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Werner Buckel, Reinhold Kleiner

Superconductivity

Fundamentals and Applications

Second, Revised and Enlarged Edition

WILEY-VCH Verlag GmbH & Co. KGaA
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Heike Kamerlingh-Onnes

Foto reprinted with courtesy of
Professor Dr. C. J. Gorter, Kamerlingh-Onnes Laboratorium, Leiden
Preface to the Second Edition

Ten years have passed since the last edition of this book – ten years in which the development of superconductivity has advanced rapidly. Therefore, it was time to completely revise Superconductivity. The structure of the book has changed in many places. However, the basic concept of describing superconductivity in the simplest possible way has been kept, in order to provide some insights into this fascinating field for non-experts.

Werner Buckel passed away just a few weeks before completion of the manuscript. I hope to have continued his book in his spirit.

I express my sincere thanks to all colleagues who have contributed to this new edition of the book, in particular Klaus SchlenGA, Rudolf Huebener, Dieter Kölle, and Michael Meyer. Thanks are due to Mrs. Marie-Luise Fenske for her great help during the literature search, Geoff C. Amor and the lectors of Wiley-VCH for many suggestions and improvements. Particularly, I want to thank Rudolf Huebener for his excellent translation of the 6th German edition.

I would like to thank the following for providing unpublished photographs: Klaus-Peter Jüngst, Research Center Karlsruhe; Jochen Mannhart and Christof Schneider, Institute of Physics, University of Augsburg; Fritz Schick, Radiology Clinic, University of Tübingen; Klaus SchlenGA, Bruker Bio Spin Inc.; Tom H. Johansen, Superconductivity Laboratory, University of Oslo; Akira Tonomura, Hitachi Ltd.; CTF Systems Inc.; Bell Laboratories, Lucent Technologies Inc.; Institute Laue-Langevin; the International Superconductivity Technology Center (ISTEC); SUMO Association; and the Railway Technical Research Institute.

Tübingen, May 2004

Reinhold Kleiner
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Preface to the First Edition

For nearly five decades superconductivity could not be explained satisfactorily. Today we have a microscopic theory that can account for many phenomena and even quantitatively describe some of them. Therefore, at least in principle, the phenomenon of superconductivity is understood.

With the construction of large superconducting magnets, the technical application of superconductivity has started. Further applications in electronics, for example, for power transmission, are being investigated intensively. In some areas of electronic measuring techniques, superconductivity has even effected a breakthrough by increasing sensitivity by several orders of magnitude, for example, for the measurement of magnetic fields.

Therefore, in the future, interest in this phenomenon will not remain restricted to physicists. Instead, more and more engineers will be exposed to this phenomenon. Furthermore, the applications will have the effect that superconductivity will move increasingly into the view of the technically interested public.

This introduction to superconductivity is addressed to all these interested non-experts. I have tried to present our basic concepts about superconductivity as clearly as possible, on purpose without mathematical formulations. Based on these concepts, many phenomena are discussed. Also applications are treated in detail.

Of course, such an introduction can only present a limited selection of ideas and facts. Each such selection necessarily must be highly subjective. Leaving out many details, I tried to present an overall picture of superconductivity and in particular of its quantum nature. It did not seem practicable to follow the historic development. Instead, the phenomena are arranged and treated according to their inner context. Hence, without any doubt, many of the outstanding pioneer works will not be given proper attention. Also the lists of references do not provide a representative cross-section of the many thousands of papers that have appeared on the subject of superconductivity. For the interested reader, only a guide to the original literature is intended. In the case of special subjects, I shall refer to a number of excellent monographs.

The book will have achieved its goal if it can help to make a growing number of people more familiar with superconductivity. Furthermore, as a short summary, perhaps it can be a small help to those who investigate questions of superconductivity.
Many people have given me support during the preparation of this book by always being available for detailed discussions about emerging problems. I have to thank all of them. In particular, I would like to thank my colleague G. Falk, who never tired of discussions and answering my questions. Sincere thanks also go to my coworkers in Karlsruhe and in Jülich, among them in particular Dr. Baumann, Dr. Gey, Dr. Hasse, Dr. Kinder, and Dr. Wittig. I sincerely thank Dr. Appleton (EEDIRDC), Dr. Schmeissner (CERN), Dr. Kirchner (Munich), Prof. Rinderer (Lausanne), Dr. Essmann (Stuttgart), Dr. Voigt (Erlangen), as well as Siemens, Vakuumschmelze, and General Electric for kindly providing photographs. I am grateful to Physik Verlag for pleasant cooperation.

Finally and most sincerely I have to thank my dear wife, who has tolerated with great patience the many evenings and weekends that I have spent exclusively on the preparation of this book.

Jülich, August 1971

Werner Buckel
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Introduction

In physics, many phenomena result from the activity of specific mutual interactions. An important example is the relation between the uncorrelated thermal motion of the atomic building blocks of matter and the ordering forces between these building blocks. With increasing temperature, the thermal motional energy eventually becomes sufficiently large compared to some relevant ordering interaction energy that the ordered state of matter, established at low temperatures, breaks down. All phase transitions, say, from the liquid to the gaseous state, as well as the construction of the atoms themselves from the elementary constituents of matter, follow this rule. Therefore, it is not surprising that often unexpected new properties of matter, which subsequently also may become important for technology, are discovered in experiments performed under extreme conditions. Superconductivity is an example of such a discovery.

In the year 1908, Heike Kamerlingh-Onnes\(^1\), Director of the Low-Temperature Laboratory at the University of Leiden, finally achieved the liquefaction of helium as the last of the noble gases. He had founded this laboratory, which became world-famous under his leadership. At atmospheric pressure the boiling point of helium is 4.2 K. It can be reduced further by pumping. The liquefaction of helium extended the available temperature range near to the absolute zero point. The first successful experiment still needed the total combined manpower of the Institute. However, before long Kamerlingh-Onnes was able to perform extended experiments at these low temperatures. At first he started an investigation of the electrical resistance of metals.

At that time ideas about the mechanism of electrical conduction were only poorly developed. It was known that it must be electrons effecting the charge transport. Also the temperature dependence of the electrical resistance of many metals had been measured, and it had been found that near room temperature the resistance decreases linearly with decreasing temperature. However, at low temperatures this decrease was found to become weaker and weaker. In principle, there were three possibilities to be discussed:

\(^1\) A biography can be found in Spektrum der Wissenschaft, May 1997, pp. 84–89 (German edition of Scientific American).
1. The resistance could approach zero value with decreasing temperature (James Dewar, 1904; Fig. 1, curve 1).
2. It could approach a finite limiting value (Heinrich Friedrich Ludwig Matthiesen, 1864; Fig. 1, curve 2).
3. It could pass through a minimum and approach infinity at very low temperatures (William Thomson = Lord Kelvin, 1902; Fig. 1, curve 3).

In particular the third possibility was favored by the idea that at sufficiently low temperatures the electrons are likely to be bound to their respective atoms. Hence, their free mobility was expected to vanish. The first possibility, according to which the resistance would approach zero at very low temperatures, was suggested by the strong decrease with decreasing temperature.

Initially, Kamerlingh-Onnes studied platinum and gold samples, since at that time he could obtain these metals with high purity. He found that during the approach to zero temperature the electrical resistance of his samples reached a finite limiting value, the so-called residual resistance, a behavior corresponding to the second possibility discussed above. The value of this residual resistance depended upon the purity of the samples. The purer the samples, the smaller was the residual resistance. After these results, Kamerlingh-Onnes expected that in the temperature range of liquid helium ideally pure platinum or gold should have a vanishingly small resistance. In a lecture at the Third International Congress of Refrigeration in Chicago in 1913, he reported on these experiments and arguments. There he said [2]: “Allowing a correction for the additive resistance I came to the conclusion that probably the resistance of absolutely pure platinum would have vanished at the boiling point of helium.” These ideas were supported further by the quantum physics rapidly developing at that time. Albert Einstein had proposed a model of crystals, according to which the vibrational energy of the crystal atoms should decrease exponentially at very low temperatures. Since the resistance of highly pure samples, according to the view of Kamerlingh-Onnes (which turned out to be perfectly correct, as we know today), is only due to this motion of the atoms, his hypothesis mentioned above appeared obvious.

In order to test these ideas, Kamerlingh-Onnes decided to study mercury, the only metal at the time that he hoped could be extremely well purified by means of multiple distillation. He estimated that at the boiling point of helium he could barely just detect the resistance of mercury with his equipment, and that at still lower temperatures it should rapidly approach a zero value.
The initial experiments carried out by Kamerlingh-Onnes, together with his coworkers Gerrit Flim, Gilles Holst, and Gerrit Dorsman, appeared to confirm these concepts. At temperatures below 4.2 K the resistance of mercury, indeed, became immeasurably small. In his lecture of 1913 Kamerlingh-Onnes summarized this phase of his experiments and ideas as follows: “With this beautiful prospect before me there was no more question of reckoning with difficulties. They were overcome and the result of the experiment was as convincing as could be.”

However, during his further experiments using improved apparatus, he soon recognized that the observed effect could not be identical to the expected decrease of resistance. The resistance change took place within a temperature interval of only a few hundredths of a degree and, hence, it resembled more a resistance jump than a continuous decrease.

Figure 2 shows the curve published by Kamerlingh-Onnes [3]. As he himself commented [2]: “At this point [slightly below 4.2 K] within some hundredths of a degree came a sudden fall not foreseen by the vibrator theory of resistance, that had framed, bringing the resistance at once less than a millionth of its original value at the melting point. . . . Mercury had passed into a new state, which on account of its extraordinary electrical properties may be called the superconductive state.”

In this way also the name for this new phenomenon had been found. The discovery came unexpectedly during experiments that were meant to test some well-founded ideas. Soon it became clear that the purity of the samples was unimportant for the vanishing of the resistance. The carefully performed experiment had uncovered a new state of matter.

Today we know that superconductivity represents a widespread phenomenon. In the Periodic Table of the elements, superconductivity occurs in many metals. Here, at atmospheric pressure, niobium is the element with the highest transition temperature of about 9 K. Thousands of superconducting compounds have been found, and this development is by no means closed.
The scientific importance of the discovery of superconductivity can be seen from the fact that in 1913 Kamerlingh-Onnes was awarded the Nobel Prize in physics. At the time hardly anybody could have foreseen the richness in fundamental questions and interesting concepts resulting from this observation, and it took nearly half a century until superconductivity was understood at least in principle.2)

The vanishing of the electrical resistance below a “critical temperature” or “transition temperature” $T_c$ is not the only unusual property of superconductors. An externally applied magnetic field can be expelled from the interior of superconductors except for a thin outer layer (“ideal diamagnetism” or “Meissner-Ochsenfeld effect”), or superconductors can concentrate the magnetic field in the form of “flux tubes”. Here the magnetic flux is quantized3) in units of the “magnetic flux quantum” $\Phi_0 = 2.07 \times 10^{-15}$ Wb. The ideal diamagnetism of superconductors was discovered by Walther Meissner and Robert Ochsenfeld in 1933. It was a big surprise, since based on the induction law one would only have expected that an ideal conductor conserves its interior magnetic field and does not expel it.

The breakthrough in the theoretical understanding of superconductivity was achieved in 1957 by the theory of John Bardeen, Leon Neil Cooper, and John Robert Schrieffer (“BCS theory”) [4]. In 1972 they were awarded the Nobel Prize in physics for their theory. They recognized that at the transition to the superconducting state the electrons condense pairwise into a new state, in which they form a coherent matter wave with a well-defined phase, following the rules of quantum mechanics. Here the interaction of the electrons is mediated by the “phonons”, the quantized vibrations of the crystal lattice.

The formation of a coherent matter wave, often referred to as a “macroscopic wave function”, represents the key property of the superconducting state. We know similar phenomena from other branches of physics. The laser is based on a coherent wave represented by photons. In the phenomenon of superfluidity below the so-called lambda point, the helium atoms condense into a coherent matter wave [5, 6]. For the isotope $^4$He the lambda point is 2.17 K, and for $^3$He it is about 3 mK. Under the proper conditions, these superfluids can flow without any friction. Furthermore, recently the condensation of gases of alkali atoms like rubidium or potassium into a coherent quantum state has also been achieved. This “Bose-Einstein condensation” was predicted by Bose and Einstein in 1925. Only in 1995 could such condensates consisting of a few thousand atoms be prepared by means of special optical and magnetic refrigeration techniques at temperatures below 1 $\mu$K [7]. Also the discov-

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2 For a summary of the history of superconductivity, we refer to monograph [M1].
3 The magnetic flux $\Phi$ through a loop of area $F$ carrying a perpendicular and spatially homogeneous flux density $B$ is given by $\Phi = BF$. In the following we denote $B$ simply by “magnetic field”. In the general case of an arbitrarily oriented and spatially inhomogeneous magnetic field $B$ one must integrate over the area of the loop, $\Phi = \int B df$. The unit of magnetic flux is the weber (Wb), and the unit of magnetic field is the tesla (T). We have 1 Wb = 1 T m$^2$. If a loop is placed at a large distance around the axis of an isolated flux tube, we have $\Phi = \Phi_0$. 

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eries of the laser, of superfluidity, and of the Bose-Einstein condensation were honored by the awards of Nobel Prizes.4)

For more than 75 years superconductivity represented specifically a low-temperature phenomenon. This changed in 1986, when J. G. Bednorz and K. A. Müller discovered superconductors based on copper oxide. For their discovery the two scientists were awarded the Nobel Prize in physics in 1987 [8]. In the September 1986 issue of the journal “Zeitschrift für Physik B”, Bednorz and Müller published a paper with the cautionary title “Possible high \( T_c \) superconductivity in the Ba-La-Cu-O system” [9]. The authors had started from the hypothesis that substances with a pronounced Jahn-Teller effect5) could be superconductors with a particularly high transition temperature \( T_c \). They first studied compounds based on nickel oxide, since Ni\(^{3+} \) in an octahedron of oxygen atoms displays a strong Jahn-Teller effect. However, within this group of substances they did not find any superconductors. Then they systematically turned to copper oxide. Cu\(^{2+} \) in an octahedron of oxygen also displays a large Jahn-Teller effect. After only a few months Bednorz and Müller had samples showing a steep drop of the electrical resistance already above 30 K. Had they found superconductors with \( T_c > 30 \text{ K} \)? After more than 10 years of stagnation this would be a major breakthrough. Surprisingly, the paper received only little attention. There were doubts that superconductivity was really observed. The samples consisted of mixtures of several phases among which there were also electrically insulating substances. Therefore, they had large values of the specific electrical resistance. It could well be possible that some phase transition within the texture caused the drop in resistance.6) Hence, a convincing proof of superconductivity in these samples was still needed.

This proof was achieved by Bednorz, Müller, and Takashige by demonstrating the existence of the Meissner-Ochsenfeld effect [10]. Figure 3 shows the key measurement of this paper. Above 40 K both samples displayed the well-known paramagnetism of metals, which is nearly independent of temperature. Around 30 K, i.e. in the same temperature range where the drop in resistance appears, during cooling in a magnetic field, an increasing diamagnetism due to the Meissner-Ochsenfeld effect can be seen, and the magnetic susceptibility turns negative.

This result was highly surprising for the scientific community, because already in the mid-1960s Bernd Matthias and his coworkers had started a systematic study of the metallic oxides (see [11]). They searched among the substances based on the

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4 To Landau in 1962 (\(^1\)He); to Townes, Basov, and Prokhorov in 1964 (laser); to Lee, Osheroff, and Richardson in 1996 (\(^3\)He); and to Cornell, Wieman, and Ketterle in 2001 (Bose-Einstein condensation).

5 The Jahn-Teller effect is understood as the displacement of an ion away from the highly symmetric position relative to its environment. In this case the degeneracy of the states of the ion is lifted and its energy is overall lowered. A strong Jahn-Teller effect indicates a strong electron-phonon interaction. So the hypothesis of Müller and Bednorz was well consistent with the BCS theory.

6 In the mid-1940s during cooling below about 70 K sharp drops of the resistance in metallic sodium-ammonia solutions were observed, which initially were interpreted in terms of superconductivity. However, in fact they were due to sodium threads precipitating from the solution [R. A. Ogg Jr.: Phys. Rev. 69, 243 and 668 (1946); 70, 93 (1946)].
transition metal oxides, such as W, Ti, Mo, and Bi. They found extremely interesting superconductors, for example, in the Ba-Pb-Bi-O system; however, no particularly high transition temperatures were found.

During the turn of 1986/87 the “gold rush” set in, when it became known that the group of S. Tanaka in Japan could exactly reproduce the results of Bednorz and Müller. Now scientists in countless laboratories all over the world began to study these new oxides. Soon this extraordinary scientific effort yielded successful results. One could show that within the La-Sr-Cu-O system superconductors with transition temperatures above 40 K could be produced [12]. Only a few weeks later transition temperatures above 80 K were observed in the Y-Ba-Cu-O system [13, 14]. During this phase new results were more often reported in press conferences than in scientific journals. The media anxiously followed this development. With superconductivity at temperatures above the boiling point of liquid nitrogen ($T = 77$ K), one could envision many important technical applications of this phenomenon.

Today we are familiar with a large series of “high-temperature superconductors” based on copper oxide. Here the most studied compounds are $\text{YBa}_2\text{Cu}_3\text{O}_7$ (also “YBCO” or “Y123”) and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ (also “BSCCO” or “Bi2212”), which display...
maximum transition temperatures around 90 K. Many compounds have transition temperatures even above 100 K. The record value is claimed by HgBa$_2$Ca$_2$Cu$_3$O$_8$, having at atmospheric pressure a $T_c$ value of 135 K and at a pressure of 30 GPa a value as high as $T_c = 164$ K. Figure 4 shows the evolution of transition temperatures since the discovery by Kamerlingh-Onnes. The jump-like increase due to the discovery of the copper oxides is particularly impressive.

In Fig. 4 we have also included the metallic compound MgB$_2$, for which surprisingly superconductivity with a transition temperature of 39 K was detected only in 2000, even though this material has been commercially available for a long time [16]. This discovery also had a great impact in physics, and many essential properties of this material have been clarified already in the subsequent years. It turned out that MgB$_2$ behaves similarly to the “classical” metallic superconductors.

In contrast to this, many properties of the high-temperature superconductors (in addition also to other superconducting compounds) are highly unusual, as we will see many times during the course of this book. Even more than 15 years after the discovery of the oxide superconductors it is still unclear how Cooper pairing is accomplished in these materials. However, it seems likely that magnetic interactions play an important role.

Due to the discovery of the cuprates, the phenomenon of superconductivity is not restricted any more to a temperature range far away from that relevant for all organic life. One hopes that one day materials are found showing this phenomenon also at room temperature or even above it.
On the other hand, low temperatures become more and more accessible for day-to-day utilization. Refrigerators and cold boxes are regular household items. Just recently, large advances have been achieved in refrigeration techniques. Modern cryo-coolers today reliably reach temperatures of 30 K, or in some cases even 4.2 K and lower [17, 18]. Also cooling with liquid nitrogen is a standard procedure in many branches of industry. Hence, superconductivity will enter our daily lives more and more, in the fields of energy technology or microelectronics, for example.

Already, for some time, with liquid helium as the cooling agent, we have utilized metallic superconductors in the medical field, for the generation of high magnetic fields in nuclear spin tomography, or in magnetic field sensors. In field tests, magnetic field sensors made from YBa$_2$Cu$_3$O$_7$ are employed for the non-destructive testing of materials or for detecting magnetic cardiac signals. In the field of energy technology, the first prototypes of cables made from high-temperature superconductors are already operating. High-temperature superconductors can be kept in a well-stabilized state above or below strong magnets. In this way a contact-free bearing and motion nearly without any friction can be achieved, which is highly attractive in many fields of technology.

This book is meant to provide an initial exposure to the phenomenon of superconductivity. Only selected aspects could be dealt with. Some subjects have had to be summarized only briefly in order to keep the size of the book within reasonable limits. However, it is hoped that the book transmits some of the fascination that superconductivity has offered now for nearly a century.

References

3 H. Kamerlingh-Onnes: Comm. Leiden 120b (1911).

In the laboratory, by means of various refrigeration methods, temperatures down to only a few millikelvin (mK) can be sustained continuously. Based on nuclear spin demagnetization, final temperatures in the microkelvin (µK) range and below are reached. For a summary, see the monographs [M32] and [M33].


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1 Fundamental Properties of Superconductors

The vanishing of the electrical resistance, the observation of ideal diamagnetism, or the appearance of quantized magnetic flux lines represent characteristic properties of superconductors that we will discuss in detail in this chapter. We will see that all of these properties can be understood, if we associate the superconducting state with a macroscopic coherent matter wave. In this chapter we will also learn about experiments convincingly demonstrating this wave property. First we turn to the feature providing the name “superconductivity”.

1.1 The Vanishing of the Electrical Resistance

The initial observation of the superconductivity of mercury raised a fundamental question about the magnitude of the decrease in resistance on entering the superconducting state. Is it correct to talk about the vanishing of the electrical resistance?

During the first investigations of superconductivity, a standard method for measuring electrical resistance was used. The electrical voltage across a sample carrying an electric current was measured. Here one could only determine that the resistance dropped by more than a factor of a thousand when the superconducting state was entered. One could only talk about the vanishing of the resistance in that the resistance fell below the sensitivity limit of the equipment and, hence, could no longer be detected. Here we must realize that in principle it is impossible to prove experimentally that the resistance has exactly zero value. Instead, experimentally, we can only find an upper limit of the resistance of a superconductor.

Of course, to understand such a phenomenon it is highly important to test with the most sensitive methods, to see if a finite residual resistance can also be found in the superconducting state. So we are dealing with the problem of measuring extremely small values of the resistance. Already in 1914 Kamerlingh-Onnes used by far the best technique for this purpose. He detected the decay of an electric current flowing in a closed superconducting ring. If an electrical resistance exists, the stored energy of such a current is transformed gradually into Joule heat. Hence, we need only monitor such a current. If it decays as a function of time, we can be certain that a resistance still exists. If such a decay is observed, one can deduce an
upper limit of the resistance from the temporal change and from the geometry of the superconducting circuit.

This method is more sensitive by many orders of magnitude than the usual current-voltage measurement. It is shown schematically in Fig. 1.1. A ring made from a superconducting material, say, from lead, is held in the normal state above the transition temperature \( T_c \). A magnetic rod serves for applying a magnetic field penetrating the ring opening. Now we cool the ring below the transition temperature \( T_c \) at which it becomes superconducting. The magnetic field 1) penetrating the opening practically remains unchanged. Subsequently we remove the magnet. This induces an electric current in the superconducting ring, since each change of the magnetic flux \( \Phi \) through the ring causes an electrical voltage along the ring. This induced voltage then generates the current.

If the resistance had exactly zero value, this current would flow without any change as a “permanent current” as long as the lead ring remained superconducting. However, if there exists a finite resistance \( R \), the current would decrease with time, following an exponential decay law. We have

\[
I(t) = I_0 e^{-\frac{R}{L} t}
\]

Here \( I_0 \) denotes the current at some time that we take as time zero; \( I(t) \) is the current at time \( t \); \( R \) is the resistance; and \( L \) is the self-induction coefficient, depending only upon the geometry of the ring. 2)

1 Throughout we will use the quantity \( B \) to describe the magnetic field and, for simplicity, refer to it as “magnetic field” instead of “magnetic flux density”. Since the magnetic fields of interest (also those within the superconductor) are generated by macroscopic currents only, we do not have to distinguish between the magnetic field \( H \) and the magnetic flux density \( B \), except for a few cases.

2 The self-induction coefficient \( L \) can be defined as the proportionality factor between the induction voltage along a conductor and the temporal change of the current passing through the conductor:

\[
U_{\text{ind}} = -L \frac{dI}{dt}.
\]

The energy stored within a ring carrying a permanent current is given by \( \frac{1}{2}LI^2 \). The temporal change of this energy is exactly equal to the Joule heating power \( RI^2 \) dissipated within the resistance. Hence, we have

\[
-\frac{d}{dt} \left( \frac{1}{2}LI^2 \right) = RI^2.
\]

One obtains the differential equation

\[
-\frac{dI}{dt} = \frac{R}{L} I,
\]

the solution of which is (1.1).
For an estimate we assume that we are dealing with a ring of 5 cm diameter made from a wire with a thickness of 1 mm. The self-induction coefficient $L$ of such a ring is about $1.3 \times 10^{-7}$ H. If the permanent current in such a ring decreases by less than 1% within an hour, we can conclude that the resistance must be smaller than $4 \times 10^{-13} \Omega$. This means that in the superconducting state the resistance has changed by more than eight orders of magnitude.

During such experiments the magnitude of the permanent current must be monitored. Initially [1] this was simply accomplished by means of a magnetic needle, its deflection in the magnetic field of the permanent current being observed. A more sensitive setup was used by Kamerlingh-Onnes and somewhat later by Tuyn [2]. It is shown schematically in Fig. 1.2. In both superconducting rings 1 and 2 a permanent current is generated by an induction process. Because of this current both rings are kept in a parallel position. If one of the rings (here the inner one) is suspended from a torsion thread and is slightly turned away from the parallel position, the torsion thread experiences a force originating from the permanent current. As a result an equilibrium position is established in which the angular moments of the permanent current and of the torsion thread balance each other. This equilibrium position can be observed very sensitively using a light beam. Any decay of the permanent current within the rings would be indicated by the light beam as a change in its equilibrium position. During all such experiments, no change of the permanent current has ever been observed.

A nice demonstration of superconducting permanent currents is shown in Fig. 1.3. A small permanent magnet that is lowered towards a superconducting lead bowl generates induction currents according to Lenz’s rule, leading to a repulsive force acting on the magnet. The induction currents support the magnet at an equilibrium height. This arrangement is referred to as a “levitated magnet”. The magnet is supported as long as the permanent currents are flowing within the lead bowl, i.e. as long as the lead remains superconducting. For high-temperature superconductors such as YBa$_2$Cu$_3$O$_7$ this demonstration can easily be performed using liquid nitrogen in regular air. Furthermore, it can also serve for levitating freely real heavyweights such as the Sumo wrestler shown in Fig. 1.4.

The most sensitive arrangements for determining an upper limit of the resistance in the superconducting state are based on geometries having an extremely small self-induction coefficient $L$, in addition to an increase in the observation time. In this way the upper limit can be lowered further. A further increase of the sensitivity is accomplished by the modern superconducting magnetic field sensors (see Sect. 7.6.4). Today we know that the jump in resistance during entry into the superconducting state amounts to at least 14 orders of magnitude [3]. Hence, in the superconducting state a metal can have a specific electrical resistance that is at most about 17 orders of magnitude smaller than the specific resistance of copper, one of

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3 For a circular ring of radius $r$ made from a wire of thickness $2d$ also with circular cross-section ($r \gg d$), we have $L = \mu_0 r \ln(8r/d) - 1.75$ with $\mu_0 = 4\pi \times 10^{-7} \text{ V s/A m}$. It follows that $R \leq \frac{-\ln 0.99 \times 1.3 \times 10^{-7}}{3.6 \times 10^3} \frac{\text{V}}{\text{A m}} \approx 3.6 \times 10^{-13} \Omega$. 


our best metallic conductors, at 300 K. Since hardly anyone has a clear idea about “17 orders of magnitude”, we also present another comparison: the difference in resistance of a metal between the superconducting and normal states is at least as large as that between copper and a standard electrical insulator.

Following this discussion it appears justified at first to assume that in the superconducting state the electrical resistance actually vanishes. However, we must point out that this statement is valid only under specific conditions. So the resistance can become finite if magnetic flux lines exist within the superconductor. Furthermore, alternating currents experience a resistance that is different from zero. We return to this subject in more detail in subsequent chapters.

Fig. 1.2 Arrangement for the observation of a permanent current (after [2]). Ring 1 is attached to the cryostat.