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Theory of Tokamak Transport

New Aspects for Nuclear Fusion Reactor Design

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Cover picture
Abstract representation of the JET Tokamak
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Library of Congress Card No.: applied for.

British Library Cataloging-in-Publication Data:
A catalogue record for this book is available from the British Library.

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the Internet at <http://dnb.ddb.de>.

Printed in the Federal Republic of Germany
Printed on acid-free paper

Typesetting Uwe Krieg, Berlin
Printing Strauss GmbH, Mörlenbach
Binding Litges & Dopf Buchbinderei GmbH, Heppenheim

ISBN-13: 978-3-527-40625-8
ISBN-10: 3-527-40625-5

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Preface

The world-wide demand for energy is growing exponentially. In the middle of the nineteenth century mankind's energy consumption was less than half a Q/century ($1Q \sim 10^{21}$ Joules), in 1851–1950 it had increased to 4Q/century, and in the half century 1951–2000 it was $\sim 15\,$Q. Estimates of the world reserves of fossil fuels keep changing — perhaps these reserves are $\sim 100\,$Q, and even if this is a substantial underestimate, it is likely that they would be depleted within a century, particularly when the rapidly increasing demand for energy from the recently industrialized nations of the East is included in the reckoning. Reactors based on the fusion of light elements may provide an almost unlimited supply of energy in the future.

There are other considerations that make the development of fusion reactors a worth-while task. As remarked recently in the New Scientist, "Burning fossil fuels and using the atmosphere as an open sewer has turned out to be a recipe for disaster. The Earth is warming and the pace is quickening." Fission reactors are likely to provide the short-term replacement for oil and gas and the development of renewable energy sources, like wind and wave power is progressing, but much too slowly. It seems unlikely that the latter will be sufficient in the long run and the supply of U^{235} is even more limited than fossil fuels, not to mention the problems of storing radioactive waste and of proliferating bomb-making capacity. Fast-breeder reactors, consuming the much more common U^{238}, could provide a long-term solution, but these reactors are potentially more vulnerable to accidents and would produce large amounts of plutonium that could be used in nuclear weapon production.

The fusion of light nuclei such as deuterium and tritium offers an alternative energy supply without the disadvantages of the fossil and fission sources. While a fusion reactor would generate some radioactive waste, this is believed to be largely short-lived and manageable. However, the serious problem with fusion is the enormous temperature required to overcome the repulsive force between colliding charged particles. The nuclei have to clash together with the speeds achieved at temperatures about 12 times hotter than the centre of the Sun, which also operates on fusion, but at densities some 10^{12} times greater than reactor values. At these enormous temperatures confining the gas long enough for appreciable fusion reactions to occur is a major problem. Strong magnetic fields provide the only possible constraint over the motions of such energetic particles, and the most successful device employing this principle is known as a *tokamak.*

A tokamak (**To**roidal **Ka**mera **Ma**gnitnaya, invented in the Soviet Union in the late 1950s) is a toroidal chamber carrying a strong toroidal magnetic field to trap a high temperature plasma. For a tokamak containing deuterium and tritium in equal parts to become a fusion reactor, temperatures exceeding 2×10^{8} K are required. The Joint European Torus (JET) at Culham Laboratory, Oxfordshire, UK, has reached more than half of the required temperature, but the triple product of the ion number density n_i, the energy confinement time τ_E and temperature T, still falls well short of the value $3\times10^{21}\,\mathrm{s\,m^{-3}\,keV}$ required for ignition; in some D-T fusion experiments in JET a value of $8.7\times10^{20}\,\mathrm{s\,m^{-3}\,keV}$ has been attained.

A survey of the situation in the journal *Nuclear Fusion*, Vol. 39, no. 12, December, 1999, commenced with the words:

> "Magnetic fusion energy research has reached the point where a tokamak burning plasma facility in which the thermonuclear heating balances (or is comparable to) transport and radiation losses for periods of 1000 s or longer can be seriously contemplated as an appropriate next step. Achieving this goal would be a major step forward, both in science and in technology, towards the ultimate goal of magnetic fusion generation of electric power with significant environmental advantages."

This volume of *Nuclear Fusion* was entirely devoted to explaining the background science and technology involved in the design of the *International Thermonuclear Experimental Reactor* (ITER), of which there are two versions: *Ignition ITER*, which has a major radius of 8.14 m and an estimated cost of 5870 million (1989) dollars and a less ambitious tokamak called *High-Q ITER*, with a major radius of 6.2 m, costing 2755 million (1989) dollars and for which ignition is not the main goal.

On 28th June, 2005, it was announced that *High-Q ITER* would be constructed at Cadarache in the south of France; it has the aim of achieving an extended burn, with a ratio of fusion power to auxiliary heating power of at least 10, and it is expected to begin operating by 2015. The parties involved are China, the European Union, Japan, the Russian Federation, South Korea, and the United States. The group emphasized "the importance of exploring the long-term potential of fusion energy as a virtually limitless, environmentally acceptable and economically competitive source of energy" and said they advocated "wide international co-operation in developing this source of energy for all mankind". It is forecast that terrestrial fusion energy is likely to become a practical energy source by $\sim$2045. Presently, there are more than 44 experimental tokamaks in laboratories around the globe, so the theory of these machines is of continuing interest and seems likely to remain so for some decades.

Whether or not the project is practicable is difficult to judge at this stage, but in view of the impending long-term energy crisis, it is important to continue the research and development, which dates from the early 1950s. Also, apart from their likely relevance to the looming energy crisis, tokamaks are useful apparatuses for a variety of experiments involving high energy phenomena, radiation, and for obtaining a better understanding of the behaviour of plasmas, which constitute more than 95% of the universe. One obvious gap in the tokamak literature concerns the *economics* of fusion reactors, not merely their cost in relation to competitive energy sources, but more importantly the energy investment required in their construction and the time over which a reactor would need to operate to recover this investment. When the basic physics and technology are better understood, this gap will need to be filled.

The last 100 pages of my text on the *Principles of Magnetoplasma Dynamics* (Clarendon Press, Oxford, 1987) were devoted to the theory of tokamak machines and since then a number of books have appeared on the subject, most notably the treatise entitled *Tokamaks* (Clarendon Press, Oxford, 3rd ed, 2004) by John Wesson and some of his colleagues working at Culham Laboratory. My aim here is to present an improved and enlarged version of my original treatment of tokamak theory, to make more comparisons of the theory with observations and to give explanations of some recently discovered phenomena. Although my theoretical approach is quite different from the accepted treatments, it has the merit of yielding good agreement with a wide range of observations and of being a 'complete' theory, in that the empirical input is negligible. When and why it departs from received tokamak theory, as set out for example in Wesson's treatise, is noted appropriately in the text, which is mainly concerned with the complexities of thermal and particle transport in toroidal geometry; for an introduction to the more straightforward MHD calculations of stability, etc., and some of the technical issues involved, besides Wesson's text there is the volume of *Nuclear Fusion* cited above, and a work by Miyamoto entitled *Fundamentals of Plasma Physics and Controlled Fusion,* (Iwanami Book Service Center, Tokyo, 1997).

The physical principle that underlies most of the theory in this text is as follows. By Fourier's law the heat flux vector $\mathbf{q}$ is related to the temperature gradient by $\mathbf{q} = -\kappa\boldsymbol{\nabla}T$, where κ is the thermal conductivity. If the gradient $\boldsymbol{\nabla}T$ is orthogonal to the magnetic field $\mathbf{B} = \mathbf{b}B$, then κ is proportional to $1/(\omega_c\tau)^2 = 1/(QB\tau/m)^2$, where Q is the particle charge, m is the particle mass, and $(\tau)^{-1}$ is the particle collision frequency. In tokamaks it is found that electrons are mainly responsible for the energy loss and the electron parameter, $1/(\omega_{ce}\tau_e)$, is typically 10^{-7}; thus the heat flux vector across the magnetic field, $\mathbf{q}_\perp$, is a mere $1/10^{14}$ times its value in the absence of a magnetic field, a circumstance that should have allowed thermonuclear temperatures to have been easily reached with ohmic or other forms of heating.

However, in a strong magnetic field there is a *transverse* heat flux, $\mathbf{q}_\wedge = -\kappa_\wedge \mathbf{b}\times\boldsymbol{\nabla}T$, in which $\kappa_\wedge \propto 1/(\omega_{ce}\tau_e)$, making $|\mathbf{q}_\wedge|$ about 10^7 times larger than $|\mathbf{q}_\perp|$. But this heat, being at right angles to the temperature gradient, normally circulates around the minor axis of the tokamak torus and makes no difference to energy confinement within the tokamak, and all would be well except for the presence of fluid shear. Shear is well-known to deflect any heat flux vector through a small angle and to create what is called a *second-order* heat flux at right angles to the primary, or first-order heat flux. The 'order' here refers to the Knudsen number k_N, which in the tokamak application is $\tau_e|\boldsymbol{\nabla}\mathbf{v}_e|$, where $\mathbf{v}_e$ is the electron fluid velocity and the gradient $|\boldsymbol{\nabla}\mathbf{v}_e|$ is a measure of its shear. Validity of macroscopic transport theory requires that $k_N \ll 1$, and in tokamaks k_N is typically ~ 0.01. On comparing the first-order heat flux $|\mathbf{q}_\perp| \propto k_N/(\omega_{ce}\tau_e)^2$ with the deflected second-order heat flux $|\mathbf{q}_\wedge{}^d| \propto k_N^2/(\omega_{ce}\tau_e)$, we see that the combination of shear and transverse diffusion removes energy from tokamaks at a rate $\sim 10^5$ times more rapidly than the early expectations, which were based on the first-order theory. Curiously, this dominant process is still ignored in the tokamak literature, despite the passage of more than twenty years since its discovery.

The deflected second-order heat flux will be directed either up or down the temperature gradient depending on whether the radial gradient of the toroidal current density, j_φ, is anti-parallel or parallel to the temperature gradient. The knowledge that there are circumstances in which heat can flow *up* the temperature gradient, allows many strange tokamak observations to be understood. Incidentally, it is very likely that this phenomenon is responsible for the extremely hot solar corona, explaining how it is possible for thermal energy to flow up plasma loops from the relatively cool $\sim 6\,000$ K photosphere to the $\sim 2\times10^6$ K corona. Although the primary concern of this book is with fusion reactors, most of the transport theory developed in the earlier chapters has applications to solar physics, for example to plasma loops, spicules, flares and corona heating.

A similar treatment of the viscous force acting in tokamak magnetoplasmas enables the radial flow velocity v_r to be determined from the second-order formula for this force, and hence the rate at which plasma is lost to the tokamak walls can be calculated. The resulting toroidal electric field, $E_\varphi \approx -v_rB_\theta$, where B_θ is the poloidal component of the magnetic field, drives a non-inductive current — called a *Lorentz current* in the text — that is additional to the induced current; substantial non-inductive currents in agreement with the Lorentz current prediction have been observed and are important for the stability and heating of the plasma.

For tokamaks there is a modified first-order theory called "neoclassical" transport, which by allowing for non-local particle excursions over large 'banana' orbits, increases $\mathbf{q}_{\perp}$ by a factor of several hundred, but this adjustment is still far too small to explain the observations. The usual approach is to speculate that turbulence is responsible for the unexpectedly large thermal transport, and the experimental results from many tokamaks operating in a variety of conditions are assembled into best-fit, empirical curves, which, while practicable for *interpolation*, provide no understanding of the physical mechanisms involved. The design calculations for ITER are based on a single, straight-line extrapolation by a factor of more than two beyond the highest points on the empirical curve for the energy confinement time τ_E. However, the presumption that turbulence is responsible for thermal transport is wrong, as is easily inferred from the observation that the voltage drop around the torus is close to its classical (non-turbulent) value.

Plasma physics is an exceedingly complex branch of macroscopic physics, especially when applied in the domain of tokamak toroidal geometry. In this situation it is too easy to allow formal equations to dominate and to impede a physical grasp of the convective and diffusive mechanisms of transport upon which the success or failure of the tokamak enterprize depends. There is no single master equation from which deductive analysis will yield good estimates of the losses of plasma energy from tokamaks. For example, the 'sheared-transverse-diffusion' transport described above and which is the basis of much of this book, cannot be deduced from Boltzmann's famous kinetic equation, which is generally supposed to cover all transport possibilities. As Eddington once remarked in a lecture at a stage where he was stressing the importance of a proper background to the analysis he was about to present:

> "I regard the introductory part of the theory as the more difficult, because we have to use our brains all the time. . . . Afterwards we can use mathematics instead."

In tokamak physics the situation is particularly demanding, for excepting some stretches of straightforward deductive analysis, physical modelling is required as an essential guide throughout.

To make the account nearly self-contained for graduate students with some experience in continuum physics, most of the background knowledge required in plasma physics, kinetic theory and thermodynamics is either provided in the text or collected as 'plasma physics notes' in the Appendix.

I am grateful to Mr D. E. T. F. Ashby, ex-Culham Laboratory, for his constructive criticism and generous help in the drafting of this book and to Dr Grant Deane of Scripps Institution of Oceanography, who took time from his research to revisit his tokamak background to give me many helpful comments.

Finally, I record with pleasure my appreciation of the help and ready support given me by the officers of the Wiley-VCH Press.

L. C. Woods

Oxford, 20 July, 2005

Lists of physical constants, plasma parameters and frequently used symbols

In SI units, the constants required in plasma theory are:

Physical Quantity	Symbol	Value	units
Electron mass	m_e	9.1095×10^{-31}	kg
Proton mass	m_p	1.6726×10^{-27}	kg
Electron charge	e	1.6022×10^{-19}	C
Boltzmann constant	k_B	1.3807×10^{-23}	$\mathrm{J\,K^{-1}}$
Permittivity (Free Space)	ϵ_0	8.8542×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability (Free Space)	μ_0	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light (Vacuum)	c	2.9979×10^{8}	$\mathrm{m\,s^{-1}}$
Proton/electron mass ratio	m_p/m_e	1.8362×10^{3}	
Temperature at 1 eV	e/k_B	1.1605×10^{4}	K
Planck constant	h	6.6262×10^{-34}	J s
Stefan-Boltzmann constant	σ	5.6703×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	8.3144	$\mathrm{J\,K^{-1}\,mol^{-1}}$

The important plasma parameters are:

Parameter	Symbol	Formula	page
Resistivity	η	$\alpha m_e/(e^2 n_e \tau_e)$	180
Cyclotron frequency (electrons)	ω_{ce}	eB/m_e	185
Thermal speed	C	$\sqrt{2k_B T/m}$	185
Larmor radius	r_L	C/ω_c	185
Coulomb logarithm	$\ln \Lambda$		182
Collision intervals	τ_e , τ_i		124
Thermal conductivity ($B = 0$)	κ	$\gamma k_B p \tau / m$	61
Magnetic diffusivity	ξ	η/μ_0	199
Magnetic Reynolds number	R_m	UL/ξ	–
Plasma frequency	ω_{pe}	$\sqrt{n_e e^2/\epsilon_0 m_e}$	179
Collisionless skin-depth	δ_e	c/ω_{pe}	–
Debye length	λ_D	$\sqrt{\epsilon_0 k_B T_e/n_e e^2}$	179

Frequently used Tokamak symbols

Symbol	Definition	page	Symbol	Definition	page
a	minor radius	4	$\mathbf{B}$	magnetic field (induction)	3
$\mathbf{b}$	$=\mathbf{B}/\lvert\mathbf{B}\rvert$		B_θ	poloidal field	1
B_φ	toroidal field	1	$\mathbf{c}$	peculiar velocity	181
D	convective derivative	182	$\mathbf{e}$	rate of strain tensor	192
$\mathbf{E}$	electric field	179	f_T	trapped fraction	41
g	passing particle fraction	43	h	specific enthalpy	11
H	velocity gradient	45	I_p	plasma current	1
$\mathbf{j}$	current density	179	$\mathcal{L}$	radiation rate	183
M	magnetic moment	38	$n_e,\ n_i$	number densities	2, 3
$p_e,\ p_i$	pressures	12	q	safety factor	8
$q_0,\ q_a$	central & surface q	8	Q	particle charge	
Q_r	radial heat flux	11	$R_0,\ R$	major radius	4, 94
s	$=B_\theta/B_\varphi$	8	T	temperature	
u	internal energy	11	$\mathbf{v}$	fluid velocity	11
V_ℓ	loop voltage drop	107	V_L	Lorentz voltage	132
y	$=(r/a)^2$	87	Z_{eff}	Z-effective	9
α	profile parameter	88	α_E	profile parameter	89
β	plasma beta	7	β_t	toroidal beta	8
β_p	poloidal beta	8	β_N	normalized beta	8
γ	profile parameter	89	δ	profile parameter	89
ε	$=r/R$	8	κ	thermal conductivity	61
η	resistivity	182	$\eta_\parallel,\ \eta_\perp$	parallel & perpendicular	182, 183
ξ	magnetic diffusivity	199	$\boldsymbol{\pi}$	viscosity tensor	184
ϱ	density	11	$\boldsymbol{\sigma}$	electrical conductivity	183
τ_E	energy confinement time	11	τ_E^*	energy replacement time	11
$\tau_e,\ \tau_i$	collision times	124	τ_φ	momentum confinement time	14
χ	thermal diffusivity	61	ω_c	cyclotron frequency	185

We shall often deviate from SI units with temperature, number density and plasma current thus:

Temperature:	T K	=	$1.1605\times10^7\,\hat{T}$,	$\hat{T}$ in keV,
Number density:	$n\,\mathrm{m}^{-3}$	=	$10^{19}n_{19}$,	n_{19} in 10^{19} units per m^{-3},
Electric current:	I_p	=	$10^6\,\hat{I}_p$,	$\hat{I}_p$ in MA.

To reference particular equations forming part of a group, we shall adopt the notation $(a.b)_{(n)}$ to indicate the n-th equation of the set $(a.b)$.

1 The quest for fusion power

This chapter introduces the basic physics and associated variables. Except for those variables cited at the foot of page XVI, SI units are almost always adopted. Pages XV and XVI have lists of physical constants, plasma parameters and frequently used symbols.

1.1 Tokamak machines

1.1.1 Topology and ignition

A tokamak is a toroidal chamber which uses a strong toroidal magnetic field, B_φ, to contain a high temperature plasma within the torus. Charged particles cannot easily move across strong magnetic fields and if the fields are closed into nested surfaces, then deuterium and tritium ions trapped in this way and colliding with sufficient energy to overcome their repulsive Coulomb potential, will fuse and liberate energy. The toroidal field is produced by external electric currents flowing in coils wound around the torus, as shown in Fig. 1.1. Superimposed on the toroidal field is a much weaker poloidal field, B_θ, generated by an electric current I_p flowing in the plasma around the torus. The plasma forms the secondary circuit of a transformer, so that I_p is induced by changing the magnetic flux B_T passing through the torus, which is usually carried by an iron core as indicated in the figure.

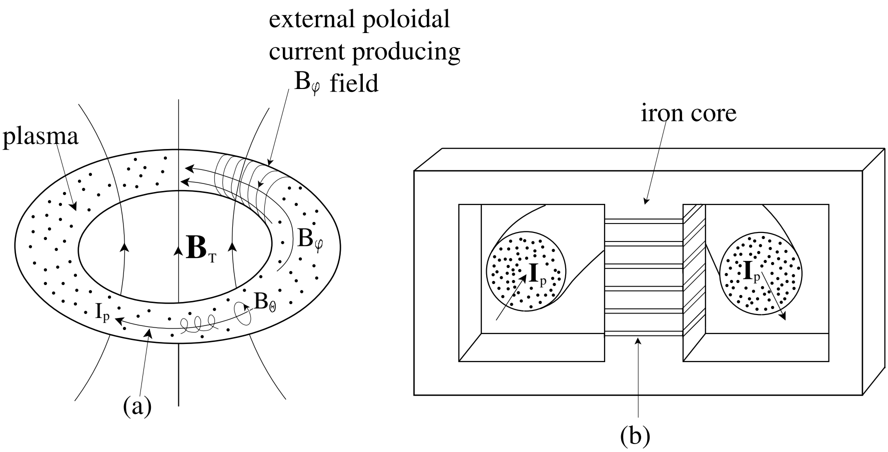

Figure 1.1: Tokamak currents and fields: (a) toroidal plasma current induced by transformer, (b) primary winding

In a plasma consisting of deuterium, or deuterium mixed with tritium, the fusion reactions

$$D^2 + D^2 \rightarrow \begin{cases} He^3 + n^1 + 3.27 \text{ MeV} \\ T^3 + H^1 + 4.03 \text{ MeV} \end{cases}$$

and

$$D^2 + He^3 \rightarrow He^4 + H^1 + 18.3 \text{ MeV}$$

$$D^2 + T^3 \rightarrow He^4 + n^1 + 17.6 \text{ MeV}$$

will occur frequently if the ion temperature, T_i, and the ion number density, n_i, are large enough. Furthermore, in a *fusion reactor* these high values of T_i and n_i must be maintained long enough for the energy liberated by fusion to more than balance the energy losses due to radiation, conduction, convection and neutron flux. Let τ_E be the time it takes these loss processes to remove all the energy from the system, then for a given value of $n_i\tau_E$ there is a minimum temperature at which the plasma is said to *ignite*, i.e. at which the liberated fusion energy is just adequate to balance all losses. As D-D plasmas require considerably higher temperatures to achieve ignition, almost all reactor proposals have concentrated on D-T fusion.

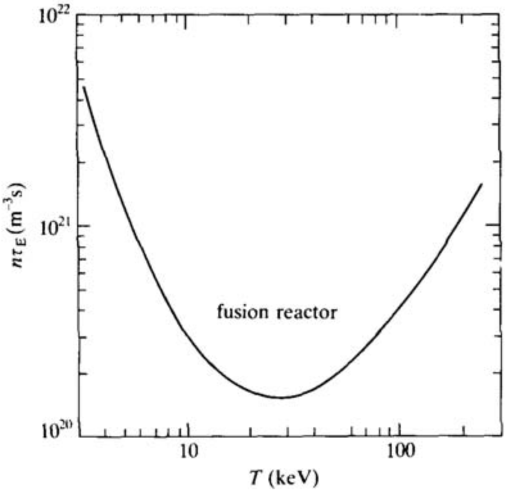

Figure 1.2: Ignition curve for a D-T plasma

Figure 1.2 shows the ignition curve for a D-T plasma. It has a minimum at a temperature of about 30 keV, where for ignition we need $n_i\tau_E > 1.5\times10^{20}\,\text{m}^{-3}\text{s}$. A slightly lower bound ($n_i\tau_E > 6\times10^{19}\,\text{m}^{-3}\text{s}$) known as Lawson's criterion (Lawson 1957) is obtained if a continuous power supply from outside the system is used to compensate transport and radiation losses. Combining the $n_e\tau_E$ value with $\hat{T} \sim 10\,\text{keV}$, we obtain

$$\tau_E n_i \hat{T} > 3\times10^{21}\,\text{s}\,\text{m}^{-3}\,\text{keV}\,, \quad (1.1)$$

which is based on the assumption that the number density and temperature profiles across the minor radius are flat. When allowance is made for typical profile shapes, and the constraint is applied to the peak values, T_0 and n_{i0} of the temperature and number density profiles, (1.1) is replaced by

$$\tau_E n_{i0}\,\hat{T}_0 > 5\times10^{21}\,\mathrm{s\,m^{-3}\,keV}\,.$$

Observations show that electron energy loses are dominant and in a pure D-T plasma, by charge neutrality, $n_i = n_e$, and so to a good approximation the left-hand side of (1.1) can be replaced by $\tau_{Ee} n_e \hat{T}_e$.

Let B denote the strength of the magnetic field[1], then for a reason explained in the first of the plasma physics notes in the Appendix, $B^2/2\mu_0$ is called the magnetic pressure, where μ_0 is the free-space permeability. An important parameter in plasma physics is the ratio of the plasma pressure p to the magnetic pressure, which is known as the plasma *beta*,

$$\beta \equiv \frac{2\mu_0 p}{B^2}\,. \tag{1.2}$$

The power output for a given magnetic field and plasma assembly is proportional to the square of beta, and for an adequate return on an energy investment in magnetic fields, it has been estimated that in a reactor β should exceed 0.1.

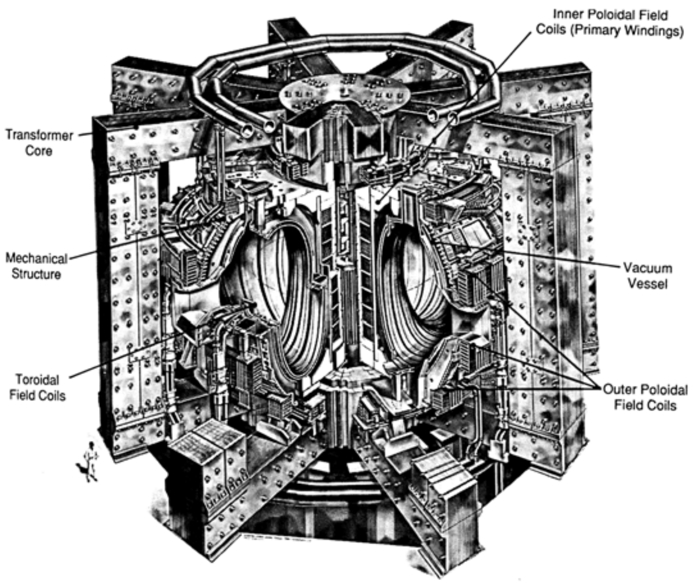

Figure 1.3: The Joint European Torus (JET)

[1] Strictly the magnetic *induction*, but the misnomer 'field' is commonly adopted in plasma physics.

1.1.2 Some early tokamaks

The advantage of the Russian tokamak machine over similar toroidal devices that were being developed in the United States and Great Britain at the same time, lay in the better stability obtained by using much stronger toroidal magnetic fields. ‘Stability’ in this context means no more than the persistence of the magnetic fields and electric currents — at least in the earlier machines — for times of the order of milliseconds. The British ZETA machine, which received much publicity in the 1950s, was so-called ‘stable’ for less than about 5 milliseconds, whereas the discharge in comparable tokamaks lasted over ten times longer.

In his review of the history of tokamak research from 1955 to 1980, Rutherford (1980) noted that this confinement device was responsible for more than half the articles published in the specialist journal *Nuclear Fusion*. The first substantial tokamak was T-3, built at the Kurchatov Institute, Moscow in the 1960s. It had a minor radius of 15 cm, a major radius of 100 cm, a toroidal magnetic field of 15 kG, and carried a plasma current of $100 - 250$ kA. In the standard notation (see Fig. 1.4), $a = 0.15\,\mathrm{m}$, $R_0 = 1\,\mathrm{m}$, $B_\varphi = 1.5\,\mathrm{T}$, $\hat{I}_p = 0.1 - 0.25\,\mathrm{MA}$.

Some twenty years later the Joint European Torus (JET) was constructed at a cost of around £200 M on the Culham site at Abingdon, England, and this is currently the largest tokamak in the world. The cross-section of the torus in JET is D-shaped, with a (horizontal) width of 2.4 m and a height of 4.2 m. Its parameters are: $a = 1.2 \times 2.1\,\mathrm{m}$, $R_0 = 3\,\mathrm{m}$, $B_\varphi = 3.5\,\mathrm{T}$, $\hat{I}_p = 5\,\mathrm{MA}$. Whereas T-3 reached electron temperatures $\sim 0.4 - 1.0\,\mathrm{keV}$ and ion temperatures $\sim 0.2\,\mathrm{keV}$ at average electron number densities of $\bar{n}_e \sim 2{\times}10^{19}\,\mathrm{m}^{-3}$ and energy confinement times of only a few milliseconds, by 1986 JET had achieved $T_e \sim 6\,\mathrm{keV}$, $T_i \sim 12\,\mathrm{keV}$, $\bar{n}_e \sim 3.5{\times}10^{19}\,\mathrm{m}^{-3}$ and $\tau_E \sim 0.9\,\mathrm{s}$, although not simultaneously. However, from (1.1) increases by factors of 3 in T_i and 5 in $n_i\tau_E$ were still required for ignition.

Wesson (2004) gives details of forty-four tokamaks built up to 1985 in England, France, Germany, Italy, Japan, USA, and USSR; Table 1.1 lists those built since 1975. Notice that under the column of the minor radius, DOUBLET III and JET have two lengths written as $a \times b$ where b is the half-height of the plasma and a is the minor radius, or half-width of the plasma; these lengths serve as a rough specification of D-shaped cross sections (e.g. JET's

Table 1.1: Typical values of tokamak parameters (not simultaneous)

Machine	year	R_0 (m)	a (m)	B_φ (T)	$\hat{I}_p$ (MA)	$\bar{n}_e$ $10^{-19}\mathrm{m}^{-3}$	$\hat{T}_{e0}$ (keV)	$\hat{T}_{i0}$ (keV)	τ_E (ms)
DITE	1975	1.17	0.26	2.7	0.2	5	0.7	0.6	14
PLT	1975	1.3	0.40	3.5	0.6	5	3	3	40
T-10	1975	1.5	0.37	4.5	0.5	4	1.4	0.7	50
DOUBLET III	1979	1.43	0.44×0.75	2.4	0.9	10	4	4	100
TFTR	1982	2.4	0.80	5.0	2.2	4	2	8	200
JET	1983	3.0	1.2×2.1	3.5	5.0	3.5	6	8	500
TEXTOR	1983	1.75	0.46	2.0	0.4	3	1.2	0.8	40
JT-60	1985	3.0	0.9	4.5	2.0	7	3	5	100
DIII-D	1986	1.67	0.67	2.1	5.0	8	26	20	160
ASDEC (upgrade)	1991	1.65	0.50	3.9	1.4	11			

vacuum vessel shown in Fig. 1.3). The elongation of the cross-section follows from a solution of the MHD equilibrium equations, which determine the magnetic field structure appropriate for a given choice of pressure and current profiles (Section 2.1). However, in this text to simplify the analysis with relatively little impact on general conclusions concerning transport, the 'elongation' variable, $\kappa = b/a$, will be taken to be unity.

1.1.3 Toroidal current

There is one evident disadvantage in the tokamak design as illustrated in Fig. 1.1, namely that its operation is necessarily pulsed because resistivity will gradually dissipate the inductive current and switch off the discharge. Quite apart from its role in heating the plasma through ohmic dissipation, a toroidal current is essential to maintain an elongated toroidal system in equilibrium, for without the B_θ field that it generates, there is a vertical instability that causes the plasma to drift in the direction of elongation. The force driving this instability results from the interaction of the poloidal field coil currents (see Fig. 1.1) and the plasma current. In some cases feedback control circuitry is necessary to maintain the plasma's position (see Wesson, 2004, p. 342).

Early tokamaks, which relied entirely on inductive currents for both heating and stabilization, were therefore designed for pulsed operation in the hope that the pulse time could be made sufficiently long for fusion to be effective; but these times are measured in seconds rather than minutes and are too short for reactor operation.

Finding other ways of continuously heating the plasma and of maintaining the stabilizing toroidal current, has been an important quest in recent tokamak research. Steady currents can be driven around the torus with radio-frequency (RF) waves and also with neutral beam injection (NBI), but there are limits to this type of 'current drive' that make it unable to generate all of the current required for a stable reactor. One such constraint, called the 'Greenwald' limit, is concerned with the avoidance of major disruptions (Section 6.2.1). For a survey of NBI current drive the reader is referred to ITER team (1999, p. 2527).

However, there is another mechanism that generates non-inductive toroidal currents. It is widely believed that a large current of this type, termed a 'bootstrap' current, can be generated simply by the existence of radial gradients in the plasma density and temperature. Observations certainly support the presence of a non-inductive current, but its origin is not the bootstrap phenomenon, for as shown in Section 3.4.3, such a current does not satisfy Ampère's law and cannot exist. In Section 5.3.2 we show that the observed non-inductive current is a result of the toroidal electric field generated by the radial flow of the plasma across the B_θ magnetic field.

Let v_D be the radial velocity of the plasma flowing across the tokamak magnetic field, then the toroidal electric field, say $E_\varphi^{\rm LR}$, driving the non-inductive current is proportional to the product $v_D B_\theta$, so the 'price' of this potentially steady current is the continual loss of plasma from the torus. Regular refueling by beam injection near the minor axis is therefore required to maintain the current, a process with its own limitations (see Section 1.4.2).

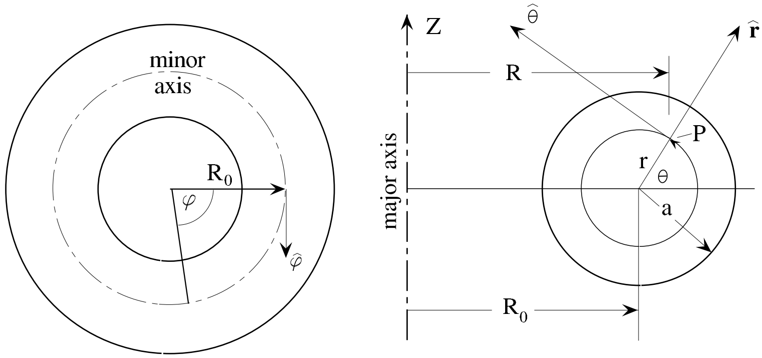

Figure 1.4: Cylindrical and local coordinates for a tokamak machine

1.2 Basic tokamak variables

1.2.1 Aspect ratio

Figure 1.4 shows the coordinate systems for a tokamak of circular cross-section. The local radial dimension lies in the range $0 < r < a$, where a is the maximum radius of the plasma. In order to prevent the plasma reaching the vacuum vessel, either a material *limiter* or a magnetic *divertor* is used, as shown in Fig. 1.5. Most tokamaks have limiters, but divertors have the merit of reducing the influx of ionized impurities into the interior of the plasma by diverting them into an outer "scrape-off" layer.

The tokamak aspect ratio, R_0/a, usually lies between 3 and 5 and as we shall see later, it has an important role in plasma energy confinement.

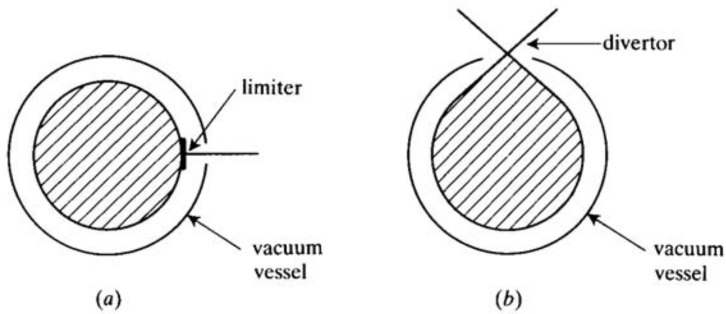

Figure 1.5: Separation of plasma from wall by (a) a limiter, (b) a divertor