

Anomalous Transport

Foundations and Applications

Edited by

Rainer Klages, Günter Radons, and Igor M. Sokolov



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*Rainer Klages, Günter Radons, and
Igor M. Sokolov*

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Cover

Dr. Dirk Brockmann, MPI Göttingen

The worldwide air traffic network, which leads to anomalous transport in human travel. Such anomalous processes can be modelled by the continuous time random walk theory of which the Montroll-Weiss equation displayed in the picture is the key ingredient.

Backcover Picture

Dr. Nickolay Korabel, Bar-Ilan University

Anomalous probability distribution function (oscillatory structure) for the positions of a subdiffusively spreading ensemble of particles generated by an intermittent map and three fit functions (other lines).

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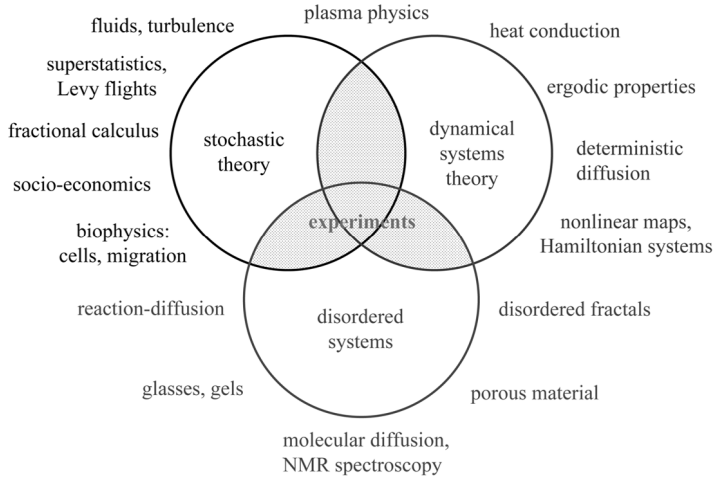
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Preface

Anomalous transport refers to nonequilibrium processes that cannot be described by using standard methods of statistical physics. This novel class of transport phenomena has recently been observed in a wide variety of complex systems such as amorphous semiconductors, plasmas, glassy materials, nanopores, biological cells and epidemic spreading. The coherent description of anomalous transport in such a broad range of systems poses a fundamental challenge to the theoretical modeling and to the mathematical language in which these models are formulated. It asks for a synergy of many different disciplines, from the mathematical theory of dynamical systems over the theory of stochastic processes to the statistical physics of disordered systems.

This book gives a comprehensive introduction to the newly emerging field of anomalous transport. It discusses particularly the important examples of anomalous particle transport in high-energy plasmas and in turbulence, ageing in glassy materials, anomalous diffusion in porous media, anomalous heat conduction and chemical reaction-diffusion as well as anomalous correlations in polymer melts. Anomalous dynamics is also observed in biological and socio-economic processes for which we include anomalous diffusion in the cell membrane and in human travel as examples. The theoretical description of such different phenomena leads, in turn, to the prediction of novel physical and mathematical properties such as sub- and superdiffusion, probability distributions with infinite moments, weak ergodicity breaking, anomalous relaxation in complex systems or subdiffusion limited reactions, topics that are all explored in this book. Anomalous transport thus nicely exemplifies the saying, freely after Tolstoi, that “all simple systems are simple in the very same manner, whereas any complex system exhibits its very own type of complexity”.

The individual chapters of this multi-author monograph are written by mathematicians, theoretical physicists and experimentalists who are all internationally recognized experts in their fields. The aim of the editors was to bring together the disciplines of *stochastic theory*, *dynamical systems theory* and *disordered systems*, as sketched in the figure shown below. These three fields



form rapidly growing research areas in themselves, which so far have developed quite independently from each other. However, in all these branches anomalous transport is moving more and more into the center of the activities. The book invites the reader to look beyond the scope of any of these three special disciplines. We editors hope that it will be thought-provoking by motivating researchers to establish cross-disciplinary connections between these neighbouring research areas. We would be particularly happy if it fosters further fruitful interactions between theory and experiment.

In contrast to original research articles and conference proceedings all book chapters are written in an introductory manner, which aims at making them accessible to graduate-level students. However, bringing together reviews from authors with so diverse scientific backgrounds, this book may also form a very useful reference for researchers already working in this field. All book chapters can in principle be read independently from each other, but the non-expert reader may strongly benefit from following the path we have suggested by our ordering of the single reviews. Since each contribution was written by different authors, the style and the type of the presentation varies from chapter to chapter. The reader may also find certain central aspects presented from different viewpoints, which in our eyes leads to a deeper understanding of the subject. We believe that altogether the book gives the reader a sensible account of what the main problems and methods in this newly emerging area of research are.

The idea for this monograph originated from the international WE Heraeus Seminar *Anomalous transport: experimental results and theoretical challenges*, organized by the three editors of this book, which took place in Bad Honnef, Germany, in July 2006. The photograph displayed after the introduction shows all participants of this meeting, including many of this book's authors. Triggered by the enthusiasm of all participants for the conference topic, this

book project got started. Unfortunately, one of the keynote speakers of this conference passed away right before the event took place. The first chapter of this book is therefore dedicated to Prof. Radu Balescu, one of the founders of the theory of nonequilibrium transport in plasmas, who before his death developed a very strong interest in anomalous dynamical processes.

The remainder of the book is organized into four main parts leading from mathematical foundations of anomalous transport over theoretical physical formulations to experimental results and applications. *Part I* introduces to *Fractional calculus and stochastic theory*. It starts with a two-fold opening: The first chapter provides the formal access road to the main topic of the book by introducing to fractional calculus, a mathematical technique that elegantly enables to deal with dynamical correlations in stochastic processes. The second chapter summarizes the physically intuitive side of the problem by sketching historical developments of anomalous transport and demonstrating basic ideas in form of simple examples. These two introductory chapters are followed by expositions elaborating on important mathematical and physical aspects of anomalous transport from the point of view of stochastic processes, which are in particular continuous time random walk theory, Lévy flights, numerical solutions of fractional diffusion equations and weak forms of ergodicity breaking.

Part II highlights *Dynamical systems and deterministic transport*. Its three chapters introduce to basic models generating anomalous deterministic transport such as intermittent one-dimensional maps, low-dimensional Hamiltonian systems exhibiting a mixed phase space and chains of (non)linear oscillators. These simple systems enable one to understand the microscopic origin of anomalous transport in terms of nonlinearities in the microscopic equations of motion. This leads to stickiness of trajectories to regular orbits or specific eigenmodes of dynamical systems by producing anomalous deterministic diffusion and anomalous heat conduction.

Part III focuses on *Anomalous transport in disordered systems*. Disorder is an appropriate characterization of many systems ranging from glasses or porous materials to biological systems such as actin networks or living cells. In this part some of the rare exact results for these systems are presented. Paradigmatic examples for anomalous behavior, such as structural glasses and random fractal structures, are considered in detail. The extension of anomalous transport in these contexts to reacting species introduces new aspects and poses new challenges, which are also elaborated in this part of the book.

The final *Part IV* reports *Applications to complex systems and experimental results*. It features a superstatistical Langevin-type theory, which identifies anomalous dynamics in experimental data of turbulence, cosmic rays and train delays. Anomalous transport by human dispersal, which is intimately related to epidemic spreading, is described subsequently. An important tool for experimentally observing anomalous molecular diffusion in porous struc-



tures as well as in polymer melts are NMR techniques, which is the subject of the following two chapters. The book concludes with new insights into the anomalous properties of the plasma membrane of biological cells, which are obtained by high-speed single-molecule tracking techniques.

We are extremely grateful to the Heraeus foundation for the financial support of the conference in Bad Honnef, which motivated us to edit this book. Particularly, we thank Dr. Ernst Dreisigacker for his advice and Jutta Lang for her help in successfully organizing this event. Thanks also go to Vera Palmer and to Ulrike Werner from Wiley-VCH publishers for their kind and efficient assistance in editing this book. R.K. acknowledges financial support by a grant from the British EPSRC under EP/E00492X/1, I.M.S. is grateful to the SFB555 – Complex Nonlinear Processes for support. We finally wish to thank our colleagues for the time and efforts they have invested in writing the individual chapters, thus making their joint expertise available in form of this multi-author monograph.

*London,
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In Memoriam: Radu Balescu

Boris Weyssow, Angelo Vulpiani, Francesco Mainardi, Raul Sánchez, and Diego del-Castillo-Negrete

This chapter is dedicated to Professor Radu Balescu, who very unexpectedly passed away on June 1, 2006, during a trip to Romania at the age of 73 years. Radu Balescu was invited as one of the keynote speakers to the conference on *anomalous transport: experimental results and theoretical challenges* in Bad Honnef, Germany, which took place about 1 month later in July 2006. This conference



Figure 1.1 Professor Radu Balescu († June 1, 2006); photo by Jacques Misguich, July 2005, Provence.

initiated the writing of the present book. Being a world class leader in the theory of nonequilibrium transport in plasmas, before his death Radu Balescu developed a very strong interest in anomalous dynamical processes. We start this chapter by including the abstract of the talk that Radu Balescu was intending to give in Bad Honnef.

Radu Balescu was planning to meet several colleagues at this conference for the very first time, with whom before he had intensive e-mail correspondences on topics of anomalous transport. These colleagues, as well as other conference participants who earlier had the privilege to meet Radu Balescu, have been invited to contribute to this book with their own obituaries. Their memories are added after a short summary of the scientific career of Radu Balescu, written by one of his closest collaborators.

1.1

Radu Balescu's Abstract for the Conference on Anomalous Transport in Bad Honnef

Statistical Mechanics and Strange Transport

Radu Balescu

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In recent years continuous time random walk (CTRW)s (CTRW) and fractional differential equations (FDE) have proved to be extremely successful modeling techniques for describing a wide range of applications for which standard diffusive transport is found experimentally to be inadequate. Yet there does not exist a complete justification of these concepts based on first principles of mechanics. A tractable starting point is provided by a "semidynamical" approach, based on a V-Langevin equation: an equation of motion of Newtonian (or Hamiltonian) type for a tracer particle moving in the presence of a random potential. Associated with it there is a "hybrid kinetic equation" (HKE) for the (stochastic) distribution function $f(x, t)$ of the positions. By standard methods of statistical mechanics an equation of evolution of the ensemble average of this function, called the "density profile" $n(x, t) = \langle f(x, t) \rangle$, is derived. The latter is, however, not closed because of its nonlocal character: on its right-hand side appears, under an ensemble average, the density profile evaluated at the fluctuating position of the particle, together with other fluctuating quantities. The usual "local approximation" provides a good description

of normal diffusive processes, but is inadequate for strange transport (sub- or supradiffusive).

Recently, Sánchez et al.¹ used an elegant method for overcoming the non-locality difficulty, based on functional integration techniques applied to the fluctuating particle trajectories (supposed to be self-similar) in order to derive a closed nonlocal equation for the density profile. Under special assumptions this equation can be reduced to a FDE.

In the present work we use a quite different approach, based on an analysis of the various types of propagators appearing in the treatment of the HKE. We introduce a nonlocal extension of an approximation similar to the Corrsin factorization assumption of turbulence theory. The result is a non-Markovian and nonlocal, formally linear equation, in which the rate of change of the density profile $n(x, t)$ at point x and time t is related to the values of this function at neighboring points $x + r$ and at past times $t - T$. Its structure is similar, but not identical (because of different approximations), to the equation of Sánchez et al. On the other hand, it can be shown that there exists an “equivalent” CTRW. The transition probability in the Montroll–Shlesinger equation describing the latter is related to the Eulerian velocity autocorrelation and to the ensemble-averaged propagator of the HKE $\langle G(x, t|x', t') \rangle$. Under certain special assumptions on the form of the latter two quantities (such as self-similar power-law forms), the equation for the density profile can be reduced to a fractional differential equation.

When viewed from a more general point of view, the equation of evolution for $n(x, t)$ is readily transformed into an equation for the average propagator $\langle G(x, t|x', t') \rangle$. The latter provides, as usual, the solution of the Dirichlet problem of the former equation for an arbitrary initial condition $n(x, 0)$. Besides its non-Markovian and nonlocal character, this equation appears to be explicitly nonlinear. Thus, not surprisingly, even in this simplest “nonlocal Corrsin-like” approach, one is faced with the complexity of a nonlinear process. A self-consistent theory of strange transport should therefore involve adequate approximation methods (such as renormalization techniques) for treating this equation.

1.2

The Scientific Career of Radu Balescu by Boris Weyssow

Professor Emeritus Radu Balescu unexpectedly passed away on June 1, 2006, during a trip in Romania at the age of 73 years. Professor at the Université Libre de Bruxelles, Member of the Royal Academy of Science, Humanities and Fine Arts of Belgium, Honorary Member of the Romanian Academy, Radu

1) see Ref. [1], the editors.

Balescu was at a very young age recognized internationally as a leader in the development of the Statistical Physics of charged particles (the well-known Balescu–Lenard collision operator) and of the theory of transport in magnetically confined plasmas.

Professor Radu Balescu was born in 1932 in Bucharest, Romania, from a Belgian mother, and acquired Belgian nationality in 1959. He studied at the Université Libre de Bruxelles and from 1957 until 1961 was the assistant of Professor Illya Prigogine (Nobel Laureate in 1977). Professor Radu Balescu made original contributions in the kinetic theory of plasmas and published his first book *Statistical Mechanics of Charged Particles* (Interscience, 465 pp) in 1963 followed in 1975 by a comprehensive treatise of statistical mechanics, *Equilibrium and Nonequilibrium Statistical Mechanics* (Wiley, 742 pp). The main purpose of this book was to achieve a presentation that would be as unified as possible with a very wide coverage of nonequilibrium theory, not treated in standard textbooks of that time. It also introduced for the first time in a textbook some new concepts, such as the renormalization group theory of critical phenomena, which turned out to be very successful in forthcoming years. This book is still very widely cited in the literature. Both this book and the previous one have been translated into Russian by leading Russian theorists.

Until becoming Emeritus in 1997, he had been Ordinary Professor at the University of Brussels. He has lectured or has been visiting Professor all over Europe, in Kyoto, Austin, and Mexico. Professor Radu Balescu was involved in the European fusion program for more than 30 years as a scientist and as the head of research unit of the ULB group in the Euratom-Belgian State Association.

In 1988, Professor Balescu published a magnum opus *Transport Processes in Plasmas: Volume 1: Classical Transport; Volume 2: Neoclassical Transport* (North-Holland, 803 pp). This set of two volumes presents a comprehensive review of the classical and the neoclassical theories of transport in plasmas, especially in the regime relevant to thermonuclear fusion. This is one of the first fully consistent presentations of the theory. It starts from first principles (Hamiltonian mechanics), going through kinetic theory, and ending in the explicit calculation of the transport coefficients and the discussion of the thermodynamic aspects of the transport. The second volume describes the nontrivial influence of the toroidal geometry (existing in magnetically confined plasmas) on the transport. Again, the main aim here was a unified presentation of the theory. It should be emphasized that these volumes encompass not only the work of many authors but that they are original in their conception and include many hundreds of pages of new consistent calculations and results. They are considered by many theorists as the current bible for plasma transport processes and has already been translated into Chinese.