Werner Vogel and Dirk-Gunnar Welsch

Quantum Optics

Third, revised and extended edition

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Quantum Optics

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Contents

Preface

The refinement of experimental techniques has greatly stimulated progress in quantum optics. Understanding of the quantum nature of matter and light has been significantly widened and new insights have been gained. A number of fundamental predictions arising from the concepts of quantum physics have been proved by means of optical methods.

In our book *Quantum Optics*, which arose from lectures that we have given for many years in Jena, Güstrow and Rostock, an attempt is made to develop the theoretical concepts of modern quantum optics, with emphasis on current research trends. It is based on our book, *Lectures on Quantum Optics* (Akademie Verlag/VCH Publishers, Berlin/New York, 1994) and its revised and enlarged second edition, *Quantum Optics – An Introduction* (Wiley-VCH, Berlin, 2001), which we wrote together with S. Wallentowitz. Taking into account representative developments in the field, in the second edition we have included new topics such as quantization of radiation in dispersing and absorbing media, quantum-state measurement and reconstruction, and quantized motion of laser-driven trapped atoms. Following this line, in the present edition we have again included new topics. The new Chapter 10 is devoted to medium-assisted electromagnetic vacuum effects, with special emphasis on spontaneous emission and van der Waals and Casimir forces. In the substantially revised and extended Chapter 8, a unified concept of measurement-based nonclassicality and entanglement criteria for bosonic systems is presented. The new measurement principles needed in this context are explained in Chapter 6. Two sections are added to Chapter 9 in which the problem of unwanted losses in quantum-state extraction from leaky optical cavities is studied. A consideration of decoherence effects in the motion of trapped atoms is added to Chapter 13.

Quantum Optics should be useful for graduate students in physics as well as for research workers who want to become familiar with the ideas of quantum optics. A basic knowledge of quantum mechanics, electrodynamics and classical statistics is assumed.

We are grateful to colleagues and students for their contributions to the research and for valuable comments on the manuscript. In particular we would like to thank S.Y. Buhmann, C. Di Fidio, T.D. Ho, T. Kampf, M. Khanbekyan, L. Knöll, C. Raabe, Th. Richter, S. Scheel, E. Shchukin, D. Vasylyev, S. Wallentowitz. Cordial thanks are due to the *Wiley-VCH* team for their helpful attitude and patience. Last but not least, we are greatly indebted to our wives for their patience with us during the period of preparing the manuscript.

W. Vogel and D.-G. Welsch Rostock and Jena, March 2006

1
Introduction

Since the first experimental demonstration of nonclassical light in 1977, quantum optics has been a very rapidly developing and growing field of modern physics. There are a number of books on the subject [e. g., Agarwal (1974); Allen and Eberly (1975); Carmichael (1993, 1998); Cohen-Tannoudji, Dupont-Roc and Grynberg (1989, 1992); Gardiner (1991); Gerry and Knight (2004); Haken (1985); Klauder and Sudarshan (1968); Loudon (1983); Louisell (1973); Mandel and Wolf (1995); Meystre and Sargent (1990); Orszag (2000); Peřina (1985, 1991); Schleich (2001); Scully and Zubairy (1997); Shore (1990); Vogel and Welsch (1994); Vogel, Welsch and Wallentowitz (2001); Walls and Milburn (1994)], and it is covered in many journals.[1] Presently, in one journal alone (Physical Review A) hundreds of articles on a broad spectrum of quantum-optical and related topics appear every year. Moreover, there are close connections to other traditional fields, such as nonlinear optics, laser spectroscopy and optoelectronics, and the boundaries have often been flexible. The recent improvements in experimental techniques allow one to control the quantum states of various systems with increasing precision. These possibilities have also stimulated the development of rapidly increasing new fields of research such as atom optics and quantum information.

The aim of this book is to describe the fundamentals of quantum optics, and to introduce the basic theoretical concepts to a depth sufficient to apply them practically and to understand and treat specialized problems which have arisen in recent research. On the basis of a general quantum-field-theoretical approach, important topics are presented in a unified manner. Keeping in mind that any real light field is due to sources, time-dependent commutation rules are considered carefully. Nonclassical light is studied and a detailed analysis of measurement schemes is given, including the effect of passive optical instruments, such as beam splitters, spectral filters and leaky cavities. From this background, the basic concepts are developed that allow one to de-

1) For example, see Europhysics Letters, European Physical Journal D, Journal of Modern Optics, Journal of Optics B, Journal of Physics A and B, Journal of the Optical Society of America B, Nature, Optics Communications, Optics Letters, Physical Review A, Physical Review Letters, Physics Letters A, Science.

termine the quantum states of various systems from measured data. Methods of quantum-state preparation are outlined for particular systems, such as propagating light fields, cavity fields and the quantized motion of a trapped atom.

Any attempt to give a complete overview on the present state of the field, together with a complete list of references, would be a hopeless venture. We have therefore decided to refer to selected work that may be useful in the context of particular topics, with special emphasis on textbooks, review articles and research-stimulating original articles. Before giving a guide to the topics covered, we mention two important fields that, apart from some basic ideas, are not considered, although they are closely related to quantum optics. These are the large fields of nonlinear optics [see, e. g., Bloembergen (1965); Boyd (1991); Peřina (1991); Schubert and Wilhelmi (1986); Shen (1984)] and laser physics and laser spectroscopy [see, e. g., Sargent, Scully and Lamb (1977); Haken (1970); Levenson and Kano (1988); Milonni and Eberly (1988); Stenholm (1984)].

1.1 From Einstein's hypothesis to photon anti-bunching

At the beginning of the last century, one of the unresolved problems in physics was the photoelectric effect. When light falls on a metallic surface, photoelectrons may be ejected (Fig. 1.1), whose energy is insensitive to the intensity,

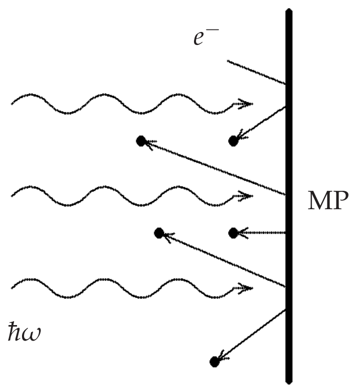

Fig. 1.1 Photoelectric effect: light of frequency ω falls on a metallic plate (MP) and ejects electrons (e^-).

but increases with the frequency of the incident light. This result is obviously in contradiction to the concepts of classical physics. From a classical point of view, one would expect the energy of the emitted electrons to increase with the light intensity. Einstein's explanation of the photoelectric effect in 1905, by postulating the existence of light quanta, photons, may be regarded as the birth of quantum optics. He assumed that light is composed of quanta of energy

$$E = \hbar\omega \tag{1.1}$$

and momentum

$$p = \hbar k = \frac{h}{\lambda}\,. \tag{1.2}$$

In this way, quantities that typically describe the wave aspects of light are related to those that describe particle aspects with the "coupling constant" between wave and particle features being given by the Planck constant $\hbar$. Hence the kinetic energy of an emitted electron, E_{kin}, is given by the difference between the energy of the absorbed photon, $\hbar\omega$, and the binding energy of the electron in the metal, E_{b}:

$$E_{\mathrm{kin}} = \hbar\omega - E_{\mathrm{b}}\,, \tag{1.3}$$

which implies that, in agreement with observations, the energy of the photoelectrons increases with the frequency of the incident light. Increasing the intensity of the light corresponds to increasing the number of light quanta falling on the metal surface, which gives rise to an increasing number of photoelectrons.

The photoelectric effect plays an important role in the photoelectric detection of light, the theory of which (Chapter 6) was developed at the end of the 1950s for classical radiation and extended to quantized radiation in the 1960s. Its experimental application has led to a deeper understanding of the statistics of light.

The invention of the laser at the beginning of the 1960s allowed qualitatively new developments in optical research and the growth of new fields such as nonlinear optics and laser spectroscopy. Intensive studies of lasers have stimulated the introduction of a series of basic theoretical concepts in quantum optics: coherent states (Chapter 3), the theory of phase-space functions (Chapter 4) and the quantum theory of damping (Chapter 5).

Modern quantum optics would be unthinkable without the availability of measurement techniques, such as the Hanbury Brown–Twiss experiment, which was first performed in 1956. By using a beam splitter and two photodetectors, the coincidences of photoelectric events were recorded and compared with the product of independently measured events (for the experimental setup see Fig. 8.1, p. 271). In the case of thermal light an excess of coincidences was observed. That is, the measured intensity correlation $G^{(2)}(\tau)$ as a function of the time delay τ, decays from its initial value at $\tau = 0$ towards a stationary value, cf. Fig. 1.2. This effect, which is called photon bunching, can

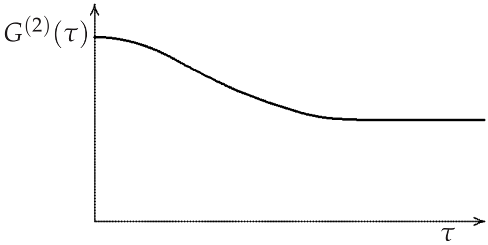

Fig. 1.2 Delay-time dependence of the intensity correlation as typically observed in a Hanbury Brown–Twiss experiment performed with light from a thermal source.

be understood by assuming that the light quanta arrive in bunches, so that the joint probability of events exceeds the product of the two probabilities measured independently of each other. Although this explanation is reasonable, it affords no proof of the existence of photons, since an intensity correlation behavior of the type observed can also be understood classically. It should be emphasized that, in the opposite case, where the measured intensity correlation has a positive initial slope (photon anti-bunching) there is no classical explanation (Chapter 8).

Notwithstanding the success of Einstein's hypothesis, the existence of photons was still a matter of discussion in the 1970s,[2] and the demonstration of photon anti-bunching in 1977 may be regarded as the first direct proof of their existence. The experimental apparatus was of the Hanbury Brown–Twiss type and the detected light was the resonance fluorescence (Chapter 11) from an atomic beam with such a low mean number of atoms that at most one atom contributed to the emitted light. Let us suppose that at a certain instant a single two-level atom that is (resonantly) driven by a laser pump is in the upper quantum state and ready to emit a photon. If the atom emits a photon, it undergoes a transition from the upper to the lower quantum state, which implies that it cannot emit a second photon simultaneously with the first one. The atom can emit a second photon only when it is again excited by the pump field. In other words, the measured intensity correlation vanishes for zero delay, $G^{(2)}(\tau \to 0) = 0$, and in the detection scheme considered there are no equal-time coincidences of photoelectric events. Note that any classical wave or wavepacket is divided by a 50%:50% beam splitter into two parts of equal intensity, which never leads to a vanishing intensity correlation at zero time delay. Photon anti-bunching is essentially a nonclassical property of light and its detection stimulated the formation of quantum optics as a specific field of research.

2) See, e. g., the paper by Karp (1976), "Test for the non-existence of photons", and the response by Mandel (1977), "Photoelectric counting measurements as a test for the existence of photons".

1.2 Nonclassical phenomena

Nonclassical phenomena, that is, phenomena that are basically quantum mechanical, have been studied intensively in quantum optics and related fields. Nonclassical light has been considered and a number of (nonlinear-optical) methods have been developed to generate it (Chapter 8). Roughly speaking, in many cases the noise in nonclassical light is reduced below some standard quantum limit (e. g., the vacuum noise level), which is usually observed in the case of ideal laser light. As already mentioned, anti-bunched light shows an intensity anti-correlation at zero time delay. Another example of nonclassical light is sub-Poissonian light, which gives rise to a photocounting distribution narrower than a Poissonian one. Sub-Poissonian light was first observed in 1983, in resonance fluorescence from a low-intensity atomic beam. If the noise of a phase-sensitive field quantity, such as the electric-field strength, is reduced (as a function of the phase parameter) below the vacuum level, then the light is called squeezed light. This was first generated in 1985 by means of four-wave mixing.

A number of specific quantum states of radiation and other bosonic systems have been studied, which can be used to define various quantum-mechanical representations of observables (Chapter 3). They may also serve as examples of typical nonclassical effects. For example, photon-number states may be regarded as reflecting particle-like features of radiation rather than wave-like features. On the contrary, when a radiation field is prepared in a coherent state, then its properties, apart from the vacuum noise, become close to those of a classical, nonfluctuating wave.

An old and troublesome problem in quantum mechanics is the description of amplitude and phase and their measurement (Chapters 3 and 7). Since the 1920s, a number of attempts have been made to introduce phase operators and phase states in the quantum theory of light. Concepts based on quantum-mechanical first-principle definitions as well as measurement-assisted definitions have been considered.

In general, a radiation field is not prepared in a pure quantum state but in a mixture of states. In this case, information on the quantum statistics of the field is contained in the density operator. Rather than representations in an orthogonal Hilbert-space basis, representations in terms of phase-space functions are frequently preferred. The concept of phase-space functions (Chapter 4) bears a formal resemblance to classical statistics and allows, to some extent, the application of methods of classical probability theory.

Generation of nonclassical states on demand offers novel possibilities of exploiting quantum features in various fields of applied physics such as measurement technology and information processing. In particular, the increasing number of experimental realizations of nonclassical states of radiation and matter requires methods for characterizing the variety of nonclassical effects to be expected (Chapter 8). In this context, the question of measurable nonclassicality criteria arises, i. e., criteria that are directly applicable to experiments. Similarly, the question of measurable criteria for entangled states must be answered – states which play a key role in quantum communication such as quantum cryptography, quantum-state teleportation and quantum computation.

The quantum nature of radiation and matter becomes obvious both in their resonant and off-resonant interaction. Whereas spontaneous emission (Chapter 10) and resonance fluorescence (Chapter 11) from a single atom and the Jaynes–Cummings-type interaction of a single atom with a high-quality cavity field (Chapter 12) are examples of resonant interaction, van der Waals and Casimir forces (Chapter 10) are typical examples of virtual-photon-assisted off-resonant interaction.

1.3 Source-attributed light

Any real radiation field may be thought of as being due to sources, which essentially determine the quantum statistics of the radiation. Quantization of the radiation field requires, in principle, quantization of the matter and the radiation-matter interaction as well (Chapter 2). As is well known, commutation relations play an important role in quantum physics. Whereas commutation relations at equal times are given from quantum-mechanical first principles, determination of the time-dependent commutation relations requires knowledge of the dynamics of the coupled light–matter system. Therefore, to study general aspects of the generation, detection and processing of quantized light (such as quantum-optical correlation functions observed in the photoelectric detection of light), it is helpful to introduce appropriate source-quantity representations of field commutators (Chapter 2).

Light detection and processing are frequently performed in a source-free region of space, and the question arises as to the conditions under which it is possible to treat a quantized radiation field as being effectively free, that is to ignore the sources when considering the radiation. A criterion for an effectively free field may be seen in the agreement of the commutation relations of the field quantities at different times with the free-field commutation relations (Chapter 2). It is worth noting that the question of whether or not the commutation relations of field quantities at different times reduce to the free-field commutation relations, can be answered by means of their source-quantity representations, that is, by expressing them in terms of free-field commutators and so-called time-delayed terms. The latter can give rise to a nonvanishing contribution when the space-time arguments of the two field quantities under consideration can be connected to each other by the propagation of light from one of the space-time points to the other through the sources. Clearly, a light field may be regarded as being effectively free when the distances of the relevant points of observation from the light source are large enough and the considered time intervals are small enough to suppress the time-delayed terms. This rule can be established not only for the full field but also for appropriately chosen (multi-mode) parts of the field, such as the incoming and outgoing fields frequently introduced in connection with experimental apparatus. For example, if a (multi-mode) part of an optical light field propagates away from the sources and cannot return to them, then it may be regarded in many cases as being effectively free, independently of the chosen space-time points.

In practice, various optical instruments, which may substantially modify the propagation of light compared with that in free space, are used and a careful consideration of the time-dependent commutation relations is necessary to actually specify the free-field conditions and the correlation functions measurable by means of standard photodetectors. Typical examples are the theory of spectral filtering of quantized light (Chapter 6) and the treatment of an optical cavity with output coupling (Chapter 9).

To derive tractable equations of motions for a coupled light–matter system, various approximation schemes have been developed and applied, such as the dipole approximation and the rotating-wave approximation (Chapter 2). Furthermore, in nonlinear optics the concept of effective Hamiltonians is widely used, for example in the treatment of multi-photon absorption and emission, parametric optical processes (e. g., the optical parametric oscillator) and multi-wave mixing.

For gaining deeper insight into the quantum nature of light–matter interactions, models that are almost exactly solvable play an important role. In particular, there have been detailed studies of the resonant interaction of a single two-level atom with a (multi-mode) radiation field in free space within the framework of optical Bloch equations (Chapter 5) to describe resonance fluorescence (Chapter 11), and of the resonant interaction of a single two-level atom with a single-mode field in a high-quality cavity on the basis of the Jaynes–Cummings model (Chapter 12).

1.4 Medium-assisted electromagnetic fields

As is already known from classical optics, the use of instruments in optical experiments needs careful examination with regard to their action on the light under study [see, e. g., Born and Wolf (1980)]. In quantum optics an additional consideration is the influence of the presence of instruments on the quantum statistics of the light. For example, let us consider a 50%:50% beam splitter oriented at 45° to an incident light beam (Fig. 1.3). In classical optics the beam splitter divides the incoming beam into two (apart from a phase shift) equal outgoing parts propagating perpendicular to each other (Fig. 1.3a), and with the same scaling factor the classical noise of the incident field is transferred to the two fields in the output channels of the beam splitter. It is intuitively clear that, in quantum optics, the noise of the vacuum in the unused input port of the beam splitter introduces additional noise in the two output beams (Fig. 1.3b). Therefore, the quantum statistics of the output fields may differ significantly from that of the input field.

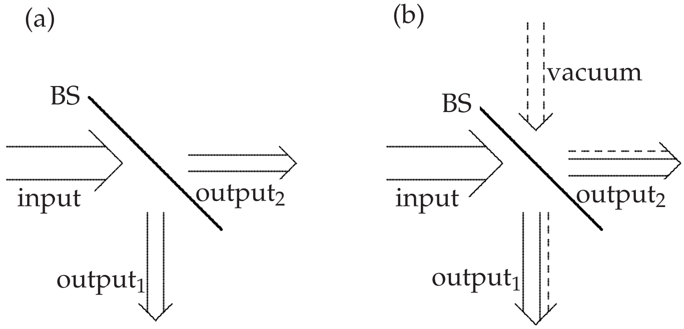

Fig. 1.3 Outline of the action of a 50%:50% beam splitter (BS). In classical optics (a) an incident light beam (input) is divided into two (apart from a phase shift) equal output beams ($output_1$, $output_2$). In quantum optics (b) the incident light beam and the quantum noise of the vacuum in the unused input port are combined to yield vacuum-noise-assisted output beams.

The above example shows that it is necessary to take into account the presence of optical instruments when considering the quantization of the radiation field. In principle, optical instruments could be included as part of the matter to which the radiation field is coupled and treated microscopically. However, in many cases, passive instruments are linearly responding macroscopic bodies that can be treated phenomenologically by introducing a spatially varying permittivity. In general, light propagation through such bodies is accompanied by dispersion and absorption, so that the permittivity is a complex function of frequency. Since in quantum physics any type of loss is unavoidably connected with fluctuations, for treatment of the effect of material absorption, quantization in an extended Hilbert space is required (Chapter 2). In some cases, in particular when the spectral range of the radiation is effectively limited to an appropriately chosen narrow interval, the effects of absorption and dispersion may become negligibly small and the description of the instruments considerably simplified. They can be modeled by bodies with real permittivities that may vary only in space. Obviously, both the time-dependent commutation relations and the quantum-statistical correlation functions of the field under study depend on the specific bodies used.

Typical examples of optical instruments whose action can be treated in this way are beam splitters and spectral filters of the Fabry–Perot type. Their main features can already be described by means of the simple model of a dielectric plate (Chapter 6). Moreover, progress in quantum optics would be unimaginable without the use of resonator-like equipment. A typical example is an optical cavity filled with an active medium and bounded by dielectric walls to allow for input and output coupling (Chapter 9). In particular, in the case of a high-quality cavity, applying the formalism of electromagnetic-field quantization in linear media naturally yields a description of the radiation field inside and outside the cavity in terms of quantum damping theory.

The formalism of electromagnetic-field quantization in linear media can also be used to treat body-assisted electromagnetic vacuum effects in a unified way (Chapter 10). Whereas the classical vacuum is the trivial state where the electromagnetic field identically vanishes, the quantum vacuum is very active and its interaction with atomic systems gives rise to a number of observable effects that are purely nonclassical. Since in the presence of macroscopic bodies the structure of the electromagnetic field is changed compared with that in free space, the electromagnetic vacuum is changed also, which can lead, e. g., to inhibition or enhancement of spontaneous emission. Moreover, forces of the van der Waals type in micro- and nano-structures can be controlled in this way.

1.5 Measurement of light statistics

To gain information on the quantum statistics of light from measured data, a careful consideration of the employed measurement scheme is needed (Chapter 6). In standard photoelectric detection of light the detection process is based on the internal photoelectric effect. In the spirit of Einstein's hypothesis, by absorbing a photon, a detector atom can undergo a transition from an initial state to a continuum of final states, ejecting a photoelectron. A combination of quantum mechanics (to treat the elementary acts of light absorption) and classical statistics (to deal with the macroscopic sample of photoelectrons produced in a chosen time interval of detection) yields the observed counting statistics in terms of either normally and time-ordered field correlation functions or the photon-number statistics.

There are various kinds of detection schemes that can be used to measure statistical properties of light that are not accessible from the photon-number statistics. On combining (single-mode) light fields by means of beam splitters before measuring the counting statistics (of the combined field), the quantum statistics of phase-sensitive light properties can be obtained. In particular, four-port homodyne detection (see Fig. 6.6, p. 206) and eight-port homodyne detection (see Fig. 6.8, p. 224) are typical examples of this measurement strategy.

If in a four-port homodyne detection one of the (single-mode) input fields is prepared in a coherent state with a sufficiently large mean number of photons, then the measured difference-count probabilities can be related to the phase-rotated quadrature probability distributions of the second input field. Using an eight-port scheme renders it possible to relate the measured joint difference-count probability to the Q function of the second input field, that is the phase-space function that applies directly to the calculation of expectation values of anti-normally ordered operator functions. Also, unbalanced homodyning is of interest since it leads to simple reconstruction methods for the quantum states.

Homodyne correlation measurements are of particular interest when a weak local oscillator is used. In this case new types of correlation properties can be observed. In principle, one may determine all normally ordered moments, including those containing unequal numbers of creation and annihilation operators. Such moments, which are not accessible by direct detection methods, are required, e. g., for implementing nonclassicality and entanglement criteria (Chapter 8).

1.6
Determination and preparation of quantum states

It is well known that the density matrix of a quantum system contains all the information necessary to completely determine its properties. Hence the determination of the density matrix from measured data is therefore an important problem (Chapter 7). The first reconstruction of a light-field density matrix from measured data was reported in 1993. Clearly, the density matrix can only be obtained from quantities that also contain the complete information on the system. For example, this information is contained in any phase-space function. Since the Q function of a (single-mode) field can be obtained from the data measured in eight-port homodyne detection (Chapter 6), the density matrix of the field can be obtained, in principle, from these data also. Moreover, knowledge of the phase-rotated quadrature probability distributions for every phase parameter in a π interval is equivalent to knowledge of any phase-space function, which implies that the density matrix can also be obtained from the phase-rotated quadrature distributions with the phase parameter varying in a π interval. Since these probability distributions can be obtained from the data measured in a succession of four-port homodyne detections (Chapter 6), the four-port homodyne detection scheme can also be used for the experimental determination of the density matrix.

An alternative way of determining the quantum state from measured data consists of a method that is local in phase space. For a radiation mode, the measurement scheme consists of unbalanced homodyning. By use of a local oscillator, the field to be measured is displaced in phase space, with the complex displacement amplitude being controlled by the phase and amplitude of the local oscillator. The resulting displacement amplitude defines the point in phase space where a chosen phase-space function can be determined locally. The phase-space function of interest is obtained in a simple manner as an appropriately weighted sum of the photon-number statistics of the displaced light field.

The basic concepts of determining the quantum state can also be modified to allow the determination of the quantum state of a high-Q cavity-field by transmission of probe atoms (Chapter 12). Moreover, methods have been developed for determining the motional quantum state of a trapped atom (Chapter 13) and the entangled state for the combined vibronic (vibrational-electronic) quantum state of an atom undergoing a quantized center-of-mass motion in a trap potential.

Appropriate methods of quantum-state preparation are needed for generating nonclassical states. The improvements of experimental techniques allowed one to prepare sophisticated quantum states such as entangled states of the Schrödinger-cat type or Einstein–Podolsky–Rosen states. Experiments of this type can be performed, for example, by using interactions of single atoms with high-Q cavity fields (Chapter 12) or by using the vibronic dynamics of trapped atoms (Chapter 13).

1.7 Quantized motion of cold atoms

The progress in developing techniques for cooling trapped atoms to extremely low temperatures has rendered it possible to visualize the quantum nature of the atomic center-of-mass motion, which is no longer hidden by thermal background noise. Control of the quantized atomic center-of-mass motion allows one to realize, e. g., atom interferometry, Bose–Einstein condensation and atom-laser like devices.[3]

If an atom is confined in a harmonic trap potential (Chapter 13), the laser-driven vibronic interaction shows some resemblance to the atom–field inter-

3) In atom lasers, the wavy nature of the atomic motion plays the role of the electromagnetic waves in conventional lasers and it is interesting to generate coherent (atomic) matter waves.

action in a high-Q cavity. An exactly solvable, nonlinear Jaynes–Cummings model is suited to describing the dynamics of the laser-driven trapped ion in the resolved sideband regime, where individual vibronic transitions are addressed by the laser. Besides the multi-quantum generalization of the standard Jaynes–Cummings model of cavity QED, there appears an additional nonlinear dependence of the interaction Hamiltonian on the vibrational excitation of the atom in the trap potential. The nonlinearity gives rise to interesting effects in the atomic dynamics, which can be employed to measure motional quantum states and prepare specific ones. In particular, it is possible to drive the motional quantum state of the atom in a nonlinear manner without affecting the electronic one.

In the first experimental realization of the nonlinear Jaynes–Cummings dynamics with a Raman-driven trapped ion, significant decoherence effects had already been observed. A detailed understanding of the underlying mechanisms is of great importance for any practical application of trapped atoms, e. g., in quantum information processing. In particular, in the case of a Raman-driven atom being cooled down to its motional ground state the decoherence is dominated by the, rarely occurring, excitation of an auxiliary electronic state used for the enhancement of the Raman coupling strength.

References

Agarwal, G.S. (1974) in *Quantum Statistical Theories of Spontaneous Emission and their Relation to Other Approaches*, ed. G. Höhler (Springer-Verlag, Berlin).

Allen, L. and J.H. Eberly (1975) *Optical Resonance and Two-Level Atoms* (Wiley, New York).

Bloembergen, N. (1965) *Nonlinear Optics* (Benjamin, New York).

Born, M. and E. Wolf (1980) *Principles of Optics* (Pergamon Press, Oxford).

Boyd, R.W. (1991) *Nonlinear Optics* (Academic Press, London).

Carmichael, H. (1993) *Lecture Notes in Physics: An Open Systems Approach to Quantum Optics* (Springer-Verlag, Berlin).

Carmichael, H. (1998) *Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations* (Springer-Verlag, Berlin).

Cohen-Tannoudji, C., J. Dupont-Roc and G. Grynberg (1989) *Photons and Atoms* (Wiley, New York).

Cohen-Tannoudji, C., J. Dupont-Roc and G. Grynberg (1992) *Atom-Photon Interactions* (Wiley, New York).

Gardiner, C.W. (1991) *Quantum Noise* (Springer-Verlag, Berlin).

Gerry, C. and P. Knight (2004) *Introductory Quantum Optics* (Cambridge University Press).

Haken, H. (1970) in *Light and Matter Ic*, ed. L. Genzel: Vol. XXV/2c of *Encyclopedia of Physics*, chief ed. S. Flügge (Springer-Verlag, Berlin).

Haken, H. (1985) *Light* (North-Holland, Amsterdam).

Karp, S. (1976) *J. Opt. Soc. Am.* **66**, 1421.

Klauder, J.R. and E.C.G. Sudarshan (1968) *Fundamentals of Quantum Optics* (Benjamin, New York).

Levenson, M.D. and S.S. Kano (1988) *Introduction to Nonlinear Laser Spectroscopy* (Academic Press, New York).

Loudon, R. (1983) *The Quantum Theory of Light* (Clarendon Press, Oxford).

Louisell, W.H. (1973) *Quantum Statistical Properties of Radiation* (Wiley, New York).

Mandel, L. (1977) *J. Opt. Soc. Am.* **67**, 1101.

Mandel, L. and E. Wolf (1995) *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge).

Meystre, P. and M. Sargent III (1990) *Elements of Quantum Optics* (Springer-Verlag, Berlin).

Milonni, P.W. and J. H. Eberly (1988) *Lasers* (Wiley, New York).

Orszag, M. (2000) *Quantum Optics* (Springer-Verlag, Berlin).

Peřina, J. (1985) *Coherence of Light* (Reidel, Dordrecht).

Peřina, J. (1991) *Quantum Statistics of Linear and Nonlinear Optical Phenomena* (Reidel, Dordrecht).

Sargent III, M., M.O. Scully and W.E. Lamb, Jr. (1977) *Laser Physics* (Addison-Wesley, Reading).

Schleich, W.P. (2001) *Quantum Optics in Phase Space* (Wiley-VCH, Berlin).

Schubert, M. and B. Wilhelmi (1986) *Nonlinear Optics and Quantum Electronics* (Wiley, New York).

Scully, M.O. and M.S. Zubairy (1997) *Quantum Optics* (Cambridge University Press, Cambridge).

Shen, Y.R. (1984) *Principles of Nonlinear Optics* (Wiley, New York).

Shore, B.W. (1990) *Theory of Coherent Atomic Excitations* (Wiley, New York).

Stenholm, S. (1984) *Foundations of Laser Spectroscopy* (Wiley, New York).

Vogel, W. and D.–G. Welsch (1994) *Lectures on Quantum Optics* (Akademie-Verlag, Berlin).

Vogel, W., D.–G. Welsch and S. Wallentowitz (2001) *Quantum Optics – An Introduction* (Wiley-VCH, Berlin).

Walls, D.F. and G.J. Milburn (1994) *Quantum Optics* (Springer-Verlag, Berlin).

2
Elements of quantum electrodynamics

In order to arrive at the basic concepts for describing the quantum effects of radiation, it is necessary to consider the quantization of the electromagnetic field attributed to atomic sources in the presence of macroscopic bodies. For example, in many cases of practical interest the (passive) optical instruments through which radiation passes can be regarded as being more or less complicated dielectric bodies. In the quantization scheme developed here we will therefore allow for the presence of a dielectric medium with space- and frequency-dependent complex permittivity satisfying the Kramers–Kronig relations. For example, a standard situation is the spectral filtering of light pro-

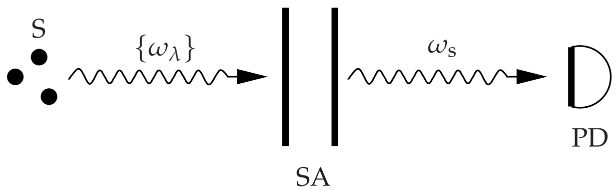

Fig. 2.1 Spectral photodetection scheme. After passing through a (Fabry–Perot-type) spectral apparatus (SA), whose spectral response function discriminates against values of the frequency ω_λ not equal to a given setting frequency ω_s, the light produced by the sources (S) falls on a photoelectric detection device (PD).

duced by some types of source, cf. Fig. 2.1. In homodyne detection a signal field is combined with a (local oscillator) reference field through a beam splitter. By means of photoelectric detection of the mixed output fields, phase information on the signal field becomes accessible. Further, resonators such as leaky optical cavities filled with optically active (nonlinear) matter are frequently used in quantum optics for generating and/or amplifying light, cf. Fig. 2.2.

Starting from the well-known classical equations of motion of microscopic electrodynamics (Section 2.1), canonical quantization of both the free electromagnetic field (Section 2.2) and the electromagnetic field with sources (Section 2.3) is performed. The theory is then extended to electrodynamics in dielectric media, transferring the powerful concepts of phenomenological clas-

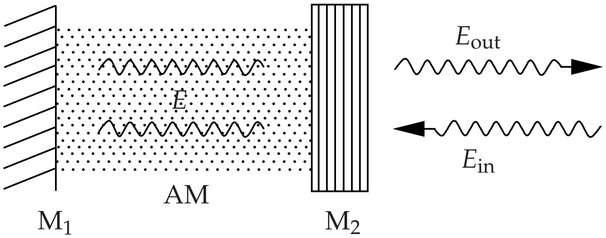

Fig. 2.2 Scheme of a resonator-like cavity bounded by a perfectly reflecting mirror M_1 and a fractionally transparent mirror M_2, the cavity being filled with optically active matter (AM). The mirror M_2 guarantees that the intra-cavity field (electric field strength E) is in contact with the incoming field (E_{in}) and the outgoing field (E_{out}), which may be utilized for subsequent optical processing.

sical electrodynamics to quantum theory (Section 2.4). With regard to the description of specific processes, frequently used concepts of approximate interaction Hamiltonians are discussed (Section 2.5). By formal solution of the Heisenberg equations of motion (Section 2.6), fundamental time-dependent commutation relations are derived (Section 2.7). This makes it possible to express observable field correlation functions in terms of source-quantity correlation functions (Section 2.8).

The standard concepts of canonical quantization are considered, for example, in the books of Cohen-Tannoudji, Dupont-Roc and Grynberg (1989), Haken (1985), Loudon (1983), Louisell (1973), Meystre and Sargent III (1990), Milonni (1994), Peřina (1991) and Schubert and Wilhelmi (1986). The concepts of inclusion in the quantization of dielectric media, are based on original work [Knöll, Vogel and Welsch (1987); Glauber and Lewenstein (1991); Huttner and Barnett (1992); Gruner and Welsch (1996); Scheel, Knöll and Welsch (1998); Ho, Buhmann, Knöll, Welsch, Scheel and Kästel (2003)].

2.1 Basic classical equations

In classical physics the electromagnetic field obeys Maxwell's equations[1]

$$\boldsymbol{\nabla}\mathbf{B}(\mathbf{r}) = 0, \tag{2.1}$$

$$\boldsymbol{\nabla} \times \mathbf{E}(\mathbf{r}) = -\dot{\mathbf{B}}(\mathbf{r}), \tag{2.2}$$

$$\boldsymbol{\nabla}\mathbf{E}(\mathbf{r}) = \varepsilon_0^{-1}\rho(\mathbf{r}), \tag{2.3}$$

$$\boldsymbol{\nabla} \times \mathbf{B}(\mathbf{r}) = \mu_0\mathbf{j}(\mathbf{r}) + \mu_0\varepsilon_0\dot{\mathbf{E}}(\mathbf{r}) \tag{2.4}$$

1) Here, and in the following, for notational convenience we denote the scalar product of two vectors simply by **ab**, the vector product by $\mathbf{a} \times \mathbf{b}$ and the tensor product by $\mathbf{a} \otimes \mathbf{b}$.