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K. Ramasubramanian
and M. S. Sriram

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ISBN 978-0-85729-035-9 e-ISBN 978-0-85729-036-6
DOI 10.1007/978-0-85729-036-6
Springer London Dordrecht Heidelberg New York

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Control Number: 2010932514

Mathematics Classification Number: 01A32, 85-03

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

समर्पणम्

नानायुक्त्यात्तशोभं न मितमनधिकं तन्त्रजालाभिनीतं
ग्रासच्छायादिकार्ये बुधभृगुविषये युक्तमार्गं दिशन्तम्।
प्राणैषीत् ग्रन्थरत्नं गणितविदमलो तन्त्रसङ्क्षिप्तिकाख्यं
गार्ग्यः श्रीनीलकण्ठः कृतबहुलकृतिर्भाग्यमेतद्बुधानाम्॥

व्याख्यालेख्यविशेषवृत्तिभरितं वो धीजुषां सम्मुदे
प्राकाश्यन्ननु तन्त्रसङ्ग्रहममुं प्रीत्या नयामोऽधुना।
श्रीमन्माधवकेरलीयगणितज्योतिर्विदां संहतेः
सन्तुष्ट्यै च समार्पयाम विबुधाः हृष्टान्तरङ्गा वयम्॥

The scholarly world is indeed fortunate that Nīlakaṇṭha—an astronomer-mathematician hailing from the Garga lineage and blessed with a clear intellect—composed among several other works a treatise called Tantrasaṅgraha, which is considered to be a gem among works [in astronomy], resplendent with a variety of yuktis, neither terse nor too elaborate, and which gives far more accurate procedures for solving the problems involving eclipses, shadow measurements, [the application of the equation of centre] in the case of Mercury and Venus, and so on.

We are now extremely happy to bring out this treatise, Tantrasaṅgraha, along with translation, annotation and detailed mathematical exposition for the intellectual delight of the scholarly community. We are also immensely pleased to dedicate this work of ours to Mādhava and the galaxy of other astronomers that this great lineage has produced.

Foreword

In the history of mathematics and astronomy in India, the Kerala school which flourished during the fourteenth–seventeenth centuries CE, has a unique position. Mādhava of Saṅgamagrāma, Parameśvara, Nīlakaṇṭha Somayājī, Jyeṣṭhadeva and Śaṅkara Vāriyar were among the luminaries of this school, which made original contributions in mathematics, formulating the infinite series for the trigonometric functions and π, that antedated similar achievements of European mathematicians by a couple of centuries. The origin of calculus also can be traced to this school.

In astronomy too, the Kerala school had significant achievements. The versatile astronomer Nīlakaṇṭha Somayājī (1444-1545) produced several works on astronomy, of which the *Tantrasaṅgraha* (about 430 verses in *anuṣṭubh* metre in eight sections or *prakaraṇas*) is a comprehensive treatise. He introduced in this elegant work a major revision of the traditional Indian planetary model, a detailed geometrical picture of which is discussed in his two small but lucid works—*Siddhāntadarpaṇa* (31 verses) and *Golasāra* (56 verses). According to Nīlakaṇṭha's geometrical picture of planetary motion, the five planets (Mercury, Venus, Mars, Jupiter and Saturn) move in eccentric orbits around the mean Sun, which in turn orbits around the Earth. Such a formulation was put forward by the European (Danish) astronomer Tycho Brahe, nearly a century later. *Tantrasaṅgraha* is also known for its other innovations introduced by Nīlakaṇṭha.

In March 2000, the Department of Theoretical Physics, University of Madras, organized a conference in collaboration with the Indian Institute of Advanced Studies, Shimla, to celebrate the 500th anniversary of *Tantrasaṅgraha*. Though the importance of this text was known to historians of Indian astronomy for quite some time, and several research papers have been published on the original ideas presented in this work, there was a great need for an accurate English translation of this seminal treatise, with detailed notes in modern notation. Profs K. Ramasubramanian and M. S. Sriram, who have the linguistic and subject expertise, have fulfilled this need admirably. As may be noted from the current volume, every attempt has been made by the authors to make the work as self-contained as possible by giving detailed explanations as well as several explanatory appendices besides a glossary and bibliography. Historians of astronomy, both Indian and foreign, are most grateful indeed to them for their devoted efforts in bringing out this publication.

The authors are already well known for their studies and publications in the area of Indian mathematics and astronomy. Together with another savant, M. D. Srinivas of the Centre for Policy Studies, Chennai, they were involved in preparing a detailed explanatory notes for *Gaṇita-yuktibhāṣā* of Jyeṣṭhadeva, which was published in two volumes by the Hindustan Book Agency, New Delhi with a critical edition of the text and English translation by K. V. Sarma, an eminent scholar who published several works on Indian astronomy. A reprint of this work was also brought out recently by Springer, the noted international publishers, to make it available for the international readership. In fact, Jyeṣṭhadeva, who was a junior contemporary of Nīlakaṇṭha, at the commencement of his work states that his aim in composing the work is to explain the calculational procedures given in *Tantrasaṇgraha*. It is but fitting, therefore, that Profs Ramasubramanian and Sriram, who were involved with the production of explanatory notes of *Gaṇita-yuktibhāṣā*, are the authors of the present volume on *Tantrasaṅgraha*.

The Hindustan Book Agency has been rendering yeoman service to scholars interested in the history of mathematics, by bringing out several volumes in its series 'Culture and History of Mathematics'. I am happy that in collaboration with Springer it is publishing the present work on *Tantrasaṅgraha*, which, I am sure, will be of great value to historians of science in general and of astronomy in particular. It is my fond hope that several other timeless works of this type will emerge from the pens of these erudite authors in future.

Bangalore
March 2010

B. V. Subbarayappa
Former President, International Union of History and Philosophy of Science

Preface

Tantrasaṅgraha composed in 1500 CE by the Kerala astronomer Nīlakaṇṭha Somayājī, has long been recognized as an important Indian text in astronomy. It is a comprehensive text which discusses all aspects of mathematical astronomy such as the computation of the longitudes and latitudes of planets, various diurnal problems, the determination of time, eclipses, the visibility of planets etc. There are two critical editions of the Sanskrit text, by S. K. Pillai in 1958 and K. V. Sarma in 1977, which between them include the commentaries *Laghu-vivṛti* in prose for the entire text, and *Yukti-dīpikā* in verses for the first four chapters, both of which are composed by Śaṅkara Vāriyar. The need has long been felt for an English translation of the work, with detailed explanatory notes in modern notation, so that the work is accessible to a larger audience. It is with this objective that we began a project on *Tantrasaṅgraha*, funded by the Indian National Science Academy (INSA), in 2000.

Meanwhile, along with M. D. Srinivas (Centre for Policy Studies, Chennai), we were involved in preparing detailed explanatory notes for *Gaṇita-yukti-bhāṣa* (GYB) of Jyeṣṭhadeva, edited and translated by K. V. Sarma, and published in 2008 by the Hindustan Book Agency and reprinted in 2009 by Springer. Though the work on GYB caused delay in the publication of the present work, it was very rewarding as GYB gives detailed explanations of most of the algorithms in *Tantrasaṅgraha*, and provides valuable insights on many topics covered in that work.

Scholars in the area of the history of astronomy in general, and Indian astronomy in particular, form the natural readership for this work. However, keeping the larger readership—anyone wanting to know the methods of Indian astronomy—in mind, we have attempted to make it as self-contained as possible, so that any motivated person with a sound background in mathematics at the final school (+2, as it is termed in India) level and interested in spherical astronomy will find it useful. We have also included a glossary of frequently occurring Sanskrit terms and several appendices that should serve to clarify many concepts relevant to the topics in the main text.

The modification of the traditional Indian planetary model by Nīlakaṇṭha in *Tantrasaṅgraha* is what attracted us to the work initially. But this topic is dealt with all too briefly in it, as it is a *Tantra* text devoted mainly to computational algorithms. However, Nīlakaṇṭha has discussed his model extensively, along with his geometrical picture of planetary motion, in other works. In collaboration with M. D. Srinivas, we had made an incisive study of this model and published a paper on it in the Indian journal *Current Science* way back in 1994. Since then we have had occasions to study this in more detail. Appendix F, on the traditional Indian planetary model and its revision by Nīlakaṇṭha Somayājī, of which M. D. Srinivas is a co-author, reflects our current understanding on this subject.

We are deeply indebted to Prof. M. D. Srinivas—our collaborator on the different aspects of studies on Indian astronomy and mathematics that we have been doing for almost two decades now—for meticulously going through the entire manuscript and offering several valuable suggestions. We would also like to acknowledge the suggestions given by the two anonymous referees for improving the manuscript. We are grateful to (the late) Prof. K. V. Sarma with whom we have had an extensive collaboration, especially during the preparation of *Yuktibhāṣā*, and who has been a source of great inspiration for us. We would like to thank Profs C. S. Seshadri and R. Sridharan of the Chennai Mathematical Institute for their continued support and encouragement. Our special thanks go to Prof. B. V. Subbarayappa, Bangalore, the doyen of the history of science in India, for having readily agreed to write the Foreword to this work.

Our heart-felt thanks are due also to Profs S. Balachandra Rao of Bangalore, and V. Srinivasan of Hyderabad (currently with the University of Madras) for their constant and vociferous support to us over all the years in all our work on Indian astronomy. We also thank Profs P. M. Mathews, G. Bhamati, M. Seetharaman, S. S. Vasan, K. Raghunathan, A. S. Vytheeswaran, R. Radhakrishnan, and Dr Sekhar Raghavan associated with the Department of Theoretical Physics, University of Madras, for their kind and active interest in our work over the years.

We would like to acknowledge the keen interest expressed by Swami Atmapriyananda, Belur Math, in promoting studies in Indian astronomy and mathematics. We are indeed grateful to Profs David Mumford of Brown University and Manjul Bhargava of Princeton University for their kind encouragement and enthusiastic support. It is a pleasure to thank Profs S. M. R. Ansari, A. K. Bag, Jitendra Bajaj, V. Balakrishnan, A. V. Balasubramanian, Rajendra Bhatia, S. G. Dani, Sinniruddha Dash, Amartya Datta, P. C. Deshmukh, P. P. Divakaran, Raghavendra Gadagkar, George Joseph, Rajesh Kocchar, S. Madhavan, Madhukar Mallayya, N. Mukunda, Roddam Narasimha, M. G. Narasimhan, Jayant Narlikar, C. K. Raju, Sundar Sarukkai, B. S. Shylaja, Navjyoti Singh, S. P. Suresh, T. Trivikraman, Mayank Vahia, Padmaja Venugopal and K. Vijayalakshmi, as well as Profs Mohammad Bagheri, Subhash Kak, Agathe Keller, Francois Patte, Kim Plofker, T. R. N. Rao, S. R. Sarma and Michio Yano for their kind interest in our work in Indian astronomy and mathematics in general, and this work in particular.

One of the authors (Ramasubramanian) would like to acknowledge the unstinting support and encouragement received from the former Director of IIT Bombay, Prof. Ashok Misra, and other IIT Bombay fraternity members, particularly Profs. S. D. Agashe, Rangan Banerjee, Jayadeva Bhat, S. M. Bhave, Amitabha

Gupta, Devang Khakhar, Malhar Kulkarni, Ravi Kulkarni, H. Narayanan, Prabhu Ramachandran, Krithi Ramamritham, H. S. Shankar, G. Sivakumar, A. K. Suresh, Sivaramakrishnan and Jugal Verma. Special thanks are also due to K. Mahesh, U. K. V. Sarma, R. Venketeswara Pai, Dinesh Mohan Joshi and Vanishri Bhat for their devoted assistance at different stages of the preparation of the manuscript. In fact, we are particularly indebted to Vanishri for spending countless hours in setting a uniform style for all the figures that appear in the book and U. K. V. Sarma for carefully proof reading the entire text.

This work is the outcome of a project sanctioned by INSA, New Delhi, during October 2000–March 2004. We would also like to place on record our gratitude to the Sir Dorabji Tata Trust and the National Academy of Sciences, India, for their financial assistance by way of projects, which was extremely useful in offering fellowship to the project staff as well as in the production of the manuscript in a camera-ready form. We are deeply indebted to INSA for the financial support as well as for readily granting the permission to publish the work. Our special thanks go to Jainendra Jain and Devendra Jain of the Hindustan Book Agency, New Delhi, for graciously coming forward to publish this work in collaboration with Springer, London.

Finally, the authors are grateful to the copy-editor(s) for going through the manuscript meticulously, and making valuable comments and suggestions.

विकृति-आश्वयुजशुक्लदशमी, कल्यब्द ५११२

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October 17, 2010

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Introduction

Tantrasaṅgraha and its importance

Tantrasaṅgraha, a comprehensive treatise on astronomy, was composed by the renowned Kerala astronomer Nīlakaṇṭha Somayājī (1444–1545 CE) of Tṛkkaṇṭiyūr. It ranks along with *Āryabhaṭīya* of Āryabhaṭa (499 CE) and *Siddhāntaśiromaṇi* of Bhāskarācārya (1150 CE) as one of the major works which significantly influenced further work on astronomy in India. In *Tantrasaṅgraha*, Nīlakaṇṭha introduced a major revision of the traditional Indian planetary model. He arrived at a unified theory of planetary latitudes and a better formulation of the equation of centre for the interior planets (Mercury and Venus) than was available, either in the earlier Indian works or in the Greco-European or Islamic traditions of astronomy, till the work of Kepler.[1] Besides this, the work also presents many important innovations in mathematical techniques related to accurate sine tables, use of series for sine and cosine functions, and a systematic treatment of spherical astronomical problems. The relations of spherical trigonometry stated here are exact, and are applied with care to diurnal problems, eclipses etc. The explanations of the procedures of *Tantrasaṅgraha* are to be found in the commentaries *Laghu-vivṛti* and *Yukti-dīpikā* by Śaṅkara Vāriyar, as well as the seminal Malayalam work *Yuktibhāṣā* of Jyeṣṭhadeva.

The present work and its context

There have been two critical editions of *Tantrasaṅgraha*, the first by Surnad Kunjan Pillai in 1958 and the second by K. V. Sarma in 1977. While the former includes the

[1] In his other works *Āryabhaṭīya-bhāṣya*, *Golasāra*, *Siddhāntadarpaṇa* and *Grahasphuṭā-nayane vikṣepavāsana*, Nīlakaṇṭha also discusses the geometrical model implied by his theory according to which the planets go around the Sun, which itself orbits around the Earth. See Appendix F for more details.

commentary *Laghu-vivṛti* in the form of prose for the whole text, the latter includes the elaborate commentary *Yukti-dīpikā* (for the first four chapters) in the form of verses.[2] Both these commentaries are by Śaṅkara Vāriyar. There is very little difference in the text between the two editions. While the main text, *Tantrasaṅgraha*, as edited by K. V. Sarma, is based on 12 manuscripts, the commentary, *Yukti-dīpikā*, is based on only four manuscripts.[3] The textual verses, as well as the references to the citations from *Laghu-vivṛti* and *Yukti-dīpikā*, that are reproduced in the present work are based on the above two editions of *Tantrasaṅgraha*.

We have gone through the entire *Laghu-vivṛti* commentary in the process of preparing the translation and explanatory notes. Some important portions of *Yukti-dīpikā* having a direct bearing on the contents of the main text have also been cited in our explanations. For the most part, *Laghu-vivṛti* gives a plain and direct description of the verses of the text in simple prose without excursions into related topics. Nevertheless, it does offer very valuable insights on several occasions and clarifies the contents of many verses, which would have been unclear otherwise. However, the commentary *Yukti-dīpikā* is of a different nature. Here Śaṅkara Vāriyar transcends the confines of immediate utility and discusses several related issues that would greatly enhance one's understanding of the subject. Many verses in *Yukti-dīpikā* reveal several aspects of the Indian thinking on astronomy and mathematics. Besides these two commentaries, we have also consulted the astronomy part of Jyeṣṭhadeva's *Yuktibhāṣā* which has proved to be extremely useful in understanding the contents of *Tantrasaṅgraha*. In fact, according to Jyeṣṭhadeva—as stated by him at the very commencement of the work—the main purpose of *Yuktibhāṣā*[4] is to elucidate the procedures enunciated in *Tantrasaṅgraha*. We have made extensive use of this work while preparing the explanatory notes on certain topics such as the planetary model, spherical astronomical problems, visibility corrections, the eclipses and so on.

There is an earlier translation of *Tantrasaṅgraha* by V. S. Narasimhan, which was published in the *Indian Journal of History of Science* as a supplement in three parts during 1998.[5] Narasimhan has also presented some explanatory notes to his translation. However, the author does not seem to have carefully studied the commentaries of Śaṅkara Vāriyar in preparing the translation. He also did not have the benefit of consulting an edited version of the astronomy part of *Yuktibhāṣā*. Often, his translation and explanations do not really bring out the exact content of the verses of *Tantrasaṅgraha*. This has been one of the motivating factors for undertaking the present work.

[2] See {TS 1958} and {TS 1977}.

[3] {TS 1977}, p. xlii.

[4] {GYB 2008}, p. 1; p. 313.

[5] {TS 1999}, pp. S1–S146.