Wolfgang P. Schleich

Quantum Optics in Phase Space

WILEY-VCH

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"FAITH MAINTAINED IS ONE OF THE GREAT GIFTS BESTOWED BY FELLOW MAN"

Dedicated to the two people who always had faith that this book would be completed

Kathy and Michael

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Preface

During the winter semester of 1992/93 I taught for the first time the course *Quantum Optics I* at the University of Ulm, which was followed by part II in the summer semester of 1993. When I offered the course a second time the University was kind enough to financially support two diplom students, Erwin Mayr and Daniel Krähmer, who had already taken this class in the previous year to transform my hand-written notes and sketches of drawings into a legible form. Erwin and Daniel have done a tremendous job. Since then I have taught this course many times and collected more and more material which was included into this manuscript by other graduate students of the Abteilung. It has served many generations of students at the University of Ulm as a first introduction to the field of quantum optics.

During one of his many visits to Ulm, Michael Poulson, a close friend from the VCH-Wiley publishing house saw the manuscript on my desk. "I want to publish these notes" was his immediate reaction. Michael had complete faith that this manuscript would eventually be turned into a publishable book. He wanted the material to be expanded to include problems, experiments and an exhaustive list of references. The goal was to convert the existing manuscript of about 150 pages into a book of about 250 pages. His trust in me was so great that he started advertising *Quantum Optics in Phase Space* before we had even signed a contract. I believe the present result satisfies the criteria Michael had put forward with one exception — the number of pages.

At Christmas of 1996 we finally signed a contract and Michael was extremely relieved. I still remember his words "now I have finally succeeded in signing you up for the book". A week later his untimely death during Christmas vacation added a new meaning to this sentence and a purposeful dimension to his faith and expectation; I was determined more than ever to deliver what I had promised.

Eventually Erwin and Daniel graduated and their new professional life did not allow them to devote more time to continue the project. Since that fateful Christmas of 1996, many students have helped me transform my class notes into various sections of the book continuing the work that Erwin and Daniel had begun. Stephan Meneghini took over and for several years he was instrumental in typing the manuscript. But also he graduated during the course of the project. In the final phase of the book his role was taken over by Florian Haug. I am enormously grateful to all of them for their assistance. What started out with 200 pages at Erwin and Daniel's departure eventually expanded and reached its present 700 page size.

Similarly, the field of quantum optics has expanded enormously over the last 10 years. This fact reflects itself in the variety of textbooks that have been published on this topic. It is impossible to represent all branches of this rapidly moving field in one single book. As a consequence many current topics are left out in the present one, such as quantum information or Bose-Einstein condensation. The main theme of the book is quantum phase space and the application of semi-classical concepts, such as WKB techniques to problems of quantum optics. In the present American hype of the "e-mail and the information highway" some people have suggested to call the book "phase-space.com."

Many friends and colleagues have read through various parts of the book and have made useful comments. In this regard I want to mention especially I. Bialynicki-Birula, J.H. Eberly, H.J. Kimble, D. Kobe, R.F. O'Connell, H. Walther, K. Wódkiewicz and E. Wolf. Special thanks go to M. König who has worked very carefully through the whole book and has made numerous constructive remarks. In the final stage all members of the Abteilung have proofread the entire book. Many thanks to G. Alber, M. Bienert, M. Cirone, O. Crasser, A. Delgado, D. Fischer, M. Freyberger, F. Haug, V. Kozlov, H. Mack, W. Merkel, G. Metikas, M. Mussinger, K. Vogel, J. Wichmann and V.P. Yakovlev. K. Vogel was also instrumental in putting the index together. I am grateful to my secretaries B. Casel, R. Knöpfle and U. Thomas who were helpful in collecting the literature.

Various chapters of the book have been tested in two lecture series given at the University of Texas at Austin. The penetrating questions of J.H. Eberly, M. Fink, D. Heinzen, J. Keto, M. Raizen, W.C. Schieve and E.C.G. Sudarshan have helped to sharpen my arguments through the extremely lively discussions during and after the classes. They have enormously helped to improve the presentation of the material. The kind hospitality of and the always friendly atmosphere at the physics department at UT Austin are greatly appreciated.

Many science organisations have supported the research summarized in the present book. In this context I want to mention especially the Deutsche Forschungsgemeinschaft and the Leibniz Program, the European Community, the Heraeus Foundation, the Humboldt Foundation and the University of Ulm. All have graciously financed my students, assistants and visitors. Many thanks to all of them.

The quiet periods in Denton, Texas with my understanding father-in-law, H.C. Phillips who always refers to me as his "blue electron son-in-law" were very conducive to completing this book. Moreover, I greatfully acknowledge the kind hospitality at the physics department at North Texas State University, Denton.

Last, but not least, I want to express my sincere thanks to my teachers. G. Süßmann, whose lectures at the Ludwig-Maximilians-Universität in München woke my interest in theoretical physics and made me change my degree from high school teacher to physicist. Süßmann's broad and deep interest in the whole field of physics and not just a special area has always impressed me and hopefully this book reflects his influence. M. O. Scully and H. Walther have introduced me to the field of quantum optics 20 years ago. I was fortunate enough to closely work with them on various problems of quantum optics and they have strongly influenced my view of the field. Through my collaborations with them I have gained many insights. A different angle of physics came through my years in Texas working with J.A. Wheeler. He taught me that many phenomena in physics become transparent when viewed using WKB techniques combined with the concept of phase space. In this sense the origin of this

book stems from my years in Austin, Texas working with John on interference in phase space.

Special thanks go to my publishers Wiley-VCH and, in particular, to the innocent successor of Michael Poulson, Michael Bär, for his patience in awaiting the final outcome of *Quantum Optics in Phase Space.* Indeed they have suffered along with me in my trials of writing a comprehensive textbook on the application of phase space to quantum optics.

Above all I want to thank my parents, who encouraged me to think deeply and who made it possible for me to get the education necessary to pursue my studies. A special thanks goes to my wife Kathy and Michael Poulson, who never gave up their faith in me that this book would ever be finished even when other people close to me have made bets that the book would never (or not) be completed before the year 2050. Michael Poulson once said "I am not worried about the book being finished because Kathy will make sure you get it done for both of us". With these fateful words he was right; may he rest in peace.

Wolfgang P. Schleich
Ulm, November 2000

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Contents

1 What's Quantum Optics?

What is quantum optics? This is a rather personal question. A well-known scientist in this field once gave the following authoritative answer: "Whatever I do defines quantum optics!" On a more objective basis one is tempted to define this branch of physics by the pun: "Quantum optics is that branch of optics where the quantum features of light matter."

Which discovery in physics marks the birthday of quantum optics? Many phenomena come to our mind. Is it the discovery of the quantum, the development of QED, or the maser/laser? Or is it none of the above?

In this chapter we answer this question in a back handed way by summarizing some path breaking experiments that define quantum optics. Admittedly this list is not complete and chosen in a rather subjective way. The rapidly moving field of quantum optics demonstrates most clearly that even after 100 years of quantum physics there is still a lot to be learned from Planck's original discovery.

1.1 On the Road to Quantum Optics

More than hundred years ago M. Planck was struggling with the experimental data of black body radiation obtained at the *Physikalisch-Technische Reichsanstalt* in Berlin by H. Rubens and F. Kurlbaum. From todays point of view these experiments look rather academic. However, they were motivated by industrial applications. Indeed, standards had to be developed in order to describe light bulbs. This need triggered one of the most important problems in the physics of the 20*th* century: Classical electromagnetic theory cannot explain the measured black body spectrum. In a desperate but courageous attempt Planck postulated that the oscillators in the walls of the cavity can only absorb and emit radiation in discrete units. This revolutionary idea of discreteness rather than a continuum provided the celebrated radiation formula and was the starting point of quantum mechanics.

Nowadays we associate the quantization with the field rather than with the mechanical oscillators in the wall. However, wave and matrix mechanics were first developed for massive particles and then, later, transferred to the electromagnetic field leading to quantized electrodynamics.

The field of quantum electrodynamics, QED, which deals with the interaction of quantized matter with quantized electromagnetic fields started with P.A.M. Dirac.

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He was the first to derive the Einstein A and B coefficients of spontaneous and induced emission. The field of QED culminated on the one hand with the experimental discovery of the level shift in the hydrogen atom by W.E. Lamb and R.C. Retherford and the measurement of the anomalous moment of the electron by H.M. Foley and P. Kusch. On the other hand the theoretical works of S. Tomonaga, J. Schwinger and R. Feynman showed how to avoid the infinities that had plagued the theory since the thirties. The incredible agreement between theory and experiment established nowadays in many QED systems confirms beyond any doubt the quantized nature of light.

The development of the ammonium maser by C.H. Townes, J. Gordon and H. Zeiger, and the laser by T. Maiman following the paper *Optical Masers* by A. Schawlow and Townes has opened the new field of quantum electronics. Motivated by the experiments on the maser and building on his own theoretical work on water-vapor absorption W.E. Lamb developed a theory of the maser during the years 1954–1956. Later he worked out a complete semi-classical theory of laser action. Independently the group of H. Haken in Stuttgart developed their own approach. In the semiclassical treatment of Lamb and Haken the electromagnetic field was described classically and the atom quantum mechanically.

Since then laser theory has come a long way from the early approaches using birth and death equations via the semiclassical theory of the laser to the fully quantized version. The three approaches to the quantum theory of the laser are the Fokker-Planck method, pursued by H. Haken and H. Risken, the noise operator method by M. Lax and W.H. Louisell and the density matrix techniques by M.O. Scully and W.E. Lamb. Earlier the quantum theory of photon counting has been developed by R. Glauber.

Unfortunately, the quantum effects of the laser were scarce. The photon statistics of the laser and the phase diffusion were the only quantum effects that could be measured.

1.2 Resonance Fluorescence

Quantum optics has received an enormous push from the phenomenon of resonance fluorescence. The light emitted from an atom which is driven by a classical monochromatic electromagnetic field shows interesting quantum effects in its spectrum and in the statistics. We now briefly review this corner stone of quantum optics.

1.2.1 Elastic Peak: Light as a Wave

Resonance fluorescence is an old problem that has been discussed for the first time in great detail by W. Heitler in his classic book "The Quantum Theory of Radiation". He pointed out that the emitted radiation has the same frequency as the incident radiation. Thus, the spectrum is a delta-function. In this sense the atom is just a driven dipole and therefore radiates with the frequency of the driving field. This elastic component in the scattered light has been observed experimentally and is shown in Fig. 1.1.

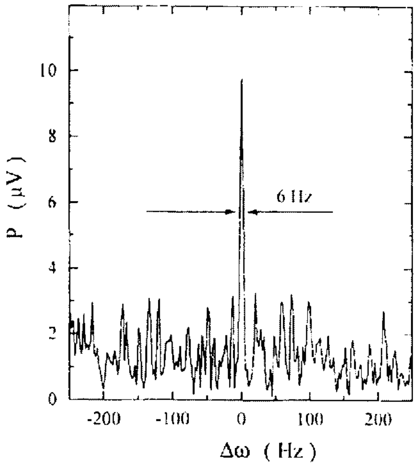

Fig. 1.1: Heterodyne spectrum of the elastic fluorescence component of a single trapped $^{24}Mg^{+}$ ion. A narrow peak emerges when the frequency difference between the heterodyne signal and the driving field vanishes. Taken from J.T. Höffges *et al.*, Opt. Comm. **133**, 170 (1997).

For this measurement a single magnesium ion was stored in a modified Paul trap shown in Fig. 1.2. A laser was driving an electronic transition of the ion and the emitted radiation was superimposed with the driving field. The resulting signal, commonly referred to as heterodyne signal, was analyzed with respect to frequency. This spectrum displays a narrow structure at the frequency of the incident radiation as shown in Fig. 1.1. Theoretically, the width of this line should be zero. However, it is determined by the spectrum of the exciting light.

In this context it is interesting to note that this experiment is also a verification of the wave nature of light. Indeed, the elastic peak is so narrow since the emitted wave has a fixed phase relation with the driving field. The emitted light is therefore a wave. This experiment clearly supports the wave rather than the particle concept. However, as we will see in the next section we can slightly rearrange the experiment such that the particle aspect of light emerges. This is one more manifestation of Bohr's principle of complementarity.

1.2.2 Mollow-Three-Peak Spectrum

However, this delta-function peak in the spectrum is only one part of the problem. In the late sixties B.R. Mollow investigated resonance fluorescence using quantum electrodynamics and found that the spectrum depends on the intensity of the incident radiation. For low intensities the Heitler result is valid. However, for larger intensities the spectrum displays a more complicated structure: In addition to this elastic delta-function peak there exist three broad, incoherent contributions centered at the incident frequency and at two side bands. They are shifted by a frequency

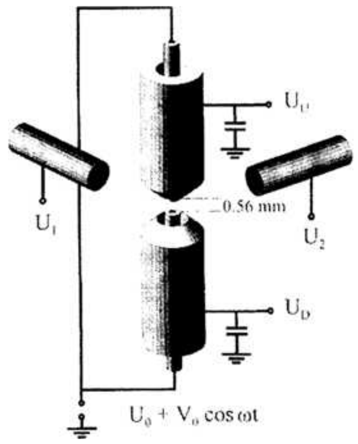

Fig. 1.2: Electrode configuration of the endcap trap. The trap consists of two co-linearly arranged cylinders corresponding to the cap electrodes of the traditional Paul trap. The ring electrode is simulated by two hollow cylinders which are concentric with each of the cylindrical endcaps. Additional electrodes allow for the compensation of stray electric field components. The open structure offers a large detection solid angle and good access for laser beams. Taken from J.T. Höffges *et al.*, Opt. Comm. **133**, 170 (1997).

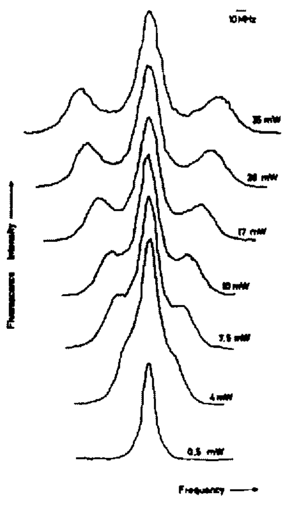

Fig. 1.3: Experimental three-peak Mollow spectrum for increasing laser intensity. We note the emergence of the side peaks. The elastic peak ideally represented by a delta-function located on top of the central peak is not shown. Taken from W. Hartig *et al.*, Z. Physik A **278**, 205 (1976).

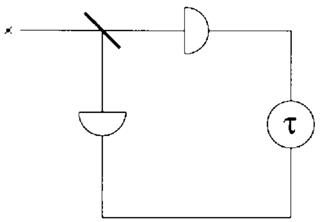

Fig. 1.4: Measurement of second order correlation function. Light from a source passes a beam splitter and falls onto two detectors. We are interested in the distribution of consecutive clicks of the two detectors. The first photon hitting a detector starts the clock, the second photon hitting the other detector stops the clock. As light source we could use a thermal light bulb, a laser or the resonance fluorescence of a single ion driven by a laser field.

determined by the electric field of the incident wave. These incoherent peaks have a different width determined by the natural line width Γ of the atom. Indeed, the central peak has a width of $\Gamma/2$ whereas the sidebands have the width $3\Gamma/4$.

This spectrum has been measured experimentally by the groups of C.R. Stroud, S. Ezekiel and H. Walther in the mid-seventies. In Fig. 1.3 we show the emergence of the three-peak Mollow spectrum for increasing laser intensity.

1.2.3 Anti-Bunching

A new chapter in the book of resonance fluorescence was opened in the mid-seventies when H. Carmichael and D.F. Walls in New Zealand, and H.J. Kimble and L. Mandel in the US independently from each other analyzed the statistics of the light. They found a time delay between two successive photons emitted from the atom. The light is anti-bunched. This behavior is in sharp contrast to thermal light where the photons come in bunches. It is interesting to note that also a laser has a non-vanishing probability for two photons arriving right after each other.

One way of measuring the effect of bunching or anti-bunching is the Hanbury Brown and Twiss arrangement shown in Fig. 1.4. Light falls through a beam-splitter onto two detectors. We can measure the delay time between two consecutive clicks on the two detectors: The first photon triggers the detector in one arm, the second photon fires the second detector in the other arm. Repeating the experiment many times we measure the distribution of delay times.

This experiment performed in the late fifties by H. Hanbury Brown and R.Q. Twiss using sunlight was the starting point of the quantum theory of photon counting pioneered by R. Glauber. For an insight into the importance of this role we refer to the Les Houches lectures of R. Glauber. Glauber's theory of photon counting is based on correlation functions of the electromagnetic field. In this formalism photon anti-bunching expresses itself in the behavior of the second order correlation function $g^{(2)}(\tau)$ as a function of the delay τ and, in particular, at $\tau = 0$.

Figure 1.5 shows the dependence of $g^{(2)}$ on the delay τ for three typical light sources, namely a thermal light source, a laser and resonance fluorescence. We note that for $\tau \to \infty$ all curves approach unity. However, the starting points for all curves,

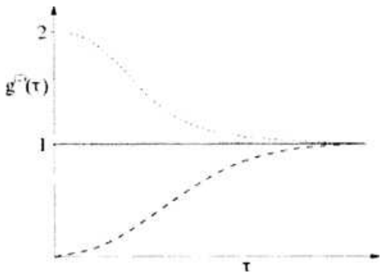

Fig. 1.5: Second order correlation function as a function of delay τ. When the light source in the Hanbury Brown and Twiss experiment is a light bulb the second order correlation function $g^{(2)}(\tau)$ (dotted line) exhibits a dominant maximum at short delay times: It is therefore more probable to find two photons right after each other rather than at larger delays. The light is bunched. When the source is a laser the light obeys Poissonian statistics and $g^{(2)}(\tau)$ is independent of the delay (solid line). However, resonance fluorescence displays quite a different behavior (dashed line): The light is antibunched since the probability that two photons follow right after each other is very small.

that is, $g^{(2)}$ at $\tau = 0$, are different. In case of thermal light it begins at $g^{(2)}(0) = 2$ and approaches unity from above. Hence, it is more likely to find two photons arriving after each other. For a laser the distribution is independent of the delay. However, the light emitted by an atom that is driven by a laser field is quite different. Here, the probability to find a photon just after one has been detected is zero and hence, $g^{(2)}(0) = 0$. Hence, the curve approaches unity from below. In this case we need the full quantum theory of radiation to describe the resonance fluorescence light.

These theoretical predictions have been verified experimentally by the groups of L. Mandel and H. Walther using atomic beams. The development of Paul traps for ions and magneto-optical traps for atoms has opened a new era in these experimental studies of resonance fluorescence. Now it is possible to observe the radiation of a single particle and thus anti-bunched light from a single ion, atom or molecule. In Fig. 1.6 we show the measured second order correlation function for resonance fluorescence of a single magnesium ion. These curves clearly show that it is highly unlikely to have two photons emitted right after each other.

The phenomenon of anti-bunching observed with the help of a single ion is particularly interesting in the context of the heterodyne experiments shown in Fig. 1.1 since in both experiments we are analyzing the same radiation. In the heterodyne measurement of the resonance fluorescence we find a narrow spectrum confirming the wave nature of the emitted light. However, when we perform a correlation experiment of the same light we observe the particle nature. Hence, resonance fluorescence provides us in this way with another striking demonstration of wave-particle duality.

Closely related to the phenomenon of anti-bunching is the effect of sub-Poissonian statistics. R. Glauber has shown that a classical current radiates an electromagnetic field in a coherent state. Its photon statistics, that is the probability to find m photons is then a Poissonian distribution. However, the photon statistics of the radiation emitted by a driven atom is narrower than a Poissonian: It enjoys sub-

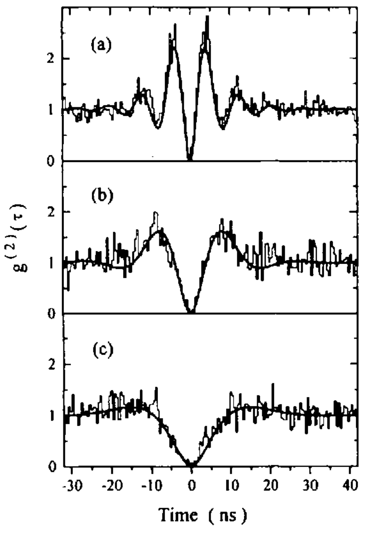

Fig. 1.6: Photon anti-bunching in the resonance fluorescence of a single $^{24}Mg^{+}$ ion stored in the endcap trap shown in Fig. 1.2. The second order correlation function $g^{(2)}(\tau)$ displays a striking minimum at zero delay times. We show the curves for three characteristic detunings. The integration time was limited by the storage time of the ion. In the last case (c) the storage time was 220 minutes. Taken from J.T. Höffges *et al.*, Opt. Comm. **133**, 170 (1997).

Poissonian statistics. This effect has been observed in the group of L. Mandel for an atomic beam, and for a single ion in the group of H. Walther.

1.3 Squeezing the Fluctuations

Recently resonance fluorescence brought out another impressive quantum effect of the radiation field. Its fluctuations are squeezed as shown by the experimental curve in Fig. 1.7.

1.3.1 What is a Squeezed State?

In order to uniquely describe the state of a classical, mechanical harmonic oscillator we need both, the amplitude and the phase of the oscillator. Likewise, we need amplitude and phase to describe uniquely the electromagnetic field. In its most elementary version we represent the electromagnetic field by a vector in complex space as shown in Fig. 1.8. Here it is worthwhile to mention that we do not mean the complete electric field vector $\vec{E}$ but only one component of it.

However, when we quantize the field a whole distribution of vectors is needed. This distribution function provides for every point of complex space a weight factor. One could imagine that this weight function represents the probability to find a specific electric field vector. Unfortunately, quantum mechanics does not allow such

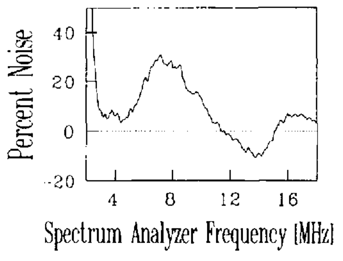

Fig. 1.7: Experimental observation of squeezing in resonance fluorescence. The dotted horizontal line marks the shot-noise limit defined by the vacuum fluctuations of the electromagnetic field. We note that in a small frequency domain above 12 MHz the fluctuations of the resonance fluorescence light go below this limit. Taken from Lu *et al.*, Phys. Rev. Lett. **81**, 3635 (1998).

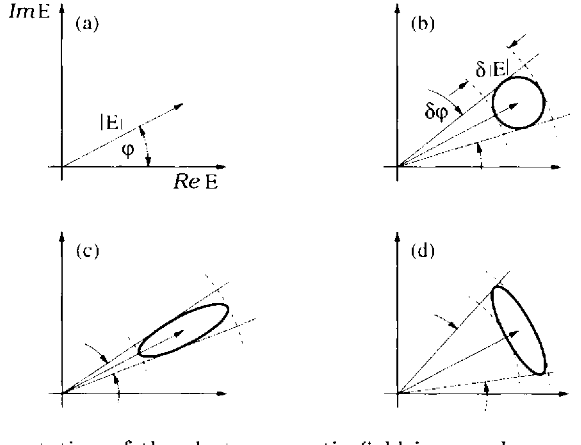

Fig. 1.8: Representation of the electromagnetic field in complex space, that is, phase space as a vector (a). Due to the quantum mechanics of the field the end point of the vector can lie anywhere in a domain of phase space with minimum area $2\pi\hbar$. This uncertainty domain can be a circle (b) which results in a symmetric distribution of the fluctuations. It can also be an ellipse with an asymmetric distribution (c,d). In this case we have squeezed either the fluctuations in phase (c) or amplitude (d). The electromagnetic field is in a squeezed state.