of the pure substance
$\Delta_{\text{fus}}H_m$	Molar enthalpy change in fusion ( $\text{J mol}^{-1}$ )
$x$	Amount of substance fraction
$F$	Melted fraction
$A$	First cryoscopic constant, defined as $A = 1/K T_m^2$ ( $K^{-1}$ ), where $K = R/\Delta_{\text{fus}}H_m$
$E_f, K_f$	First cryoscopic constant, in the literature $E_f$ or $K_f = 1/A$
$\Delta_{\text{fus}}T_{tp}$	Melting range (K)
$\mu$	Chemical potential ( $\text{J mol}^{-1}$ )
$X$	“Driving capability” of sealed-cell ( $\text{kg K mol}^{-1}$ )
$T_\lambda$	$^4\text{He}$ lambda point temperature
$\Delta_{\text{vap}}H_m$	Molar enthalpy of vaporization

## Chapter 3

$B(T)$	Second virial coefficient ( $\text{m}^3 \text{ mol}^{-1}$ )
$C(T)$	Third virial coefficient ( $\text{m}^6 \text{ mol}^{-2}$ )

$T_b, p_b, n_b$	Values of $T$ , $p$ or $n$ for the gas thermometer bulb
$N_d, V_d$	Amount of substance and volume for the dead volume, including $n_r, V_r$ in the room temperature dead volume, and $n_c, V_c$ for the capillary tube
$K$	Adsorption coefficient or factor
$\tau$	Adsorbed film thickness (measured in number of layers)
$d$	Capillary diameter
$h$	Capillary axial coordinate
$H$	(Portion of the) capillary length
$p^*$	Pressure value measured at the manometer (or room-temperature diaphragm) level
$\Delta p_{\text{aer}}$	Aerostatic pressure head
$\Delta p_{\text{TM}}$	Thermomolecular pressure difference
$K$	Compression modulus (Pa)
$\alpha_1$	Linear (thermal) expansion coefficient ( $\text{K}^{-1}$ )
$\alpha$	Atomic polarizability, where $\alpha_0 =$ in vacuum
$\Delta V/V_0$	Volume relative change
$\delta p$	Pressure error due to manometer uncertainty
$c$	Velocity of sound ( $\text{m s}^{-1}$ , meter per second), where in vacuo $c_0 = 299\,792\,458 \text{ m s}^{-1}$ exactly
$\varepsilon$	Relative permittivity (dielectric constant), where in vacuo $\varepsilon_0 = 1/\mu_0 c_0^2$
$A_\varepsilon$	Molar polarizability
$b, c$	Dielectric virial coefficients
$C$	Electrical capacitance (F, farad)
$\gamma$	Relative capacitance change
$n$	Refractive index, defined as $n = c_0/c$
$E_g$	Adiabatic bulk modulus of the gas (Pa)
$E$	Young's modulus of elasticity (Pa)
$g$	Acceleration due to gravity ( $\text{m s}^{-2}$ )

## Chapter 4

$p_i$	Partial pressure
$p_s^0$	Pressure of the pure solvent
$K_H$	Henry's law constant
$n^L, m^L, V^L$	Quantity of substance, mass and volume of the liquid phase
$n^V, m^V, V^V$	Same as the vapor phase
$n_b, m_b, V_b$	Same in the bulb
$n_r, m_r, V_r$	Same in the room temperature volume
$n_c, m_c, V_c$	Same in the connecting tube
$V_T$	Volume of the room-temperature filling reservoir
$p_f$	Condensation pressure at thermometer filling temperature $T_f$
$H$	Magnetic field strength ( $\text{A m}^{-1}$ , ampere per metre)
$s^L$	Surface area of the liquid phase

$\dot{n}$	Molar flow rate of refluxing film ( $\text{mol s}^{-1}$ )
$h_f$	Elevation of the superfluid film above the liquid surface
$\Delta T_K$	Temperature difference caused by the Kapitza effect

## Chapter 5

$T_{\min}, p_{\min}$	Temperature and pressure at the point of minimum of the melting curve
$T_A$	Temperature of the second-order transition from normal liquid to superfluid A phase
$T_B$	Temperature of the first-order transition from superfluid A and B phases
$T_S$	Temperature of the first-order transition in solid $^3\text{He}$ to the ferromagnetically ordered phase
$p_m$	Melting pressure
$\eta$	Dynamic viscosity (Pa s, pascal per second)

## Chapter 6

$X_x$	Gap in a gas-filled heat switch
$L$	Mean free path of gas molecules, where $\Lambda_0$ is at high temperature
$K_n$	Knudsen number
$C$	Conductance in the molecular regime ( $\text{L s}^{-1}$ , liter per second)

## Chapter 7

$p$	Pressure
$p_0$	Vacuum reference pressure
$p_{\text{atm}}$	Atmospheric pressure
$p_v$	Mercury vapor pressure
$p_j$	Operating value of the jacket pressure in a controlled clearance piston-cylinder unit
$p_z$	Jacket pressure value for zero clearance
$\Delta_p$	Differential pressure
$p_L$	Line pressure
$t$	Temperature expressed in $^{\circ}\text{C}$
$T$	Temperature expressed in K
$t_{\text{ref}}$	Reference temperature (generally $20^{\circ}\text{C}$ )
$\rho_f(t, p)$	Density of a fluid
$\rho_o$	Mercury density at $20^{\circ}\text{C}$ and 101 325 Pa
$\rho(t, p_{\text{atm}})$	Mercury density at temperature $t$ and pressure $p_{\text{atm}} = 101\,325\text{ Pa}$

$\rho(t, (p + p_0)/2)$ or $\rho_{\text{Hg}}(t, (p + p_0)/2)$	Mercury density at temperature $t$ and at average pressure between $p$ and $p_0$
$\rho_{\text{gas}}(t, p)$	Density of a gas at temperature $t$ and pressure $p$ (e.g., for nitrogen it will be $\rho_{\text{N}_2}(t, p)$ )
$\rho_{\text{a}}$	Air density
$\gamma(t)$	Surface tension of a fluid at temperature $t$
$\eta(t, p)$	Dynamic viscosity of a fluid at temperature $t$ and pressure $p$
$g_{\text{L}}$	Local acceleration of gravity
$h$	Difference in height between liquids (case of liquid columns), or distance of a reference level from a fixed position (case of pressure balances)
$h$	Relative humidity ( $h = 0.5$ represents a rate of relative humidity of 50 %)
$H$	Helmoltz thermodynamic free energy
$U$	Energy
$S$	Entropy
$V$ and $V_0$	Volumes
$K_{\text{T}}$	Isothermal compressibility
$K_{\text{S}}$	Adiabatic compressibility
$n(t, p)$	Refractive index (gaseous media must be specified)
$N$	Number of interferometric fringes
$\lambda$	Wavelength of a light source in a gas media
$\lambda_{\text{v}}$	Wavelength of a light source in vacuum
$C_{\text{i}}$	Refractive index correction
$c$	Speed of sound in mercury
$L$	Length
$l$ or $l_0$	Engagement length of a piston in a cylinder
$K_{\text{Hg}}$	Mercury compressibility
$M$	Molar mass of a gas
$R$	Molar gas constant ( $R = 8.314\,4621(75) \text{ J mol}^{-1} \text{ K}^{-1}$ ) (CODATA, 2010)
$x_{\text{v}}$	Mole fraction of water vapor
$x_{\text{CO}_2}$	Mole fraction of carbon dioxide
$Z$	Compressibility factor for moist air (see Table F.4)
$m_i$	Mass of the $i$ th weight piece referred to its density $\rho_{mi}$
$\rho_{mi}$	Density of the $i$ th weight
$m_{\text{icon}}$	Conventional mass value referred to the conventional density $\rho_{m_{\text{icon}}}$
$\rho_{m_{\text{icon}}}$	Conventional density ( $8000 \text{ kg m}^{-3}$ )
$F$	Gravitational force
$A_0(t_{\text{ref}}, p_{\text{atm}})$	Effective area of a piston-cylinder unit measured at the reference temperature $t_{\text{ref}}$ and at atmospheric pressure $p_{\text{atm}}$
$\lambda$	Pressure distortion coefficient of a piston-cylinder unit (case of pressure balances)
$\lambda_{\text{p}}$	Pressure distortion coefficient of the piston (case of pressure balances)

$\alpha_p$ and $\alpha_c$	Linear expansion coefficients of a piston and cylinder respectively
$A_e(t, p)$	Effective area of a piston-cylinder unit at temperature $t$ and pressure $p$
$h_0$	Radial clearance for a piston-cylinder unit at the position $x = 0$
$h(x)$	Radial clearance for a piston-cylinder unit at the position $x$ ( $x$ can change between 0 and $L$ )
$u(x)$	Piston deviations from the ideal reference radius
$U(x)$	Cylinder deviations from the ideal reference radius
$U_r$	Radial displacements (generally on the outer surface of a cylinder)
$\nu_p, \nu_c$	Poisson coefficients for piston and cylinder respectively
$E_p, E_c$	Elasticity moduli (Young) for piston and cylinder respectively
$r_0$	Piston radius at the position $x = 0$
$r_c$	Average inner radius of the cylinder
$r_p$	Average radius of the piston
$R_c$	Average outer radius of the cylinder
$A_p(t, p_{atm})$	Effective area of the piston at temperature $t$ and atmospheric pressure
$d$	Jacket pressure coefficient in a controlled clearance piston-cylinder unit
$A_e(t, p, p_j)$	Effective area of a controlled clearance piston-cylinder unit at pressure $p$ , temperature $t$ and jacket pressure $p_j$
$\Delta m$ or $\Delta m_a$	Added mass on a pressure balance during differential pressure calibration, typically at a high line pressure
$\sigma^2$	Estimated variance
$\sigma$	Estimated standard deviation (it can be related to a fitting or to a number of repeated measurements of a physical quantity, or to the combination of different $\sigma$ 's for quantities related between them by a mathematical function)
$p_{sv}(t)$	Saturation vapor pressure of water (see Table F.3.)
$f(p, t)$	Enhancement factor (see Table F.2.)
$G$	Shear modulus

## Chapter 8

Symbols previously reported may be used in the present chapter, but are not all listed here.

$A_e(t, p)$	Effective area of a piston-cylinder unit at temperature $t$ and pressure $p$
$F$	Force due to the application of a pressure $p$ to a piston-cylinder unit of effective area $A_e$ (in first approximation $F = p \times A_e$ )
$p$	Pressure
$p_L$	Line pressure
$\Delta p$	Differential pressure
$p_{tr.}$	Pressure reading of a transducer
$p_{pr. std.}$	Pressure value measured by a primary standard



$p_{\text{pr. balance}}$ OR $p_{\text{p. gauge}}$	Pressure value measured by a pressure balance
$(p_{\text{tr.}} - p_{\text{pr. std.}})$	Pressure calibration difference
$(p_{\text{pr. std.}} - p_{\text{tr.}})$	Pressure calibration correction
$t$	Temperature expressed in °C
$T$	Temperature expressed in K
$T$	(Tesla), SI unit of magnetic flux density
$n$	Number of measurements
$x_i$	Result of the $i$ th measurement of the physical quantity $x$ ( $i$ variable from 1 to $n$ )
$\bar{x}$	Arithmetic mean of the $n$ measurement results
$s$	Experimental standard deviation
$p = f(X)$	Interpolating equation of a transducer
$X$	Output signal of a transducer, frequently corrected for its zero output value
$GF$	Gauge factor ( $GF = dR/R$ for a strain gauge transducer)
$\varepsilon$	Strain
$C$	Electrical capacitance
f.s.	Full scale
LVDT	Linear variable differential transformer
$R$	Electrical resistance

## Chapter 9

Symbols previously reported may be used in the present chapter, but are not all listed here.

$p$	Pressure
$t$	Temperature in °C
$T$	Temperature in K
t.p.	Triple point
$p_{\text{tp}}$	Triple point pressure
$T_{\text{tp}}$	Triple point temperature
v.p.	Vapor pressure
c.p.	Critical point
m.l.	Melting line
$T_{\text{tp, Hg}}$	Mercury triple point (234.3156 K of ITS-90)
$d$	$= T - T_{\text{tp, Hg}}$
$F$	Melted fraction ( $1/F = 1$ is equivalent to 100% melted, i.e., liquid point)
$R$	Electrical resistance

## Chapter 10

Symbols previously reported may be used in the present chapter, but are not all listed here.

$p_c$	Cold pressure
$p_w$	Warm pressure
$p_0$	$= (p_c + p_w)/2$
$T_c$	Cold temperature
$T_w$	Warm temperature
$T_0$	$= (T_c + T_w)/2$
$d$	Tube diameter
$r$	Tube radius
$L$	Tube length
$\lambda$	Mean free path of gas molecules
$\eta(T, p)$	Gas viscosity at temperature $T$ and pressure $p$
$\eta_0(T_0, p)$	Gas viscosity at temperature $T_0$ and pressure $p$
$M$	Molar mass of a gas
$R$	Molar gas constant, $R = 8.314\,4621\text{ J mol}^{-1}\text{ K}^{-1}$
$R'$	$= (T_w/T_c)^{1/2}$
$R''$	$= P_w/p_c$
$D$	Collisional diameter of a gas molecule
$\Phi_g$	Pressure-shifting factor
$f$	Momentum accommodation coefficient
$\gamma$	Geometry factor
$N_G$	Number of gas–gas collisions
$N_W$	Number of gas–surface collisions
$\Delta_p$	Thermo molecular pressure correction calculated on the basis of Weber–Schmidt equation
$\delta(\Delta p)$	Difference between the Weber–Schmidt equation and experimental values of the thermo molecular pressure correction

## Appendix A

$T_{90}, t_{90}$	Kelvin and Celsius temperature for ITS-90 scale
$T_{68}, t_{68}$	Kelvin and Celsius temperature for IPTS-68 scale
$T_{76}$	Kelvin temperature for EPT-76 scale
$W(T_{90})$	Ratio $R(T_{90})/R(273.16\text{ K})$
$W_r(T_{90})$	ITS-90 reference function

Symbols used specifically in particular areas of application or used to define constants or calculation parameters are explained in the text.

## Notes to the Reader

1. All temperature data are referred to the new International Temperature Scale 1990 (ITS-90). This is correct except when otherwise stated, because in some cases there are real needs to express some relevant data with reference to the former IPTS-68 temperature scale.
2. All the uncertainties are declared at the one sigma level ( $u_c$  in the GUM notation), except when otherwise stated because sometimes we are reporting data from literature where uncertainties either are not declared or it is impossible to recalculate the one sigma level uncertainty. The expanded uncertainty  $U = ku_c$  ( $k \approx 2$ ) is also used.
3. All notations are made according to IUPAC recommendations (IUPAC Green Book, third edition, 2009–2010). In some cases, however, American rather than International spelling has been used (e.g., meter rather than metre and liter rather than litre).

# Part I

## Temperature Measurements in the Range from 0.1 to 300 K

### Introduction

In Part I, modern methods which are based on the use of substances gaseous at room temperature, for measuring temperatures lower than 0 °C, are described. The lower limit of the temperature range where these substances can be used is arbitrarily set at  $\approx 0.1$  K, but  $^3\text{He}$  melting-curve thermometry, described in Chap. 5, extends down to  $\approx 0.001$  K.

Figure 1.7 at the end of Chap. 1 shows the typical range for each of these gas-based types of thermometry. The present state-of-the-art allows a top measurement accuracy better than  $\pm 100$   $\mu\text{K}$  for all of them. A recent short digest of the advances in cryogenic thermometry in the last 50 years can be found in Pavese (2006).

Each of the fixed points described in Chap. 2 realizes a single temperature value. Gas thermometry, described in its various forms below 0 °C in Chap. 3, can be used as well above room temperature. Vapor pressure thermometry too, described in Chap. 4, can be used above room temperature: each substance spans only a narrow interval of the whole temperature range, and in some intervals no substance is available.

Finally, Chap. 6 describes thermostats that are used for performing all these thermometric measurements and temperature controls, and devices that are based on the use of gases.

# Chapter 1

## The Concept of Temperature

This monograph is intended for the use of low-temperature experimentalists, as well as those individuals interested in one or more aspects of thermometry. The concept of temperature, therefore, will only be given a brief introduction and review in this section. For a more complete treatment, the reader is directed to the textbooks listed in the section “Further Readings Part I” after the References.

However, an introduction of the concept of temperature seems desirable for two reasons, one general and one specific. In general, each course or textbook presenting an introduction to thermodynamics or thermometry makes the choice of introducing only one of the several methods of defining temperature. As a consequence, the student or the reader obtains an oversimplified impression of the problems involved with this basic physical quantity, missing some of its more subtle features and developing little sensitivity in dealing experimentally with the associated problems. We will, therefore, provide a brief account of several different approaches to its definition, in order to stimulate further understanding and study.

The range of temperatures considered in cryogenics appears quite small, a few hundred kelvins, when compared with the millions of degrees required to attain the plasma region. However, absolute zero in nature can only be approached asymptotically. Therefore, a logarithmic scale of temperature values is a more realistic way to portray the temperature scale (Fig. 1.1). Since absolute zero is approached as infinity, it becomes similar to the upper boundary of the temperature scale. In the very low temperature range, conditions far away from human experience occur. Temperatures far below the minimum existing in nature (background cosmic radiation, 2.75 K) are commonly attained in laboratories, where equilibrium temperature values may be different for either the lattice or the sublattice (e.g., spin) populations and specific temperature gradients may occur during heat transfer (Kapitza conductance). These conditions directly affect the capability of measuring temperature. Since this text is concerned with temperature measurements based on gases, the concept of the “ideal gas” will be introduced, and before dealing with “the real thing,” the limits and limitations of this model relative to the definition of temperature will be elucidated.

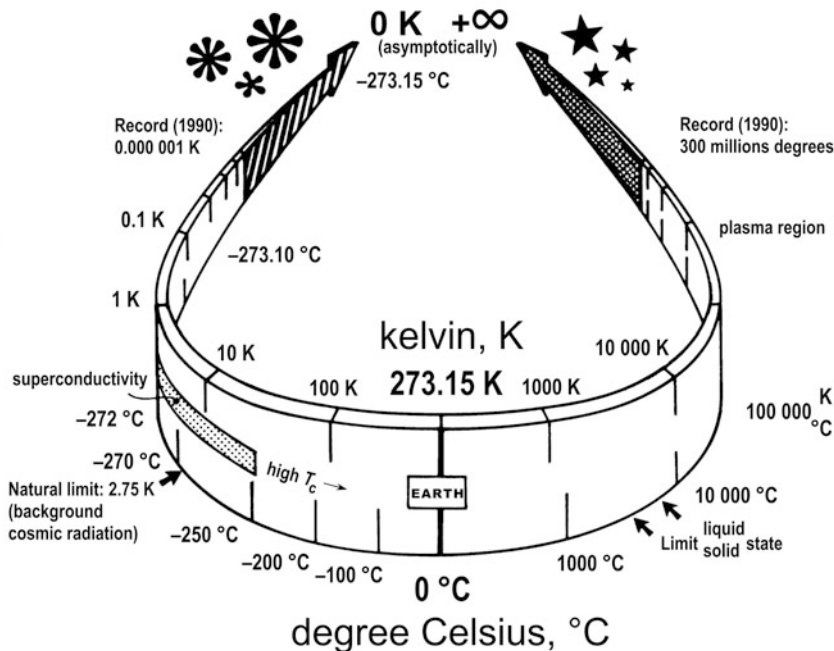


Fig. 1.1 The logarithmic natural scale of temperatures

## 1.1 Definitions of Temperature

It has long been recognized that people—including scientists and teachers—are divided into two immiscible clusters: “les esprits de justesse” and “les esprits de géométrie” (Pascal 1670). Each of them believes that there is only one approach for explaining the concept of temperature. They follow approaches that are quite different, although consistent. We call the former the *phenomenological approach*, the latter the *axiomatic approach*. A short account will also be given of a third approach, the *microscopic approach*, which is based on the structure of matter.

### 1.1.1 The Phenomenological Approach

In history, the phenomenological approach came first, but its introduction has not been very straightforward. In fact, the concept of temperature was not separated from the concept of heat until the middle of nineteenth century. This occurred shortly after heat was recognized as not being a substance—the caloric—but energy (a recent concept too). In this respect, Joule’s experiment first published in 1845, is traditionally considered crucial. It recognized the relationship between heat and