CHAOTIC AND FRACTAL DYNAMICS

An Introduction for Applied Scientists and Engineers

FRANCIS C. MOON
Mechanical and Aerospace Engineering
Cornell University
Ithaca, New York

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To my grandchildren
Anneliese and Calum
Sources of new creative chaos in my world

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PREFACE

Had anyone predicted that new discoveries could be made in dynamics 300 years after publication of Newton's *Principia*, they would have been thought naive or foolish. Yet, in the decade 1977–1987, new phenomena in nonlinear dynamics were discovered, principal among these being chaotic and unpredictable behavior from apparently deterministic systems. Since publication in 1987 of *Chaotic Vibrations*, the first edition of this book, new discoveries in dynamics have been made in many of the sciences, including biology. And, what should be of special interest for the applied scientist or engineer is the emergence of applications of the new ideas in chaotic dynamics and fractals. Chaotic dynamics has been known to be a common occurrence in fluid mechanics, and turbulence remains one of the unsolved problems of classical physics. However, it is now generally accepted that unpredictable dynamics can be found quite easily in simple electrical and mechanical systems as well as in other physical systems.

The purpose of this book is to help translate the new mathematical ideas in nonlinear dynamics into language that engineers and scientists can use and apply to physical systems. Many fine books have been written on chaos, fractals, and nonlinear dynamics (see e.g., Appendix D), but most have focused on the mathematical principles. Many readers of the first edition cited the inclusion of many physical examples as an important feature of the book, and they have urged me to keep the physical nature of chaos as a hallmark of any new edition. The decision to make a substantial rewrite of *Chaotic Vibrations* was based on feedback from a number of readers. They asked for more tutorial material on maps or difference equations and fractals, and they wanted some problems so that the book could be used as a basis of a course.

In this book I have tried to start from a background that a B.S. engineering or science graduate would have; namely, ordinary differential equations and some intermediate-level dynamics and vibrations or system dynamics courses. I have also taken the view of an experimentalist, namely that the book should provide some tools to measure, predict, and quantify chaotic dynamics in physical systems.

Chapter 1 includes an introduction to classical nonlinear dynamics; however, if the book is used as a text, additional supplemental material is recommended. Chapter 2 presents an experimentalist's view of chaotic dynamics along with some simple tools such as the Poincaré map. Chapter 3 introduces maps and is entirely new. It is an attempt to summarize the basic concepts of coupled iterative difference equations as they relate to chaotic dynamics. Chapter 4 is a much expanded litany of physical applications with lots of references to experimental observations of chaos along with the appropriate mathematical models. Many readers have found the discussion of experimental methods (Chapter 5) to be useful, and this too has been expanded. If Chapter 2 asks the question, "How do we recognize chaos?," then in Chapter 6 we ask, "How do we predict when chaos will occur?" Topics such as period doubling, homoclinic bifurcations, Shilni'kov chaos, and Lyaponov exponents are discussed here. The treatment of fractals has been much expanded in the new Chapter 7, including an introduction to multifractals. One of the new directions in chaos research has been in spatiotemporal dynamics. An introduction to some of the simple models of spatially extended systems including dynamics of chain systems and Lagrangian chaos are discussed in Chapter 8. Finally, in Appendix C, an expanded list of chaotic toys and experiments is presented; a guide to some of the more popular books on chaos and fractals is given in Appendix D.

Although over 100 new references have been included in this new edition, it became clear that the tremendous growth in papers on chaos and fractals in the last few years would make it impossible to cover all the significant papers. I apologize to those researchers whose fine contributions have not been cited, especially those who took the time to send me papers, photos, and software. The inclusion of more of the papers from my own Cornell research laboratory must be interpreted as an author's vanity and not any measure of their relative importance to the field.

I have written this new edition not only because of the success of the first, but because I believe the new ideas of chaos and fractals are important to the fields of applied and engineering dynamics. It is already evident that these new geometric and topological concepts have become part of the laboratory tools in dynamics analysis in the same way that Fourier analysis became an important part of engineering systems dynamics decades ago. Already, these tools have found application in areas such as machine noise, impact printer dynamics, nonlinear circuit design, laser instabilities, mixing of chemicals, and even in understanding the dynamics of the human heart. This book is only an introduction to the subject, and it is hoped that interested students would be inspired to explore the more advanced aspects of chaos and fractals, not only for its potential application, but for the fascination and beauty of the basic mathematical ideas which underlie this subject.

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ACKNOWLEDGMENTS

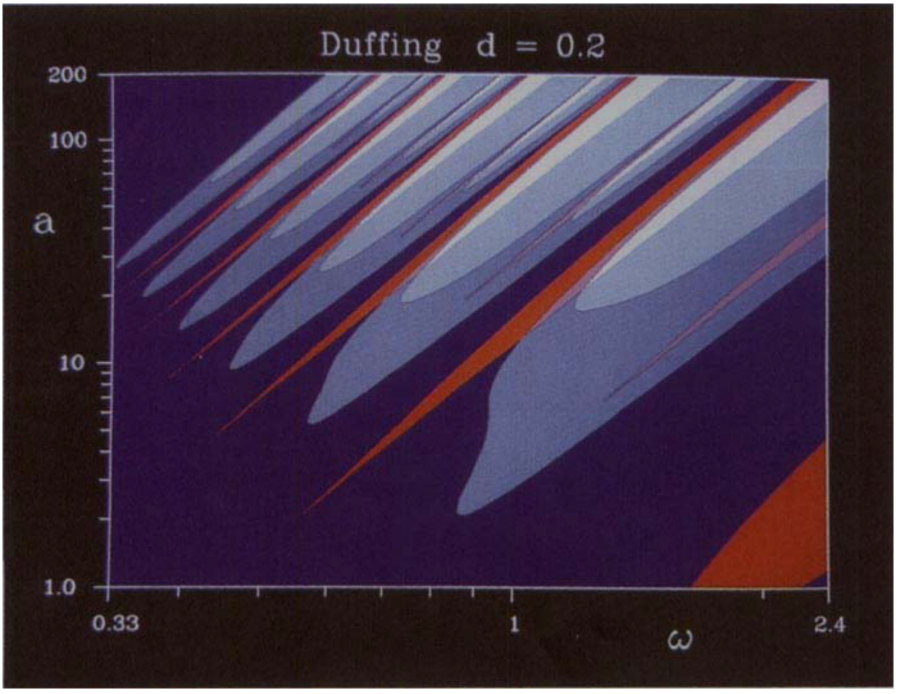
Many of the examples presented in this book reflect one and a half decades of research on nonlinear dynamics at Cornell University, especially in the Nonlinear Dynamics and Magneto-Mechanics Laboratory. This unique facility has had many contributors and sponsors. First, I must thank my colleagues at Cornell in Theoretical and Applied Mechanics, John Guckenheimer, Phillip Holmes, Subrata Mukherjee, and Richard Rand, who have always been a source of advice and criticism to myself and other students of the Laboratory. Any deliberate lack of mathematical rigor in this book, however, must be blamed on me.

With regard to the sponsors, I acknowledge generous support from the Air Force Office of Scientific Research, the Army Research Office, the Department of Energy, IBM Corporation, the National Science Foundation, the National Aeronautics and Space Administration, and the Office of Naval Research. I am especially grateful for the long-time support of the Air Force Office of Scientific Research.

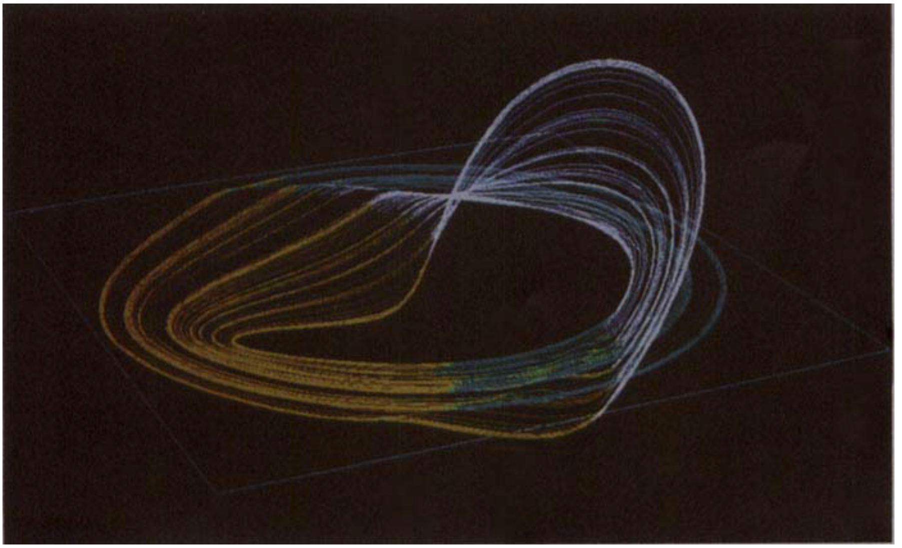
Among the present and former graduate students who have contributed much to the life of this laboratory are G. S. Copeland (David Taylor Naval Lab), J. Cusumano (Penn State University), M. Davies, B. F. Feeny (Institut für Robotik, ETH Zürich), M. Golnaraghi (University of Waterloo), P.-Y. Chen (Chung-Shan Institute of Science and Technology, Taiwan), C.-K. Lee (IBM San José), G.-X. Li (Heroux, Inc., Montreal), G. Muntean, O. O'Reilly, and P. Schubring (U. California, Berkley). Many undergraduate students have made contributions, including P. Khoury (University of California at Berkeley).

Among the many visiting faculty and postdoctoral researchers who have worked with me are F. Benedettinni (Universitá Degli Studi di L'Aquila, Italy), M. El Naschie (Cambridge University), M. Païdoussis (McGill University), C. H. Pak (Inha University, Korea), K. Sato (Hachinohe National College of Technology, Japan), and T. Valkering (University of Twente, The Netherlands).

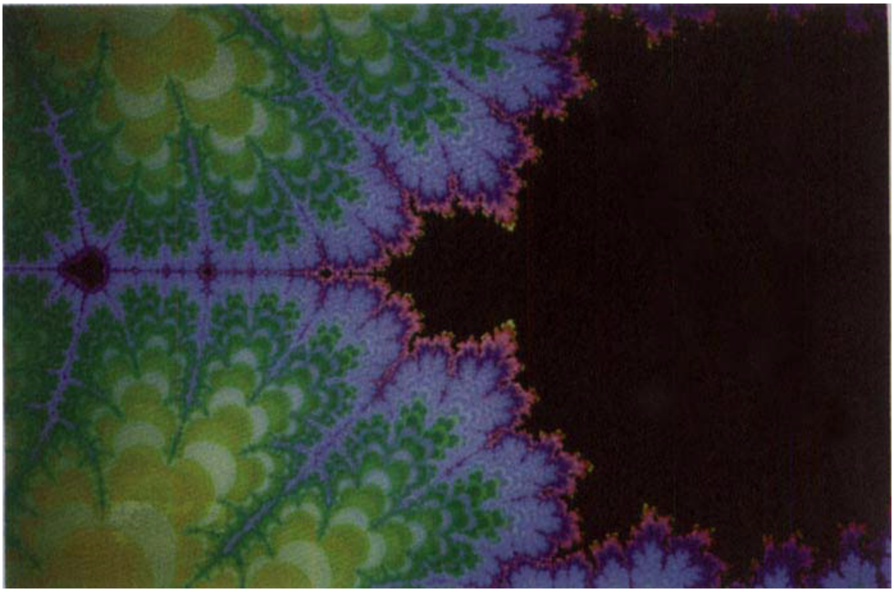
Finally, I want to acknowledge the contribution of two staff members of this Laboratory: Cora Lee Jackson, who deciphered the scribblings of me and my students in typing our many reports and papers and who typed the final manuscript, and William Holmes, who kept computers and research equipment in top shape and who helped design many of the experiments.



Color Plate 1 Dynamic regimes for the forced motion of a particle in a cubic force field; $\ddot{x} + d\dot{x} + x + x^3 = a \cos \omega t$. Dark blue: symmetric, period -1 motion; lighter blues: two asymmetric period -1 and period -2 motions; red: other period -1 solutions. (Courtesy of Professor W. Lauterborn, Technische Hochschule, Darmstadt, FRG; copyright U. Parlitz C. Scheffczyk, W. Lauterborn.)

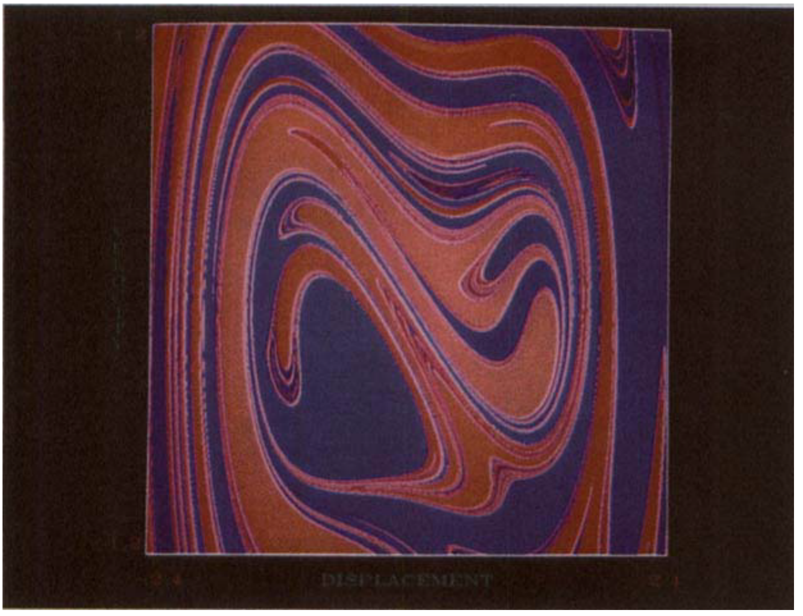


Color Plate 2 Experimental chaotic attractor for periodic forcing of a mass under dry friction. [From Feeny and Moon (1992).]

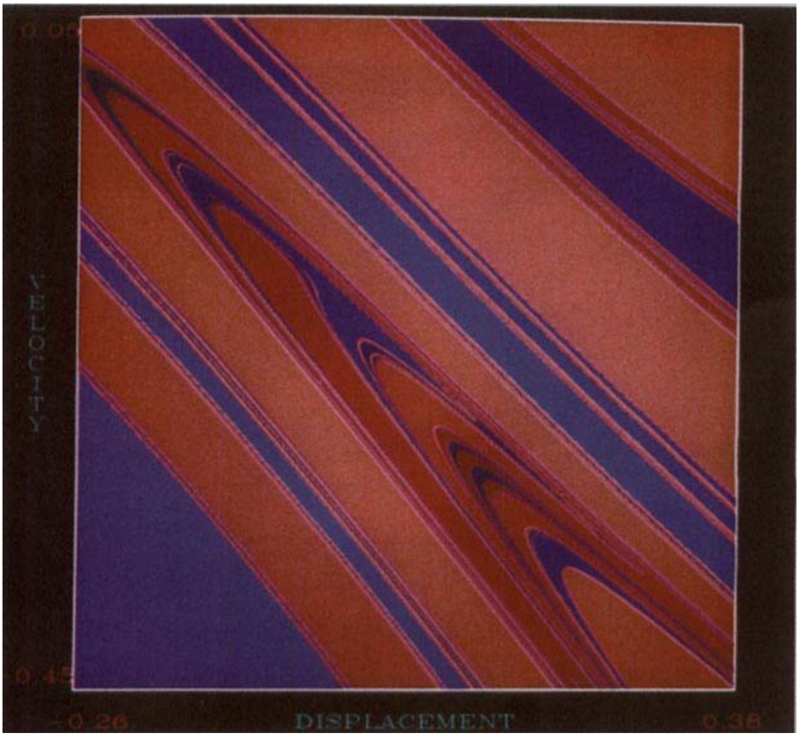


Color Plate 3 Mandelbrot set (black). Each color represents the same number of iterations to escape to "infinity."

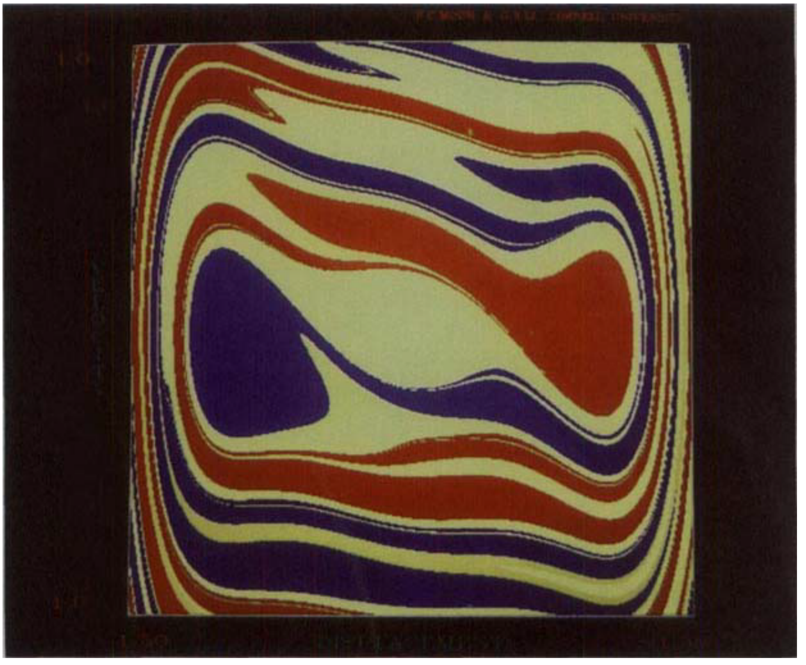
Color Plate 4 Basin of attraction for the periodically forced motions of a particle in a one-well potential with a saddle point (dark purple). Other colors indicate a measure of time to escape out of the well, see Chapter 6 and Thompson (1989b). (Graphic courtesy of Professor J. Cusumano, Pennsylvania State University.)



Color Plate 5 Basins of attraction for the periodically forced motions of a particle in a two-well potential. [Based on Moon and Li (1985).]

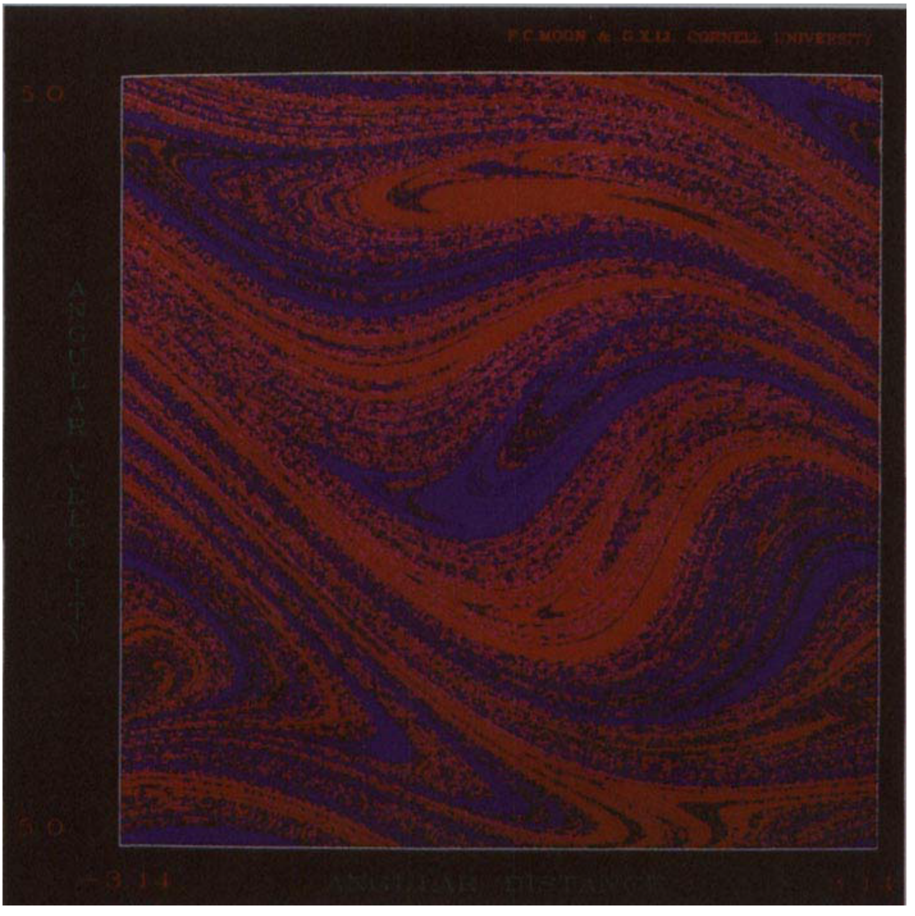


Color Plate 6 Enlargement of the central section of Color Plate 5 for motion in a two-well potential. [Based on Moon and Li (1985).]



Color Plate 7 Basins of attraction for periodically forced motions of a particle in a three-well potential – close to the homoclinic criterion. [From Li and Moon (1990a).]

Color Plate 8 Basins of attraction for three-well potential problem in Color Plate 7–force greater than homoclinic criteria. [From Li and Moon (1990a).]

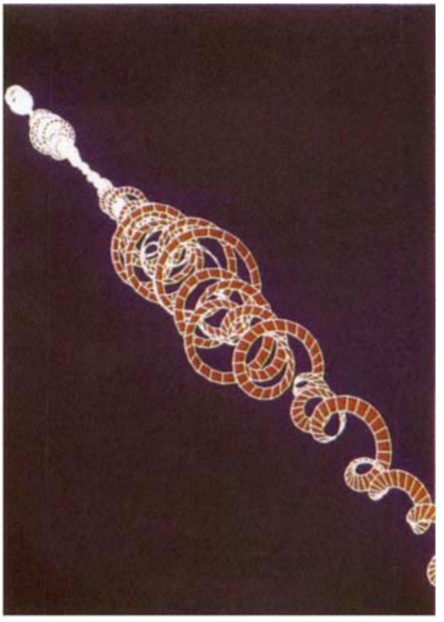


Color Plate 9 Basins of attraction for the parametrically forced magnetic pendulum; blue, clockwise; red counterclockwise. [Color graphic, G.-X. Li, based on Moon, Cusumano, and Holmes (1987).]

Color Plate 10 Lagrangian chaotic mixing of dye in a fluid between two oscillating cylinders. (Courtesy of Professor J. M. Ottino, Northwestern University, Evanston, Illinois.)

Color Plate 11 Lagrangian chaotic mixing of dye in a fluid between two oscillating cylinders. (Courtesy of Professor J. M. Ottino, Northwestern University, Evanston, Illinois.)

Color Plate 12 Spatially periodic deformation of the elastica. [From Davies and Moon (1992).]



Color Plate 13 Spatially quasiperiodic deformation of the elastica. [From Davies and Moon (1992).]

Color Plate 14 Spatially chaotic deformation of the elastica. [From Davies and Moon (1992).]

Color Plate 15 Alternating torque mixing of elasto-plastic solids. [From Feeny et al. (1992).]

Color Plate 16 Alternating torque mixing of elasto-plastic solids. [From Feeny et al. (1992).]