Linear and Nonlinear Structural Mechanics

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To our wives
Samirah and Lifien

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PREFACE

In the last few decades engineering materials have gone through different phases and can be categorized into four groups: composite materials, smart materials, micro- and nano-materials, and materials for Gossamer (or ultra-lightweight deployable/inflatable) space structures. The factors that distinguish them are the ability for tailoring; coupling of mechanical, electrical, magnetic, and/or thermal fields; tremendous size decrease; and tremendous increase in size but decrease in mass density. Research in continuum mechanics has followed and advanced through these phases, but the major challenge is still the modeling and analysis of structures built with such materials. Although modeling of such structures built with materials in different groups requires consideration of different effects, all these structures can be modeled as continuum media, especially as cables, beams, plates, and/or shells. However, such structures are usually nonlinear by nature or, in some cases, by design.

For example, thin-walled structures play an important role in the design of aircraft structures because they are often designed to operate in the postbuckling range in order to reduce structural weight. In recent years, the rapid development and use of huge deployable/inflatable structures in aerospace and space exploration has stimulated extensive research into fully nonlinear modeling and analysis, thermal buckling, and control of highly flexible structures. Also, the increasing use of laminated composite materials in modern structures has stimulated the development of refined structural theories that can account for nonclassical effects, such as transverse shear stresses, interlaminar peeling stresses, torsional warping, free-edge effects, and warping restraint effects. Nonlinear problems considered in such structures are mostly those of postbuckling analysis, prediction of stability, and flutter analysis. However, the nonlinear dynamics of such structures have to be ascertained in order to design and control them. Hence, nonlinear modeling and analysis of structures becomes a complex but important step in advancing the design and optimization of modern structural systems.

This book presents mathematically consistent and systematic derivations of comprehensive structural theories developed by the authors as well as well-known linear and nonlinear structural theories in the literature, details the physical meaning of linear and nonlinear structural mechanics, shows how to perform nonlinear structural analysis, points out important nonlinear structural dynamic behaviors, and provides ready-to-use governing equations and boundary conditions, ranging from simple linear ones to complex nonlinear ones, for strings, cables, beams, plates, and shells. The major goal of this book is to close the gap between the practicing engineer and the applied mathematician in the modeling and analysis of geometrically nonlinear structures. This book is written in a common vector-based mathematical language that is understandable by most engineering students. A unique unified approach, more general than those found in most structural mechanics books, is used to model geometric nonlinearities of structures. As a result, the reader can readily extend the methods to formulate and analyze different and/or more complex structures. This book is intended to be a graduate-level text and a reference book for graduate students and structural engineers in mechanical, civil, and aeronautical engineering or in applied mechanics who have had courses in mechanics of materials, ordinary and partial differential equations, and vibrations.

The text is organized into nine chapters. Chapter 1 is essentially an introduction to modeling issues, dynamic characteristics of linear and nonlinear discrete systems, and methods for analyzing linear and nonlinear continuous systems. Chapter 2 presents a self-contained treatment of the basic principles of structural mechanics. Chapter 3 presents linear and geometrically exact formulations, nonlinear analysis, and nonlinear dynamics of taut strings, cables, and bars. Chapter 4 presents linear and geometrically exact formulations of beams. Chapter 5 presents nonlinear analysis and dynamics of different beams, including microbeams for MEMS devices. Chapter 6 presents the mathematics needed for geometrically exact modeling of plates and shells. Chapter 7 presents linear and geometrically exact formulations of plates. Chapter 8 presents nonlinear analysis and dynamics of different plates, including MEMS-based microplates and thermally loaded circular and annular plates. Chapter 9 presents linear and geometrically exact formulations, nonlinear analysis, and nonlinear dynamics of shells. It also includes the nonlinear dynamics of circular cylindrical shells and spherical shells. A long list of references, by no means complete or up-to-date but consisting of most of the important articles in the literature, is provided in the Bibliography at the end of the book.

The authors wish to acknowledge with great appreciation the many valuable suggestions from their colleagues. In particular, the authors thank Dr. Haider Arafat for his valuable comments, thorough proofreading of the entire manuscript, and preparing many of the tables and illustrations. Also, the authors thank Drs. Pramod Malatkar and Eihab Abdel-Rahman for their valuable comments and thorough proofreading of parts of the manuscript. We wish also to thank Mr. Nader Nayfeh for editing and preparing the postscript files for some of the illustrations. Many results presented in this book were obtained under research grants supported by NSF, NASA, ARO, and AFOSR; the support is gratefully acknowledged. Last but not least, the authors want to express their appreciation to Mrs. Sally Shrader for repeatedly typing, correcting, and beautifying the manuscript.

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INTRODUCTION

Mechanics of elastic structures includes linear and nonlinear modeling, statics, dynamics, buckling, postbuckling, flutter, stability, and analyses of stresses, strains, and failure. This book is primarily concerned with the nonlinear modeling and dynamics of elastic structures.

In the literature, there are many different theories of beams, plates, and shells, and the number of structural theories is increasing because of the increase in the use of computers and new structural materials and applications. Moreover, the demand of materials by today's technologies has become so diverse that it often cannot be met by single-component materials. Also, structural engineers are confronted by the challenge of strict requirements of high vehicle performance, less materials, less weight, high safety factors, etc. These requirements cannot be met, in general, except by the use of composites. Although composite materials have characteristics that are better than those of isotropic materials and hence can meet complex design requirements, some nonclassical structural effects, such as transverse shear strains, peeling stresses, free-edge effects, and warping restraint effects, which are usually neglected in isotropic materials, are significant in these materials. Hence, modeling of modern composite structures has become a complex but important step in design, and refined structural theories are needed for the analysis of composite structures.

1.1 STRUCTURAL ELEMENTS

In terms of geometries and loading conditions, structures can be divided into six groups: cables, bars, beams, membranes, plates, and shells. Cables are one-dimensional structures, which can only sustain extensional loads. The buckling loads of cables are zero, and hence they cannot sustain compression loads. Strings are pre-tensioned and initially straight cables.

Bars are one-dimensional structures which can sustain extensional, compressional, and torsional loads. If a bar is only subjected to longitudinal tensile loads, it is usually called a rod. If a bar is only subjected to longitudinal compressive loads, it is usually called a column. Rods and columns are two-force members, and trusses consist of bars. Beams are structures having one dimension much larger than the other two and primarily subjected to lateral loads, resulting in bending of their reference axes. A general beam should be able to sustain extension, compression, bending, transverse shear (flexure), and twisting loads. In other words, cables, strings, bars, rods, and columns are special cases of a general beam, and arches are initially curved beams.

Plates are initially flat structures having two dimensions much larger than the third and can sustain extension, compression, inplane shear, bending, twisting, and transverse shear loads. A membrane is a two-dimensional structure, which can only sustain extensional and inplane shear loads. The membrane stresses of a plate can contribute significantly to its strength. Shells are initially curved structures having two dimensions much larger than the third and can sustain extension, compression, inplane shear, bending, twisting, and transverse shear loads. Shells are the most general engineering structures; they include plates and membranes as special cases. Because the initial curvatures of a shell offer some geometric stiffnesses, the strength of a shell depends on its geometry as well as material.

The complexity of a dynamical system depends on whether the relation between the input and the output is linear or nonlinear, the number of independent and dependent variables used in the system modeling, and the number of parameters. To describe the motion of a discrete dynamical system consisting of N isolated particles, one needs $3N$ dependent variables $u_i(t)$, $v_i(t)$, and $w_i(t)$, $i = 1, 2, \ldots, N$, where u_i, v_i, and w_i are the displacement components of the ith particle along three perpendicular directions and t denotes time. Such a system is called a $3N$-degree-of-freedom system and a $6N$-dimensional system because $3N$ dependent variables are used and second-order time derivatives of all dependent variables are involved.

Any continuous dynamical system can be described by an infinite number of mathematical particles having a volume $dx\, dy\, dz$ if a Cartesian coordinate system xyz is used, and the displacement components of the ith particle are $u(x_i, y_i, z_i, t)$, $v(x_i, y_i, z_i, t)$, and $w(x_i, y_i, z_i, t)$, where (x_i, y_i, z_i) is the location of the ith particle at $t = 0$. For a continuous system without fracture, the distance between two adjacent particles is infinitesimal and particle displacements can only vary continuously from particle to particle. Hence, only three dependent variables u, v, and w are needed but they are continuous functions of three independent spatial variables x, y, and z and time t; that is,

$$u = u(x, y, z, t), \; v = v(x, y, z, t), \; w = w(x, y, z, t) \tag{1.1.1}$$

If the dependence of u, v, and w on x, y, z, and t can be separated as

$$u = U(x, y, z)q(t), \; v = V(x, y, z)q(t), \; w = W(x, y, z)q(t) \tag{1.1.2}$$

and if the spatial functions U, V, and W are known, then there is only one unknown dependent variable $q(t)$, which is called a generalized coordinate in structural engineering, and the system has a single degree of freedom; $q(t)$ is governed by a second-order differential equation. However, the spatial functions U, V, and W are load-dependent or unknown or even do not exist (in other words, they are time-dependent functions). Hence, to find the solution of a continuous medium subjected to a general load, engineers usually use an infinite number of assumed, known spatial functions to approximate the solution as

$$u = \sum_{i=1}^{\infty} U_i(x, y, z) q_i(t), \ v = \sum_{i=1}^{\infty} V_i(x, y, z) q_i(t), \ w = \sum_{i=1}^{\infty} W_i(x, y, z) q_i(t) \tag{1.1.3}$$

In order to have a convergent solution for a general loading condition, one needs to choose the spatial functions from a set of complete functions.

Structures are three-dimensional continuous systems. Structural engineers usually use linear eigenfunctions, which are obtained from the unforced undamped governing equations and are called mode shapes, as the spatial functions. Because an infinite number of dependent variables $q_i(t)$ is involved, any continuous system has infinite degrees of freedom.

Although solving a three-dimensional structural problem may not be impossible, it may require an insurmountable amount of work. Fortunately, most structural elements have one or two of their geometric dimensions much smaller than the others, and hence their motions can be described by the particles on a reference line in the case of cables, bars, and beams or a reference plane in the case of membranes, plates, and shells. Their spatial functions depend on one or two independent variables. However, in addition to the displacements u, v, and w of a general point on the reference line or plane, extra displacement variables are needed in order to describe the motion of a general point that is not on the reference line or plane, which results in different structural theories. Moreover, depending on the type of loading and/or the geometric constraints on the structure, one can adopt some more assumptions about the stress distribution and/or the displacement distribution to simplify the model.

Mathematical modeling is a form of an approximation theory; different models are the result of adopting different assumptions. Simple or rough models are easy to solve, but their accuracy in predicting the system behavior may be poor. To improve the accuracy of structural models, researchers developed refined structural theories by relaxing some of the constraints on the displacement and/or stress field representations. However, relaxing constraints results in an increase in the number and order of the governing equations. In other words, to solve problems modeled by refined theories requires more effort. Consequently, choosing an appropriate model for a specific problem is critical in the analysis.

There are no rules about how to choose a right model, and hence one can only make a decision based on experience, accuracy requirement, and the goal of the analysis. For example, in the dynamic analysis of a pendulum, it can be treated as a rigid body if the forcing frequency is far below its first natural bending frequency. When the forcing frequency is close to or above its first natural bending frequency, the pendulum needs to be treated as an Euler-Bernoulli beam, where transverse shear strains are neglected. When the forcing frequency is higher than the first bending frequency, the shear effect may become important and Timoshenko's beam theory or a higher-order shear-deformation theory is required in order to have accurate results. Moreover, to study the initiation of delamination of laminated composite plates, one needs to choose a plate model that is able to accurately predict the interlaminar shear and peeling stresses because composite laminates are weak in shear.

1.2 NONLINEARITIES

Nonlinear systems are those for which the principle of superposition does not hold. Nature abounds with nonlinear systems; in fact they are the rule rather than the exception. The sources of nonlinearities can be material or constitutive, geometric, inertia, body forces, or friction. The constitutive nonlinearity occurs when the stresses are nonlinear functions of the strains. The geometric nonlinearity is associated with large deformations in solids, such as beams, plates, frames, and shells, resulting in nonlinear strain-displacement relations (e.g., mid-plane stretching, large curvatures of structural elements, large strains, and large rotations of elements). The inertia nonlinearity may be caused by the presence of concentrated or distributed masses; in a Lagrangian formulation, the kinetic energy is a function of the generalized coordinates as well as their rates and, in fluid flow, the acceleration includes a nonlinear convective term. Other examples include Coriolis and centripetal accelerations. The nonlinear body forces are essentially magnetic and electric forces. The friction nonlinearity occurs because the friction force is a nonlinear function of the displacement and velocity, such as dry friction and backlash.

The nonlinearities may appear in the governing partial-differential equations, or the boundary conditions, or both. To some extent, the form of the nonlinearity appearing in the equations and boundary conditions depends on the coordinate system used and the orientation of the body forces, such as gravity. Examples of nonlinear boundary conditions include free surfaces in fluids and deformation-dependent constraints.

The nonlinearities considered in this book are primarily geometric arising from large rotations and displacements and electric arising from the proximity of two plate electrodes to each other. To include the effects of material nonlinearities, one only needs to replace the constant stiffnesses with displacement-dependent ones; they are usually obtained from experiments.

Suppose that a metallic panel is subjected to a longitudinal compressive load. As long as the panel remains flat, it is in equilibrium, and it can only fail by crushing; that is, the compressive stress exceeds the stress that the material can withstand. However, it is well known that panels, whose length is much larger than its thickness, may bend before it fails by crushing. This phenomenon is called buckling or elastic instability, which can occur in bars, beams, plates, and shells. It results in an unproportional increase in the displacement resulting from a small increase in the load. In spite of this, the postbuckling strength of thin-walled structures plays an important role in the design of aircraft structures because conventional aircraft structural elements are often designed to operate in the postbuckling range. To determine the postbuckling behavior, one needs to develop a more inclusive, geometrically nonlinear theory. Hence, nonlinear problems considered in the theory of elastic structures are mostly those of postbuckling analysis, prediction of stability, and nonlinear panel flutter analysis.

In recent years, the rapid developments in aerospace exploration have stimulated extensive research into the dynamics and control of large flexible space structures, such as solar collectors, dish antennas, radar arrays, long truss structures, space telescopes, and space stations. Because these structures are characterized by low flexural rigidities, weak material dampings, and interconnections of rigid and flexible parts, and because there is no air damping in space, maneuvers often lead to destructive large-amplitude vibrations, which introduce excessive material fatigue and affect the operational accuracy of such structures. The tasks of controlling the rotation and high-pointing accuracy and eliminating the structural vibrations in a finite period of time pose difficult control problems, which require theoretical and computational advances. From the dynamic point of view, a great disadvantage of such flexible structures is that their natural frequencies are clustered in very narrow bands, making them more prone to becoming involved in resonant vibrations that cannot be easily controlled. Moreover, flexible structures can undergo large displacements without exceeding the elastic limit. Consequently, the responses of flexible structures exhibit many complicated vibration phenomena, such as multiple solutions, jumps, hysteresis, modal interactions, flutter, chaos, and transfer of energy from high-frequency to low-frequency modes.

To design strategies for the control of large-amplitude structural vibrations, one needs to understand their nonlinear static and dynamic behavior, including modal couplings (transfer of energy among the structure modes) and static and dynamic instabilities. These require accurate nonlinear structural models.

The modeling of structural systems can be divided into three groups: (1) linear modeling, (2) pseudo nonlinear modeling, and (3) nonlinear modeling. In linear modeling, both static and dynamic behaviors of a structure are described by linear models whose static and dynamic solutions are unique. A linear static model can predict the onset of static (or geometric) bifurcation (e.g., buckling), but cannot give the magnitude of buckled displacements. In pseudo nonlinear modeling, the static behavior is described by a nonlinear model, but the dynamic behavior is described by a linear model. A nonlinear static model can predict the magnitudes of buckled displacements of a structure. Then, a linear dynamic model around the static equilibrium position is used to perform dynamic stability analyses and predict the onset of dynamic bifurcations. However, such linear models cannot predict the amplitudes of limit cycles or the presence and character of chaotic attractors, which usually occur after dynamic bifurcations. We note that the parameters of the linear dynamic model will generally depend upon the static (equilibrium) model, and there may be several static equilibria. In nonlinear modeling, both the statics and dynamics are described by nonlinear models. Several distinct possible dynamic equilibria may coexist, and the one observed depends on the static equilibria, the system parameters, and initial conditions.

1.3 COMPOSITE MATERIALS

Because of their high strength-to-weight ratio, long fatigue life, resistance to corrosion, high damping, structural simplicity, and possible use for aeroelastic tailoring, advanced laminated structures made of fiber-reinforced composite materials, such as boron-epoxy, graphite-epoxy, and boron-aluminum, have emerged as primary materials for advanced aerospace vehicle structures, marine structures, large space structures, automotive parts, helicopter rotor blades, turbine blades, and robot manipulators. They show great promise for improved performance. Moreover, the inherent anisotropy is an important property of composite materials and one of the basic reasons for their success because it offers linear, elastic couplings among bending, extension, torsion, and shearing motions, thereby making it possible to satisfy sophisticated design criteria, such as aeroelastic tailoring. For example, the extension-twisting coupling produces different twist distributions along the rotors of a two-speed helicopter when the system rotates at different speeds. Moreover, the bending-twisting coupling produces a pitch-flap stability of helicopter rotor blades.

Flutter of aircraft wings occurs because the speed of flow affects the amplitude ratios and phase shifts between bending and torsional motions of the wing in such a way that energy can be absorbed by the wings from the airstream passing by, resulting in self-excited or self-sustained oscillations. Moreover, experiments (Fung, 1969) on cantilever wings show that the flexural movements at all points across the span are approximately in phase with one another, and likewise the torsional movements are all approximately in phase, but the flexure is considerably out of phase of the torsional movement. This phase difference is apparently the main factor that is responsible for the occurrence of flutter. Hence, the bending-torsion coupling characteristics of composite beams can be used to suppress flutter because bending and torsional vibrations of a composite beam with bending-twisting coupling are forced to be in phase by the fiber-matrix mechanism. One well known example of using the bending-twisting coupling effect is the X-29 demonstrator aircraft; the composite skin of its forward-swept wing has a built-in structural and aerodynamic stability.

However, nonclassical structural effects, such as shear deformations, transverse normal stresses (peeling stresses), warping restraint effects, and boundary-layer effects, can be very significant in composite materials although they are usually negligible in isotropic materials. Because composite structures exhibit relatively weak rigidity in the transverse shear, shear deformations are significant in such materials and need to be included in the study of free vibrations of moderately thick plates, forced vibration amplitudes and stress distributions, high-frequency responses, short-wavelength waves, and localized impacts. Also, peeling stresses can be significant because of non-uniform distributions of Poisson's ratios. Moreover, St. Venant's principle is usually assumed to be valid in the analysis of isotropic structures. This principle states that stresses at a point that is at a sufficient distance from the loading end depend only on the magnitude of the applied load and are practically independent of the manner in which the load is distributed over the end. It also implies that a system of loads having zero resultant forces and moments (i.e., a self-equilibrating stress system) produces a strain field that is negligible at a point that is away from the loading end (Iesan, 1987). But, for highly anisotropic and heterogeneous materials, such a self-equilibrating stress system can result in nontrivial strains with long decay lengths, which are the so-called boundary-layer solutions. To study these nonclassical effects, one requires new, refined structural theories. However, to include these nonclassical effects in the modeling of composite structures is not an easy task, especially if geometric nonlinearities are also involved. Moreover, elastic couplings make it difficult or even impossible to obtain exact linear solutions for some simple structural problems.

In the analysis of composite structures, a macromechanics approach is conventionally used. In this approach perfect bonding between the fibers and matrices is assumed, the material is assumed to be uniform, and the mechanical properties are obtained by taking the average of the properties of the constituent fibers and matrices. However, for real composite structures, there are many problems, which include imperfect bonding, nonuniform distributions of fibers, initially crooked fibers that make the structure behave like a hardening-type material, the existence of gas bubbles at the interfaces of fibers and matrices, local stress concentrations, local elastic-plastic behaviors, cracks, delamination, etc. Hence, one needs to use a micromechanics analysis to obtain valid material and structural properties, or even a statistical approach to account for variations in the many unknown factors and manufacturing processes.

The fundamental mechanics of composite materials can be found in the books by Jones (1975), Christensen (1979), Tsai and Hahn (1980), Whitney (1987), Vinson and Sierakowski (1986), and Reddy (1997, 2003).

1.4 DAMPING

Damping arises from the removal of energy by dissipation or radiation. Dissipative forces in structures can be the result of either internal or external damping. External damping includes aerodynamic and hydrodynamic drag and dissipation in the supports of structures. The drag may be linear or nonlinear. Aerodynamic damping was found to be significant for high-amplitude vibrations of beams of low damping and high modulus of elasticity. Anderson, Nayfeh, and Balachandran (1996) experimentally found the nonlinear aerodynamic damping to be significant for large-amplitude first-mode vibrations of slender parametrically excited beams. Internal damping is usually studied by modeling the mechanisms of energy dissipation in materials. Internal damping mechanisms include thermoelastic, hysteretic, Coulombic, magnetoelastic, and dislocation unpinning and grain boundary relaxation of metals and alloys. For most structural metals, such as steel and aluminum, the energy dissipated per cycle is independent of the frequency over a wide frequency range and is proportional to the square of the amplitude of vibration, and the shape of the hysteretic curve remains unchanged with amplitude and is independent of the strain rate. Internal damping, which fits this classification, is called solid or structural damping and its equivalent linear viscous damping is proportional to the inverse of the frequency of vibration (Thomson, 1981). However, the damping ratios of some structures may exhibit both frequency and amplitude dependence. Moreover, the aerodynamic and hydrodynamic drag is not easily modeled because of structure-fluid interactions.

Although damping forces are small in comparison with the elastic and inertia forces in many applications of structural vibration and wave theory and the influence of damping on structural mode shapes is usually small, damping can be important in controlling the amplitudes of vibration under conditions of steady-state resonance and stationary random excitations. Damping has significant influence on the response amplitudes and phases near resonance and plays a crucial role in fixing the borderline between stability and instability in many dynamical systems. Moreover, damping can significantly affect structural nonlinear responses.

The damping ratios of composite materials, especially nonmetal composites, are much higher than those of structural steels. All damping ratios in the experimental results of Schultz and Tsai (1968) and Ray and Bert (1969) are in the range 0.02% to 2.8%. Using the modified Kennedy-Pancu method (Pendered and Bishop, 1963), Siu and Bert (1974) obtained analytically the damping ratios of laminated boron-epoxy plates with free edges and various orientation angles. The obtained damping ratios are in the range 0.09% to 3.31%. For isotropic materials, the experimental results (e.g., Schultz and Tsai, 1968; Baz and Poh, 1989) usually show that modal dampings decrease with mode number. But for composite beams, the damping ratios may increase with mode number (Schultz and Tsai, 1968). Adams and Bacon (1973) found that the damping ratio of composite materials can be as high as 5.5%. The highest experimentally determined damping ratio of boron-epoxy plates is 5.3% (Clary, 1972). Adams et al. (1969) indicated that the damping capacity of composites under torsion is higher than that under flexure. A comprehensive mathematical technique was developed by Ni and Adams (1984) for predicting the damping of laminated composite beams. They showed that the torsional motion induced by bending-twisting coupling may result in high modal damping ratios for flexural vibrations. Experimental results obtained by Adams and Bacon (1973) show that the damping ratios of composite materials increase with temperature.

Saravanos and Chamis (1990a,b, 1991) showed that damping of composites depends on an array of micromechanics and laminate parameters, including constituent material properties, fiber volume ratios, ply angles, ply thicknesses, ply stacking sequences, temperature, moisture, and existing damage. Damping in composites is also anisotropic, but it exhibits an anisotropy trend that is opposite to that of the stiffness and strength, being minimum in the direction of the fibers and maximum in the transverse direction and in shear. Moreover, metal matrix composites can also undergo energy dissipation at the fiber/matrix interface due to interfacial slip, microplasticity of the matrix, dislocation breakaway, and microcracking at or near the fiber/matrix interface.

The damping mechanisms of most structures are unknown because the sources of energy loss are too complicated. Practicing structural engineers usually use the concepts of modal damping and proportional damping obtained experimentally from modal testing (Ewins, 1984).

1.5 DYNAMIC CHARACTERISTICS OF LINEAR DISCRETE SYSTEMS

Linear systems are those for which the principle of superposition holds. In linear equations of motion, there are no terms containing products of different dependent variables or powers of any dependent variable. Discrete systems are governed by ordinary-differential equations.

A linear single-degree-of-freedom system is characterized by its natural frequency and damping ratio. A linear multi-degree-of-freedom system is characterized by its natural frequencies, modal damping ratios, and mode shapes. Moreover, the response frequency under a single harmonic excitation is the same as the excitation frequency, and the response amplitude is unique and independent of the initial conditions.

1.5.1 One-Degree-of-Freedom Systems

In Figure 1.5.1, we show a typical single-degree-of-freedom spring-mass-damper system, where $x(t)$ denotes the displacement of the mass m from its static equilibrium position, k denotes the spring constant, c denotes the damping coefficient, and F and Ω denote the forcing amplitude and frequency, respectively. Using Newton's second law, we obtain the governing equation as

$$m\ddot{x} + c\dot{x} + kx = F \sin \Omega t \tag{1.5.1}$$

We take the initial conditions in the form

$$x(0) = x_0, \quad \dot{x}(0) = \dot{x}_0 \tag{1.5.2}$$

When m, c, and k are constants, the system is referred to as time-invariant.

The solution of (1.5.1) consists of a particular solution x_p (i.e., the steady-state solution) and a complementary function x_c (i.e., the transient solution), which is the solution of the homogeneous part of (1.5.1). To determine the complementary function $x_c(t)$, we substitute

$$x_c = e^{st} \tag{1.5.3}$$

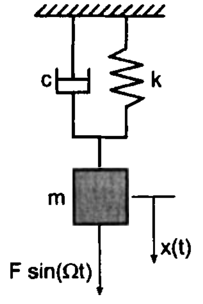

Fig. 1.5.1 A one-degree-of-freedom spring-mass-damper system.

into (1.5.1) with $F = 0$ and obtain

$$(ms^2 + cs + k)e^{st} = 0 \tag{1.5.4}$$

Since e^{st} is time varying, we have

$$ms^2 + cs + k = 0 \tag{1.5.5}$$

which is called the characteristic equation. Hence,

$$s_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} \tag{1.5.6}$$

We consider the case of positive damping; that is, $c > 0$. There are two types of solutions depending on the sign of the discriminant $D \equiv c^2 - 4mk$. When D is positive, s_1 and s_2 are negative real numbers because m and k are positive, and hence x_c is an exponentially decaying function. In this case, one speaks of an overdamped system. When D is negative, s_1 and s_2 are complex conjugate with negative real part. Hence, x_c oscillates while it decays exponentially. When $D = 0$, $s_1 = s_2 = -c/2m$, and hence x_c decays exponentially, and one speaks of a critically damped system. The value c_c of c that renders $D = 0$ is referred to as the critical damping coefficient. It is given by

$$c_c = 2\sqrt{mk} \tag{1.5.7}$$

Because structural materials usually have small dampings, we only consider the case $D = c^2 - 4mk < 0$. In this case, one speaks of an underdamped system.

Next, we define two linear free-oscillation frequencies: undamped and damped natural frequencies. When $c = 0$, (1.5.6) reduces to

$$s_{1,2} = \pm i\sqrt{\frac{k}{m}}$$

where $i \equiv \sqrt{-1}$. Hence, x_c is a harmonically oscillatory function with the frequency $\sqrt{k/m}$, which is called the undamped natural frequency ω_n, where

$$\omega_n \equiv \sqrt{\frac{k}{m}} \tag{1.5.8a}$$

When $c \neq 0$, we rewrite (1.5.6) as

$$s_{1,2} = -\frac{c}{2m} \pm i\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}} \tag{1.5.8b}$$

The absolute value of the imaginary part of s_1 and s_2 is usually referred to as the damped natural frequency ω_d; that is,

$$\omega_d = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}} \tag{1.5.8c}$$