Sensors

Volume 6

Optical Sensors

VCH

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A Comprehensive Survey

Edited by
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J. Hesse (Zeiss, Oberkochen, FRG)
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Preface to the Series

The economic realities of productivity, quality, and reliability for the industrial societies of the 21st century are placing major demands on existing manufacturing technologies. To meet both present and anticipated requirements, new and improved methods are needed. It is now recognized that these methods must be based on the powerful techniques employing computer-assisted information systems and production methods. To be effective, the measurement, electronics and control components, and sub-systems, in particular sensors and sensor systems, have to be developed in parallel as part of computer-controlled manufacturing systems. Full computer compatibility of all components and systems must be aimed for. This strategy will, however, not be easy to implement, as seen from previous experience. One major aspect of meeting future requirements will be to systematize sensor research and development.

Intensive efforts to develop sensors with computer-compatible output signals began in the mid 1970's; relatively late compared to computer and electronic measurement peripherals. The rapidity of the development in recent years has been quite remarkable but its dynamism is affected by the many positive and negative aspects of any rapidly emerging technology. The positive aspect is that the field is advancing as a result of the infusion of inventive and financial capital. The downside is that these investments are distributed over the broad field of measurement technology consisting of many individual topics, a wide range of devices, and a short period of development. As a consequence, it is not surprising that sensor science and technology still lacks systematics. For these reasons, it is not only the user who has difficulties in classifying the flood of emerging technological developments and solutions, but also the research and development scientists and engineers.

The aim of "Sensors" is to give a survey of the latest state of technology and to prepare the ground for a future systematics of sensor research and technology. For these reasons the publishers and the editors have decided that the division of the handbook into several volumes should be based on physical and technical principles.

Volume 1 (editors: T. Grandke/Siemens (FRG) and W. H. Ko/Case Western Reserve University (USA)) deals with general aspects and fundamentals: physical principles, basic technologies, and general applications.

Volume 2 and 3 (editors: W. Göpel/Tübingen University (FRG), T. A. Jones †/Health and Safety Executive (UK), M. Kleitz/LIESG-ENSEEG (France), I. Lundström/Linköping University (Sweden) and T. Seiyama/Tokuyama Soda Co. (Japan)) concentrate on chemical and biochemical sensors.

Volume 4 (editors: J. Scholz/Sensycon (FRG) and T. Ricolfi/Consiglio Nazionale Delle Ricerche (Italy)) refers to thermal sensors.

Volume 5 (editors: R. Boll/Vacuumschmelze (FRG) and K. J. Overshott/Gwent College (UK)) deals with magnetic sensors.

Volume 6 (editors: E. Wagner and K. Spenner/Fraunhofer-Gesellschaft (FRG) and R. Dändliker/Neuchâtel University (Switzerland)) treats optical sensors.

Volume 7 (editors: N. F. de Rooij/Neuchâtel University (Switzerland), B. Kloeck/Hitachi (Japan) and H. H. Bau/University of Pennsylvania (USA)) presents mechanical sensors.

Each volume is, in general, divided into the following three parts: specific physical and technological fundamentals and relevant measuring parameters; types of sensors and their technologies; most important applications and discussion of emerging trends.

It is planned to close the series with a volume containing a cumulated index.

The series editors wish to thank their colleagues who have contributed to this important enterprise whether in editing or writing articles. Thank is also due to Dipl.-Phys. W. Greulich, Dr. M. Weller, and Mrs. N. Banerjea-Schultz of VCH for their support in bringing this series into existence.

W. Göpel, Tübingen J. Hesse, Oberkochen J. N. Zemel, Philadelphia, PA

August 1991

Preface to Volume 6 of "Sensors"

Optical sensors are used in numerous scientific and technical applications. The recent increase of the utilization of optical sensors in research and development, in automation and all kinds of measuring systems justified the dedication of a volume on this topic in "Sensors". This volume gives a survey of the various measuring techniques in the wide field of optics. Fundamentals, technical aspects and applications are demonstrated. Classical optical instruments, already described in many physical textbooks, are completely omitted in order to allow a broader treatment of the various sensing techniques and their physical and technical limits.

This book is directed at interested students, engineers and scientists who require a profound background in optical sensing. They become acquainted with the present state-of-the art. Complicated mathematics, however, have been omitted; formulae are only given where necessary. The important optic laws and fundamentals are presented in a form easy to understand. Light propagation, its measurement and the principles of photoelectric conversion are described in Chapters 2, 4, and 5. A large variety of different components are used in sensor technology today. A survey of light sources, detectors and different kinds of optical parts is given in Chapters 3, 6, 7, 8, and 9. The availability of semiconductor light sources, detectors, and image sensors has provided us with revolutionary opportunities for new and cost-effective optical sensors. Different detection schemes depending on wavelength, phase, and pulsetime are shown in Chapters 10 to 14. Instruments approved in industry and noval concepts of optical sensors are treated. Fiber and integrated optics as more recent techniques are presented in Chapters 15, 16, 17, 20, and 21. The different techniques of optical sensing as machine vision and signal processing with its applications are described in Chapter 18 and 19. A further important aspect of optical sensors is the determination of surface morphology and deformation. Such measuring systems based on intensity or phase measurements are shown in Chapters 22 and 23.

We thank all the authors for their cooperation and the time-consuming work of writing such profound articles. We acknowledge the support and patience of the VCH staff in publishing this volume. In particular, we gratefully acknowledge the assistance of Mrs. N. Banerjea-Schultz and Mr. W. Greulich.

Elmar Wagner, René Dändliker, and Karl Spenner
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1 Fundamentals of Electromagnetic Waves

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Contents

1.1 Introduction

Optical sensing became a key technique for many measuring tasks. The manifold use of optics, however, requires a good understanding of the basic laws of light propagation. The principal objective of this article is to provide a unified and comprehensive overview of optical radiation and its propagation. The number of equations is kept small, complicated mathematics have been omitted and only basic laws have been compiled in order to facilitate the reading. A number of textbooks are given as references in order to provide more detailed information. Particular attention is paid to wave phenomena described by Maxwell equations. The most essential laws of planewave propagation, interference, refraction and reflection are discussed. Before the main properties of light are described, as we understand them today, a short historical overview shows the long and troublesome way from the first preception of seeing to modern optics.

Historical overview: In ancient times the observation of the sun and the stars with their periodical movements was the first beginning of natural science. Astronomical knowledge was specially important for seafaring men and tradesmen who had to cross the oceans or the deserts. When the Greeks started to expand and to found their colonnies around the coast lines of the Mediterranean sea, the University of Alexandria became a famous center of optical science. The Greek scientist Thales (640–546 BC) determined the height of a pyramid from the length of its shadow; he measured the shadow of a stick and calculated the unknown height by proportionality. He determined the significant parameters of the orbit of the sun and could predict the solar eclipse in 585 BC. The excellent book entitled "Elements of Geometry" published by Euclid (315–250 BC) has been the basis of many optical and geometrical problems over two thousand years. Light reflection and refraction was already studied by Ptolemy (70–147) of Alexandria. He measured the angles of incidence and refraction at different materials, for example, at air and water, and found an approximated expression valid for small angles of incidence.

The ideas about optical vision at that time represented by Democrit (460–370) and further advocated by Plato (428–347) and Euclid show the very general problem of many ancient philosophers and scientists who did not try to verify a hypothesis by experiments. They believed the eye itself emanates a stream of particles, a devine fire, which is combined with solar rays at the seen object and returns then to the eye. This eye ray emission hypothesis was accepted by many contempories over thousand years until the Arabian physicist Alhazen (965–1020) recognized that the light beam penetrates from the illuminated object in the eye. He also determined the height of the earth atmosphere. He measured the length of twilight in the evening hours by the right assumption that the scattered light of the sky causes the twilight after sunset. He also investigated reflection from nonplanar surfaces and formulated Alhazen's law of reflection that the incident and the reflected ray lie in the same plane. He worked on refraction of light at boundaries with different materials. His discoveries showed deep insights in the field of optics and were an optical guide until a rebirth of science begun during the Renaissance.

In the fifteenth century the great Leonardo da Vinci (1452–1519), wellknown as architect and sculptor, was also engaged in physical studies. He worked on different optical problems and built the first pinhole camera. Roughly one hundred years later a key invention was made by Galileo Galilei (1565-1642). He constructed the first astronomical telescope, the cornerstone for modern astronomy. With its help, the heliocentric system with planetary orbits was verified by Kepler (1571-1630).

The refraction of light at the boundary of two different materials, ie, air and water, has been studied by the ancient Greeks, by Alhazen and Kepler, but the first correct formular was found by the Dutch Willebrod Snell (1519-1626). Snell's law is one of the most basic laws of light propagation.

In the following time the scientists began to describe the observations by mathematical formulas; a new physical and more exact concept was born. Pierre Fermat (1601-1665) formulated his principle that light rays always propagate the shortest optical path length minimizing the transit time. This law nicely explained the observations of reflected and refracted light, whereby Fermat already concluded that the speed of light in a dense medium is lower than in air and, as a consequence, the light is refracted towards the normal of the boundary surface.

At this time, the nature of light was unknown. René Descartes (1596-1650) proposed the idea of a luminiferous ether. The ether should be a very tenous fluid-like medium and can penetrate through any transparent medium. A modified concept was later discussed as a carrier medium for electromagnetic waves until the beginning of the twentieth century.

Some scientists postulated light beams as a jet of particles. Isaak Newton (1642-1726) had also been in favor of this theory, but he conceeded that the corpuscular concept does not well explain diffraction and interference. He explained refraction by attractive forces between particles and the refracting medium. The attractive force is stronger in a denser medium and, therefore, the ray is more bent towards the normal. As a consequence of this theory, Newton believed that the light speed is higher in the denser medium, contrary to Fermat's law. The dispersion phenomena on different glass prisms was explained by Newton with the help of the different mass of the particles. He stated that dispersion of the refractive index follows his concept of refraction, whereas attractive forces bend the particles. For this reason, Newton believed that lenses without chromatic aberation are impossible and began the development of his reflector telescope.

Christian Huygens (1629-1695) could not agree with Newton's concept of particles and postulated that light is a propagating wave. He nicely explained Snell's law of refraction; every point of a wavefront generates a spherical wave whose propagating speed is lower in a dense medium. The velocity change causes the light refraction at the boundary of two different media. Thomas Young (1773-1829) in Scotland, following the wave concept, explained the phenomena of interference. He could describe the interference pattern of "Newton's rings" or of a double pinhole in terms of the path difference of two monochromatic waves by multiples of one wavelength. From the spacing of the interference pattern he determined the wavelength for the different colors of light. Jean Fresnel (1788-1827) intensively studied interference patterns. He measured intensity variations behind different apertures. He realized that Huygen's principle can explain the diffraction pattern only if the wavephase is also taken into account. He introduced the concept of constructive and destructive interference and derived mathematical expression to describe the observed fringe pattern. He already proposed the transverse character of light waves and showed that the ordinary and extraordinary rays of double refraction do not interfere. Faraday's (1791-1867) discovery of the connection between light and magnetism gave a strong indication as to the electromagnetic nature of light. A complete mathematical description of the electromagnetic character of light was found by James Maxwell (1831-1879). Abraham Michelson (1852-1931) and Williams Morley

(1831–1923) performed very accurate measurements of the speed of light. They were able to show that light is not connected with an ether, as was first proposed by Descartes and which has been an open question since that time. The speed of light was found to be constant and independent from the movement of the system considered. This result inspired Einstein (1879–1955) to develop his theory of relativity.

1.2 Optical Radiation

The optical radiation is part of the electromagnetic radiation, which extends over a continuous spectrum from low frequency radio waves to high energy cosmic rays. The propagation of electromagnetic waves is completely described by a system of differential equations, known as Maxwell's equations, and by the electromagnetic properties of the medium. The electromagnetic radiation has a continuous spectrum; the different spectral ranges are fixed by convention and there exists no discontinuity in the spectrum. The optical spectrum is imbedded between the high energy X-rays and the microwaves. The relatively wide range of ultraviolet and infrared radiation is usually further divided into different spectra according to the different techniques of observation. Figure 1-1 shows that the optical spectrum includes the ultraviolet radiation in the short wavelength range, the visible at wavelengths between 0.38 µm and 0.75 µm and the infrared at longer wavelengths. The visible light is only a very small part of the electromagnetic wave spectrum. The visible spectrum is determined by the sensitivity of the human eye. The optical impression is caused by the absorption of the wave in the retina of the eye. The frequency of the wave determines the physiological impression of a certain color.

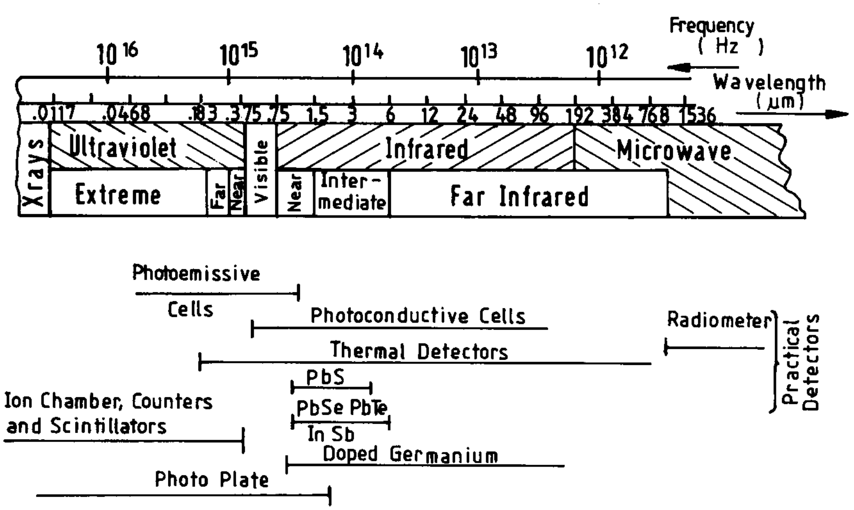

Figure 1-1. The optical spectrum can be classified in different regions whereby the visible region is only a very small part of it.

In general, optical radiation is produced by transitions between different energy levels in atoms or molecules. Transitions between rotational or vibrational levels in molecules produce infrared radiation, whereas electronic transitions emit visible or ultraviolet waves. The energy E of the optical radiation is given by Einstein's relation

$$E = h\nu \tag{1-1}$$

where ν is the frequency of the wave and $h = 6.6261 \times 10^{-34}\ \mathrm{Ws}^2$ is Planck's constant. Equation (1-1) says that the energy of optical radiation is represented by quanta, called photons. This corpuscular nature of light is complementary to the classical description of light by waves and manifests itself whenever light is emitted or absorbed. For the propagation of the radiation, however, the wave character is relevant.

Light propagates in vacuum with a speed of $c = 3 \times 10^8$ m/s. This is a universal constant. In other media the velocity is different and depends on the frequency of the radiation. This effect is known as dispersion. In this case one has also to distinguish between the phase velocity and the group velocity. The phase velocity describes how fast the phase fronts of the wave propagate, whereas the group velocity is relevant for the propagation of information (wave packets). The ratio $n = c/v$ of the speed of light c in vacuum and the phase velocity v in the medium is called index of refraction. The phase velocity v may be smaller or larger than the speed of light in vacuum. This is not inconsistent with the principle of special relativity, which states that information cannot travel at velocities larger than the speed of light in vacuum. As a consequence, the speed of light in vacuum is the largest possible group velocity.

1.3 Electromagnetic Waves

1.3.1 Electromagnetic Theory

The propagation of electromagnetic waves is completely described by Maxwell's equations and by the electromagnetic properties of the medium. The basic laws of electromagnetism were found by Coulomb, Ampère, Gauss, Faraday, Biot and Savart. Maxwell combined the different observations to a consistent set of differential equations by introducing four vector fields, which depend on space $\boldsymbol{r}$ and time t, namely the electric field $\boldsymbol{E}(\boldsymbol{r}, t)$, the magnetic field $\boldsymbol{H}(\boldsymbol{r}, t)$, the electric displacement $\boldsymbol{D}(\boldsymbol{r}, t)$ and the magnetic induction $\boldsymbol{B}(\boldsymbol{r}, t)$. In SI units Maxwell's equations take the form

$$\operatorname{curl} \boldsymbol{E} + \dot{\boldsymbol{B}} = 0 \tag{1-2}$$

$$\operatorname{div} \boldsymbol{B} = 0 \tag{1-3}$$

$$\operatorname{div} \boldsymbol{D} = \rho \tag{1-4}$$

$$\operatorname{curl} \boldsymbol{H} = \dot{\boldsymbol{D}} + \boldsymbol{j} \tag{1-5}$$

where ρ is the electrical charge density and $\boldsymbol{j}$ is the electrical current density.

To determine the field vectors from Maxwell's equations for a given distribution of currents and charges, these equations must be supplemented by relations which describe the behavior of substances under the influence of the electromagnetic field. These relations are known as material equations or constitutive equations of the medium. In general they are rather complicated; but if the material is isotropic and at rest they take the simple form

$$\boldsymbol{D} = \varepsilon_0 \varepsilon \boldsymbol{E} \tag{1-6}$$

$$\boldsymbol{B} = \mu_0 \mu \boldsymbol{H} \tag{1-7}$$

where ε_0 is the electric permittivity of vacuum, ε is the relative permittivity of the medium, μ_0 is the magnetic permeability of vacuum, and μ is the relative permeability of the medium. The value of μ_0 is defined as $4\pi \times 10^{-7}$ Vs, whereas the value of ε_0 is experimentally determined to be 8.85×10^{-12} As/Vm.

For transparent, charge free, homogeneous and isotropic media, like liquids and glasses, ε and μ are constant (div $\varepsilon = 0$, div $\mu = 0$) and both the current density and the charge density are zero ($\boldsymbol{i} = 0$, $\rho = 0$). The Maxwell equations take then the simple form

$$\text{curl } \boldsymbol{E} + \mu_0 \mu \frac{\partial \boldsymbol{H}}{\partial t} = 0 \tag{1-8}$$

$$\text{curl } \boldsymbol{H} - \varepsilon_0 \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} = 0 \tag{1-9}$$

$$\text{div } \boldsymbol{E} = 0, \quad \text{div } \boldsymbol{H} = 0 \tag{1-10}$$

From Equations (1-8), (1-9) and (1-10) the wave equation

$$\Delta \boldsymbol{E} - \varepsilon_0 \varepsilon \mu_0 \mu \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \tag{1-11}$$

for the electric field in an isotropic, homogeneous, transparent and insulating medium can be derived. This expression can still be used for quasi-homogeneous media where ε and μ are nearly constant over one wavelength, ie, if $\lambda \cdot \text{grad } \varepsilon \ll 1$ and $\lambda \cdot \text{grad } \mu \ll 1$.

The simplest solution is a plane wave, which can be expressed by sinosoidally varying functions in time t and space $\boldsymbol{r}$. The mathematical formulation of harmonic waves often makes use of the complex function formalism which simplifies the calculation. We will therefore describe the harmonic electric field $\boldsymbol{E}$ by

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 \cos(\omega t - \boldsymbol{k}\boldsymbol{r} - \Phi_0) = \boldsymbol{E}_0 \,\text{Re}\,\{\exp[\mathrm{i}(\omega t - \boldsymbol{k}\boldsymbol{r} - \Phi_0)]\} \tag{1-12}$$

where $\boldsymbol{E}_0$ is the amplitude, $\boldsymbol{k}$ is the wave vector, Φ_0 the phase, and ω the angular frequency, which is related to the frequency ν by

$$\omega = 2\pi\nu\,. \tag{1-13}$$

For convenience, the sign "Re" is often not explicitly written, but it is always understood that only the real part of the complex function is meant. The phase velocity v of such a wave is given by

$$v = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\omega}{|\boldsymbol{k}|^2}\boldsymbol{k} . \tag{1-14}$$

When we insert this plane wave into the wave equation (1-11), we get the relation

$$|\boldsymbol{k}|^2 - \varepsilon_0 \varepsilon \mu_0 \mu \omega^2 = 0 \tag{1-15}$$

which yields with Equation (1-14)

$$|\boldsymbol{v}| = v = c/n \tag{1-16}$$

where

$$c = 1/\sqrt{\varepsilon_0 \mu_0} \tag{1-17}$$

is the velocity of radiation in vacuum and

$$n = \sqrt{\varepsilon \mu} \tag{1-18}$$

is the refractive index. The speed of light in vacuum is a universal constant. Its value has been fixed exactly to be $c = 2.99792458 \times 10^8$ m/s. Since the physical unit of time, the second, can be reproduced very accurately, the unit of length, the meter, is defined accordingly. Finally, the exact value for ε_0 is also defined by c through Equation (1-17). The length of the wave vector $\boldsymbol{k}$ is then given by

$$|\boldsymbol{k}| = nk = n\frac{\omega}{c} = n\frac{2\pi}{\lambda} , \tag{1-19}$$

where k is the wavenumber and λ the wavelength in vacuum. From the Maxwell equations one finds also that the wave vector $\boldsymbol{k}$, the electric field $\boldsymbol{E}$, and the magnetic field $\boldsymbol{H}$ are mutually orthogonal; electromagnetic waves in a homogeneous medium are transverse waves. The orientation of the amplitude $\boldsymbol{E}_0$ of the electric field defines the polarization.

For a monochromatic wave, ie, for a single frequency ν, we can execute in the wave equation (1-11) the differentiation with respect to time. Then we get the Helmholtz wave Equation (1-20)

$$\Delta \boldsymbol{E}_\omega(\boldsymbol{r}) + n^2 k^2 \boldsymbol{E}_\omega(\boldsymbol{r}) = 0 . \tag{1-20}$$

This time independent wave differential equation describes the distribution of the waves in space and can also be used to calculate the propagation of light in inhomogeneous media, where the refractive index n is a function of the position in space.

The energy flux of an electromagnetic wave can also be determined from Maxwell's equations and is given by the Poynting vector

$$\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H} . \tag{1-21}$$

The high frequency of optical radiation does not allow to measure the time dependent fields directly. Only the time averaged value of the Poynting vector, which is called the light intensity, can be observed. Using the complex representation of waves, we get then

$$\langle \boldsymbol{P} \rangle = \frac{1}{T} \int_{t}^{t+T} P(t')\,\mathrm{d}t' = \frac{1}{2} (\boldsymbol{E} \times \boldsymbol{H}^*) . \qquad (1\text{-}22)$$

Since $\boldsymbol{E}$ and $\boldsymbol{H}$ are perpendicular and $|\boldsymbol{H}| = \sqrt{\varepsilon_0 \varepsilon / \mu_0 \mu}\ |\boldsymbol{E}|$ the intensity I becomes

$$I = \frac{1}{2} \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}}\ |\boldsymbol{E}|^2 . \qquad (1\text{-}23)$$

For optical frequencies, where μ is close to 1 and $\sqrt{\varepsilon} = n$, we get for the intensity

$$I = \frac{n}{2 Z_0} E^2 , \qquad Z_0 = 377\,\Omega . \qquad (1\text{-}24)$$

Polychromatic radiation is described by superposition of waves at different frequencies.

1.3.2 Interference and Coherence

The superposition of two light beams from a single source can produce intensity variations which are called interference. Wave optics is based on the principle of superposition stating that wave vectors can be added. This is true for all media where the field equations can be assumed as being linear.

The energy flux of two superposed beams is the sum of their Poynting vectors

$$\begin{aligned} \boldsymbol{P} &= \boldsymbol{P}_1 + \boldsymbol{P}_2 = (\boldsymbol{E}_1 + \boldsymbol{E}_2) \times (\boldsymbol{H}_1 + \boldsymbol{H}_2) = \\ &= \boldsymbol{E}_1 \times \boldsymbol{H}_1 + \boldsymbol{E}_2 \times \boldsymbol{H}_2 + \boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1 . \end{aligned} \qquad (1\text{-}25)$$

The last two terms which cross-correlate the two beams are called interference terms. Only when the wavefunction E_1 is perpendicular to H_2, ie, when E_1 and E_2 are equally polarized, interference occurs, otherwise the energy is the algebraic sum of both beams. If both beams are equally polarized, then the intensity becomes with Equation (1-23)

$$I = \frac{1}{2} \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}}\ [|\boldsymbol{E}_1|^2 + |\boldsymbol{E}_2|^2 + 2\,\mathrm{Re}\,\{\boldsymbol{E}_1 \boldsymbol{E}_2^*\}] . \qquad (1\text{-}26)$$

Two superposed plane waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ with equal amplitudes

$$\boldsymbol{E}_1 = \boldsymbol{E}_0 \exp\{\mathrm{i}\,[\omega t - \boldsymbol{k}_1 \boldsymbol{r} - \Phi_1]\}$$

$$\boldsymbol{E}_2 = \boldsymbol{E}_0 \exp\{\mathrm{i}[\omega t - \boldsymbol{k}_2 \boldsymbol{r} - \Phi_2]\}$$

gives then an intensity pattern

$$I = \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}}\, E_0^2 [1 + \cos(\Delta \boldsymbol{k} \boldsymbol{r} + \Delta \Phi)] \tag{1-27}$$

with

$$\Delta \boldsymbol{k} = \boldsymbol{k}_1 - \boldsymbol{k}_2 , \qquad \Delta \Phi = \Delta \Phi_1 - \Delta \Phi_2 .$$

The amplitude factor shows a harmonic standing wave. The intensity is zero for

$$\Delta \boldsymbol{k} \cdot \boldsymbol{r} + \Delta \Phi = (2m + 1) \frac{\pi}{2} , \tag{1-28}$$

where m is an integer (0, 1, 2, ...). The difference $\Delta \boldsymbol{k}$ can be caused by slightly different angles of the two beams, the difference $\Delta \Phi$ by a different optical ray path, eg, by optical beam splitters in an interferometer. Such an interference pattern can be generated only if both beams have the same wavelength; this means if the source is monochromatic.

Interference occurs only when the two waves are coherent. A real physical source is never strictly monochromatic, it has a spectral width $\Delta \nu$. The coherence length L is then

$$L = \frac{c}{\Delta \nu} . \tag{1-29}$$

The coherence length limits the optical path difference over which interference can be obtained. The coherence length can experimentally be determined with a Michelson interferometer. The different aspects of interferometry and its application for extremely sensitive measurements are discussed in Chapter 13.

1.3.3 Reflection and Refraction

A plane wave incident at an angle θ_i on a dielectric interface, as shown in Figure 1-2, is usually partly refracted and partly reflected. The refraction angle θ_t and the reflection angle θ_r can be determined from the boundary condition at the interface. Both wave vectors $\boldsymbol{k}_r$ and

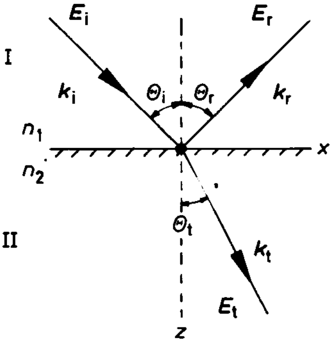

Figure 1-2.
Reflection and refraction at a boundary between two media with the index of refraction n_1 and n_2.

k_t of the reflected and the refracted wave remain in the plane of incidence which is defined by the normal to the surface, z-direction, and the incident wave vector k_i. The electric field along the x-axis must have the same phase velocity for the incident and for the reflected or refracted part of the wave. This yields the reflection law

$$|k_i| = |k_r| \text{ or } \theta_i = \theta_r . \tag{1-30}$$

The corresponding refraction law is

$$|k_i| = |k_t| \text{ or } n_1 \sin\theta_i = n_2 \sin\theta_t , \tag{1-31}$$

where n_1 and n_2 are the refractive indices in media I and II. Equation (1-31) is known as Snell's law. The amount of the reflected intensity depends on the polarization of the electric field. If the electric field of the incident wave is perpendicular to the plane of incidence (TE-wave), then the reflectivity is

$$R_\perp = \left(\frac{E_r}{E_i}\right)^2_{TE} = \left(\frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}\right)^2 = \left(\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}\right)^2 . \tag{1-32}$$

If the incident field vector is parallel to the incident plane (TM-wave) then the reflectivity is

$$R_\parallel = \left(\frac{E_r}{E_i}\right)^2_{TM} = \left(\frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}\right)^2 = \left(\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}\right)^2 . \tag{1-33}$$

The corresponding intensity transmittances are

$$T_\perp = \left(\frac{I_t}{I_i}\right)_{TE} = \frac{4 n_1 n_2 \cos\theta_i \cos\theta_t}{(n_1 \cos\theta_i + n_2 \cos\theta_t)^2} , \tag{1-34}$$

$$T_\parallel = \left(\frac{I_t}{I_i}\right)_{TM} = \frac{4 n_1 n_2 \cos\theta_i \cos\theta_t}{(n_2 \cos\theta_i + n_1 \cos\theta_t)^2} . \tag{1-35}$$

These equations are known as the Fresnel formulae. For randomly polarized light the reflectivity R_r is given by

$$R_r = \frac{R_\parallel + R_\perp}{2} . \tag{1-36}$$

The reflectivity as a function of the angle of incidence is shown in Figure 1-3 for polarization parallel and perpendicular to the plane of incidence. The refraction indices have been assumed to be $n_1 = 1$ and $n_2 = 1.5$, which is typical for an air-glass interface.

According to Equation (1-33) the reflectivity R of the the polarization parallel to the plane of incidence becomes zero at the Brewster angle θ_{iB}, given by the condition

$$\tan\theta_{iB} = \frac{n_2}{n_1} . \tag{1-37}$$

At the Brewster angle the reflected light is polarized perpendicular to the plane of incidence.

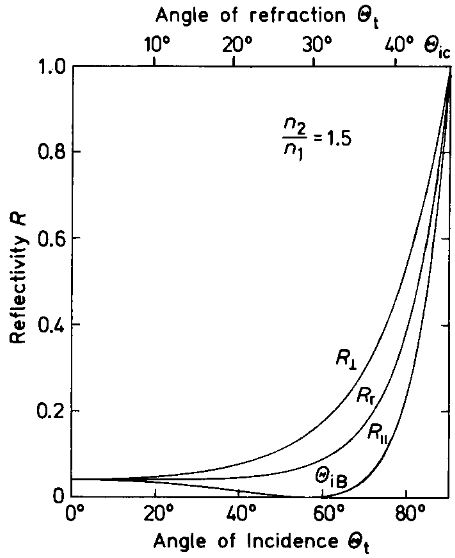

Figure 1-3.
Reflectivity R as a function of the angle of incidence for a light beam polarized perpendicularly and parallel to the plane of incidence and for an randomly polarized beam. θ_{iB} is the Brewster angle, where $R_{\parallel} = 0$ and θ_{ic} is the critical angle of total reflection.

When light is incident on the interface from the optically denser medium, ie, $n_1 > n_2$, then total internal reflection occurs if θ_i exceeds the critical angle θ_{ic}, given by

$$\sin\theta_{ic} = \frac{n_2}{n_1}. \tag{1-38}$$

The reflectivities are unity for both polarizations, ie, $R_{\parallel} = R_{\perp} = 1$ and $T_{\parallel} = T_{\perp} = 0$. However, the electric field penetrates into the medium of lower refractive index with an exponentially decreasing amplitude. The penetration depth is given by

$$\zeta = \frac{\lambda}{2\pi\sqrt{n_1^2 \sin^2\theta_i - n_2^2}}. \tag{1-39}$$

This field is not a real propagating wave. It is called an inhomogeneous or an evanescent wave. Although the reflected intensity is unity for all angles greater than the critical angle independently of the polarization, the phase of the totally reflected wave is different for the two orthogonal polarizations and depends on the angle of incidence θ_i. The phase shifts $\Phi_{\parallel}$ and $\Phi_{\perp}$ for the polarizations perpendicular and parallel to the plane of incidence, respectively, are found to be

$$\tan\frac{\Phi_{\perp}}{2} = \frac{\sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_1\cos\theta_i}, \qquad \tan\frac{\Phi_{\parallel}}{2} = \frac{n_1^2}{n_2^2}\tan\frac{\Phi_{\perp}}{2}. \tag{1-40}$$

As a consequence, if the incident wave is linearly polarized such that the orientation of the optical field has both perpendicular and parallel components, then there will be a phase difference between these two reflected components. This results in elliptical polarization (see Section 1.3.6).

1.3.4 Attenuation of Waves

So far, light was considered in transparent non-conductive media. In the presence of a conductivity σ, the electric field generates an electric current j according to Ohm's law

$$j = \sigma E . \tag{1-41}$$

For a harmonic time dependence

$$E = E_0 \exp(\mathrm{i}\omega t) \tag{1-42}$$

the Maxwell equation (1-5) gives

$$\operatorname{curl} H = \varepsilon_0 \varepsilon \frac{\partial E}{\partial t} + \sigma E = \varepsilon_0 \left(\varepsilon - \mathrm{i}\frac{\sigma}{\varepsilon_0 \omega}\right) \frac{\partial E}{\partial t} . \tag{1-43}$$

This equation can be reduced to the non-conducting case of Equation (1-9) by the introduction of the complex permittivity $\bar{\varepsilon}$

$$\bar{\varepsilon} = \varepsilon - \mathrm{i}\frac{\sigma}{\varepsilon_0 \omega} . \tag{1-44}$$

We further define a complex index of refraction corresponding to Equation (1-44)

$$\bar{n} = \sqrt{\bar{\varepsilon}} = n(1 - \mathrm{i}\kappa) . \tag{1-45}$$

The wave equation in a conducting medium is now described by the complex index of refraction and yields

$$E = E_0 \exp\{\mathrm{i}(\omega t - \bar{n} k_0 r)\} = E_0 \exp\{-n\kappa k_0 r\} \exp\{\mathrm{i}(\omega t - n k_0 r)\} , \tag{1-46}$$

where k_0 is the wavevector in vacuum with the wavenumber $k_0 = \omega/c$. The exponential decrease of the wave amplitude corresponds to an attenuation of the intensity in the direction of propagation z as

$$I(z) = I(0) \exp(-2n\kappa z) = I(0) \exp(-\alpha z) , \tag{1-47}$$

where α is the linear absorption coefficient.

Since we can take care of the absorption in the Maxwell equation by the complex index of refraction, the refraction and reflection laws shown in the last section are also valid for conducting media when we use the complex values. For metals the conductivity $\kappa \gg 1$ and therefore we obtain from Equation (1-45) and (1-33) for the reflectivity of metallic mirrors

$$R = \left(\frac{E_r}{E_i}\right)^2 \approx 1 . \tag{1-48}$$

1.3.5 Optical Dispersion

The electromagnetic radiation is significantly affected by the interaction with matter. When the radiation travels through gases, liquids or solids, wavelength and velocity are altered. The refractive index becomes a function of wavelength. The variation $dn/d\lambda$ is called optical dispersion.

In the optical frequency range, the light frequency is so high that ions usually do not respond to the wavefield. The electrons, however, give rise to a frequency dependent change of the permittivity. The frequency dependence of the refractive index n is sketched in Figure 1-4. Above the plasma frequency ω_p of the electrons the material is transparent and the refractive index increases with ω. This is known as "normal dispersion". Below the plasma frequency ω_p the electric field of the wave is strongly attenuated by the medium, the imaginary part κ of the refractive index (Equation (1-43)) becomes important, and we have a region of "anomalous dispersion" where n decreases with frequency. Real and imaginary part of the refractive index are not independant. They are related by the Kramers-Kronig relations.

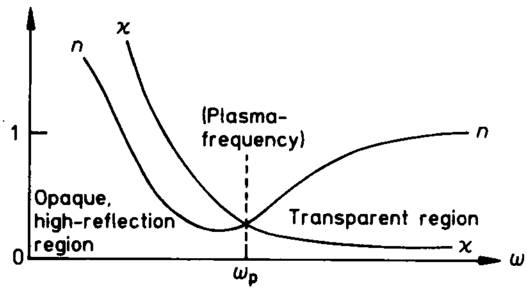

Figure 1-4.
Index of refraction n and extinction index κ versus frequency ω (for a metal)

For optical materials the dispersion is usually given as a function of wavelength rather than frequency, namely by $n(\lambda)$. As a result of the dispersion the laws of refraction will change with the wavelength. This introduces the problem of chromatic aberrations for lenses and imaging systems (see Chapter 3). Dispersion can also be used to analyze the light spectrum. Certain glasses with high dispersion are used in prism spectrometers. The resolving power $\Delta\lambda/\lambda$ of a prism with 10 cm base length is about 10^4. This is much less compared with those of a good diffraction gratings (see Chapter 3).

The dispersion $n(\lambda)$ describes the dependence of the phase velocity v on the wavelength through the relation

$$v(\lambda) = c/n(\lambda)\,, \tag{1-49}$$

where c is the speed of light in vacuum. The phase velocity v describes how fast the phase fronts of the wave propagate, whereas the group velocity v_g is relevant for the propagation of information. The ratio $n_g = c/v_g$ of the speed of light c in vacuum and the phase velocity v_g in the medium is called group index. The group index is obtained from the dispersion $n(\lambda)$ by the relation

$$n_g(\lambda) = n(\lambda) - \lambda dn/d\lambda\,. \tag{1-50}$$

The change of the group velocity with wavelength or frequency introduces the problem of pulse dispersion or pulse spreading in optical communication systems.

1.3.6 Birefringence

In the previous discussion of light propagation an isotropic medium was assumed. In anisotropic media, such as dielectric crystals, the permittivity is a tensor and the constitutive Equation (1-6) for the dielectric displacement becomes

$$D_i = \varepsilon_0 \sum \varepsilon_{ik} E_k \, . \tag{1-51}$$

When the coordinate system coincides with the principle axes of the crystal, the dielectric tensor becomes diagonal and Equation (1-51) simplifies to

$$D_x = \varepsilon_0 \varepsilon_{11} E_x \, , \quad D_y = \varepsilon_0 \varepsilon_{22} E_y \, , \quad D_z = \varepsilon_0 \varepsilon_{33} E_z \, . \tag{1-52}$$

Introducing Equation (1-52) instead of Equation (1-6) into Maxwell's equations (1-2)–(1-5) yields the wave equation for anisotropic materials. Solving the wave equation shows that to every direction of propagation correspond two definite direction of the electric displacement $\boldsymbol{D}$ which propagate with, in general, two different phase velocities. This effect is called birefringence. The two definite $\boldsymbol{D}$ directions are orthogonal and define two linear polarizations, which are the only two polarizations to propagate independently in the anisotropic material.

We now introduce a geometrical construction which permits us to determine the two polarization directions and their velocities. We construct an ellipsoid with axes of half lengths $n_x = \sqrt{\varepsilon_{11}}$, $n_y = \sqrt{\varepsilon_{22}}$, $n_z = \sqrt{\varepsilon_{33}}$ along the x, y and z-directions respectively (see Figure 1-5). The equation of such an ellipsoid is

$$x^2/n_x^2 + y^2/n_y^2 + z^2/n_z^2 = 1 \, . \tag{1-53}$$

The above ellipsoid is referred to as the index ellipsoid. For a given direction of wave propagation $\boldsymbol{k}$ one draws a plane perpendicular to $\boldsymbol{k}$ and passing through the center of the ellipsoid. This plane intersects the ellipsoid in an ellipse; the directions of the major and the minor axes correspond to the $\boldsymbol{D}$ directions of the two linearly polarized waves and their lengths give the respective refractive indices.

When all three permittivities ε_{11}, ε_{22} and ε_{33} are different, there are two directions of propagation for which the intersection ellipse becomes a circle, which means that the refractive index is the same for all polarization directions, the birefringence disappears. The corresponding directions of propagation are called the optical axes of the crystal and are not orthogonal. These crystals are called biaxial. If

$$\varepsilon_{11} = \varepsilon_{22} \neq \varepsilon_{33} \tag{1-54}$$

the index ellipsoid becomes an ellipsoid of revolution around the z-axis and the z-axis is the only optical axis; we have an uniaxial crystal. This is the case for crystals which have a single axis of threefold, fourfold, or sixfold symmetry, ie, calcite.