Heinz Georg Schuster and Wolfram Just

Deterministic Chaos

An Introduction

Fourth, Revised and Enlarged Edition

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Cover Picture
Detail of the “tail” in Plate IX (after Peitgen and Richter)

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Preface

Since 1994 when the last edition of the present monograph was published, the field of Nonlinear Science has developed tremendously. It is nowadays no longer possible to give a comprehensible introduction into, and a balanced overview of the different branches within this field. Following the general practice of the previous editions it is the scope of this fourth augmented edition to introduce aspects of Nonlinear Dynamics at a level which is accessible to a wide audience. We have intensified and added three new topics:

– Control of chaos is one of the most popular branches of Nonlinear Science. As a particular new aspect we have included a comprehensive discussion of time-delayed feedback control which is widely used in applications.

– Topics in synchronization became recently quite popular from a fundamental as well as an applied point of view. We introduce basic concepts as well as novel notions like phase synchronization or strange nonchaotic, attractors, at an elementary level.

– Spatiotemporal chaos covers a wide range of topics, from classical fields in physics such as hydrodynamics to current research topics in theoretical biophysics, which are commonly related with the nonlinear dynamics of a large number of degrees of freedom. We introduce here basic features of relevant model systems as well as selected concepts for quantitative analysis. But our exposition is far from complete.

The fourth edition benefits from data and figures that have been provided by several colleagues, in particular by R. Klages, J. Kurths, A. Pikovski, H. Posch, and M. Rosenblum. It is a pleasure to thank E. Schöll for his kind hospitality during a stay at Berlin University of Technology, where parts of the new edition were written. We are indebted to the publisher, in particular to Dr. M. Bär and R. Schulz, for their continual help in preparing the manuscript. Despite the remarkable support from various people the present edition could still contain mistakes. We apologize in advance for such inconsistencies and we invite the reader to report to us any deficiencies.

Kiel/London, October 2004
H. G. Schuster, W. Just

Preface to the Third Edition

Since the last edition of this book in 1989 the field of deterministic chaos has continued to grow. Within the wealth of new results there are three major trends.

– Unstable periodic orbits have been rediscovered as building blocks of chaotic dynamics, especially through the work of Cvitanovich *et al.* (1990). They developed an expansion of physical averages in terms of primitive cycles (see also Appendix H).

– Exploiting the concept of unstable periodic orbits, Ott, Grebogi and Yorke demonstrated in 1990 that deterministic chaos can be controlled. They found that small time-dependent changes in the control parameter of the system can stabilize previously unstable periodic cycles in such a way that the system becomes nonchaotic (see Chapter 10).

– There are new theoretical and experimental results in the field of quantum chaos which are described excellently in the new books by Gutzwiller (1990), Haake (1991) and Reichl (1992).

During the preparation of the new edition, J. C. Gruel helped with the pictures of the new chapter, Mrs. H. Heimann typed the new text, M. Poulson and R. Wengenmayr from VCH Publishers took care of the editorial work. H. J. Stockmann and H. J. Stein contributed the fascinating pictures of simulations of quantum chaos in microwave resonators. I would like to thank all these people for their cooperation and patience.

Kiel, August 1994
H. G. Schuster

Preface to the Second Edition

This is a revised and updated version of the first edition, to which new sections on sensitive parameter dependence, fat fractals, characterization of attractors by scaling indices, the Farey tree, and the notion of global universality have been added. I thank P. C. T. de Boer, J. L. Grant, P. Grassberger, W. Greulich, F. Kaspar, K. Pawelzik, K. Schmidt, and S. Smid for helpful hints and remarks, and Mrs. Adlfinger and Mrs. Boffo for their patient help with the manuscript.

Kiel, August 1987
H. G. Schuster

Preface to the First Edition

Daily experience shows that, for many physical systems, small changes in the initial conditions lead to small changes in the outcome. If we drive a car and turn the steering wheel only a little, our course will differ only slightly from that which the car would have taken without this change. But there are cases for which the opposite of this rule is true: For a coin which is placed on its rim, a slight touch is sufficient to determine the side on which it will fall. Thus the sequence of heads and tails which we obtain when tossing a coin exhibits an irregular or chaotic behavior in time, because extremely small changes in the initial conditions can lead to completely different outcomes. It has become clear in recent years, partly triggered by the studies of nonlinear systems using high-speed computers, that a sensitive dependence on the initial conditions, which results in a chaotic time behavior, is by no means exceptional but is a typical property of many systems. Such behavior has, for example, been found in periodically stimulated cardiac cells, in electronic circuits, at the onset of turbulence in fluids and gases, in chemical reactions, in lasers, etc. Mathematically, all nonlinear dynamical systems with more than two degrees of freedom, i. e.,, especially many biological, meteorological or economic models, can display chaos and, therefore, become unpredictable over longer time scales. "Deterministic chaos" is now a very active field of research with many exciting results. Methods have been developed to classify different types of chaos, and it has been discovered that many systems show, as a function of an external control parameter, similar transitions from order to chaos. This universal behavior is reminiscent of ordinary second-order phase transitions, and the introduction of renormalization and scaling methods from statistical mechanics has brought new perspectives into the study of deterministic chaos. It is the aim of this book to provide a self-contained introduction to this field from a physicist's point of view. The book grew out of a series of lectures, which I gave during the summer terms of 1982 and 1983 at the University of Frankfurt, and it requires no knowledge which a graduate student in physics would not have. A glance at the table of contents shows that new concepts such as the Kolmogorov entropy, strange attractors, etc., or new techniques such as the functional renormalization group, are introduced at an elementary level. On the other hand, I hope that there is enough material for research workers who want to know, for example, how deterministic chaos can be distinguished experimentally from white noise, or who want to learn how to apply their knowledge about equilibrium phase transitions to the study of (nonequilibrium) transitions from order to chaos. During the preparation of this book the manuscripts, preprints and discussion, the remarks of G. Eilenberger, K. Kehr, H. Leschke, W. Selke, and M. Schmutz were of great help. P. Berge, M. Dubois, W. Lauterborn, W. Martienssen, G. Pfister and their coworkers supplied several, partly unpublished, pictures of their experiments. H. O. Peitgen, P. H. Richter and their group gave permission to include some of their most fascinating computer pictures in this book (see cover and Section 6.4). All contributions are gratefully appreciated. Furthermore, I want to thank W. Greulich, D. Hackenbracht, M. Heise, L. L. Hirst, R. Liebmann, I. Neil, and especially I. Procaccia for carefully reading parts of the manuscript and for useful criticism and comments. I also acknowledge illuminating discussions with V. Emery, P. Grassberger, D. Grempel, S. Grossmann, S. Fishman, and H. Horner. It is a pleasure to thank R. Hornreich for the kind hospitality extended to me during a stay at the Weizmann Institute, where several chapters of this book were written, with the support of the Minerva foundation. Last but not least, I thank Mrs. Boffo and Mrs. Knolle for their excellent assistance in preparing the illustrations and the text.

Frankfurt, October 1984
H. G. Schuster

Legends to Plates I–XX

Many of these plates are part of Chapter 6. Accordingly, references mentioned in the legends are to be found on pages 89–125.

I. *Biperiodic flow in a Bénard experiment*. Figs. 1–8 show interferometric pictures of a Bénard cell in the biperiodic regime; that is, there are two incommensurate frequencies in the power spectrum (see also pages 2.1.2–2.1.2). The time between successive pictures is 10 s. The first period lasts 40 s after which the "mouth" in the middle of the pictures repeats itself (see Figs. 1 and 5). But the details, e. g., in the upper right corners of Figs. 1 and 5 are not the same; that is, the motion is not simply periodic. (From a film taken by P. Berge and M. Dubois, CEN Saclay, Gif-sur-Yvette, France.) [page xv]

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III. *Taylor instability*. a) Formation of rolls, b) the rolls start oscillating, c) a more complicated oscillatory motion, d) chaos. (After Pfister, 1984; see also pages 133–135.) [page xvi]

IV. *Disturbed heartbeats*. The voltage difference (black) across the cell membrane of one cell of an aggregate of heart cells from embryonic chicken shows a) phase locking with the stimulating pulse and b) irregular dynamics, displaying escape or interpolation beats if the time between successive periodic stimuli (red) is changed from 240 ms in a) to 560 ms in b). (After Glass *et al.*, 1983; see also page 7.3.3.) [page xvii]

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VII. *The Cassini division*. The ring of Saturn (a) shows a major gap (b), the so-called Cassini division, because the motion on this orbit is unstable (see also Fig. 142 on page 174).

(NASA pictures no. P-23068 and P-23207 with permission from Bildarchiv, Baader Planetarium.) [page xviii]

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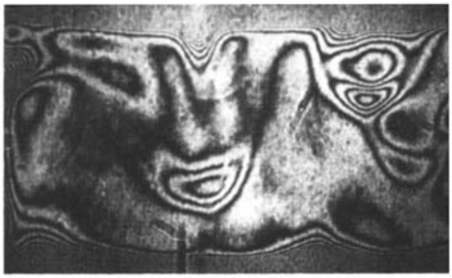
1



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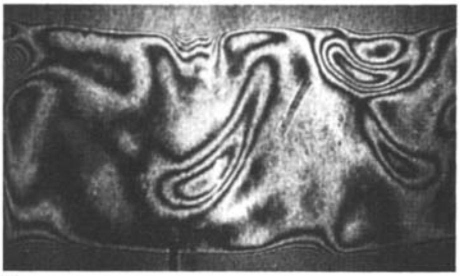

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Plate I.

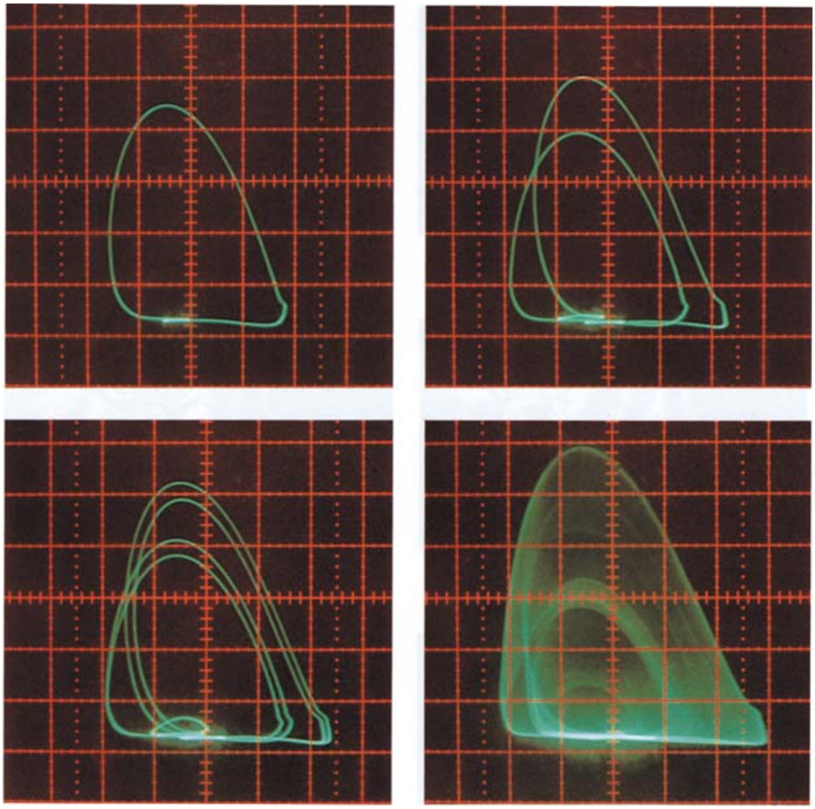
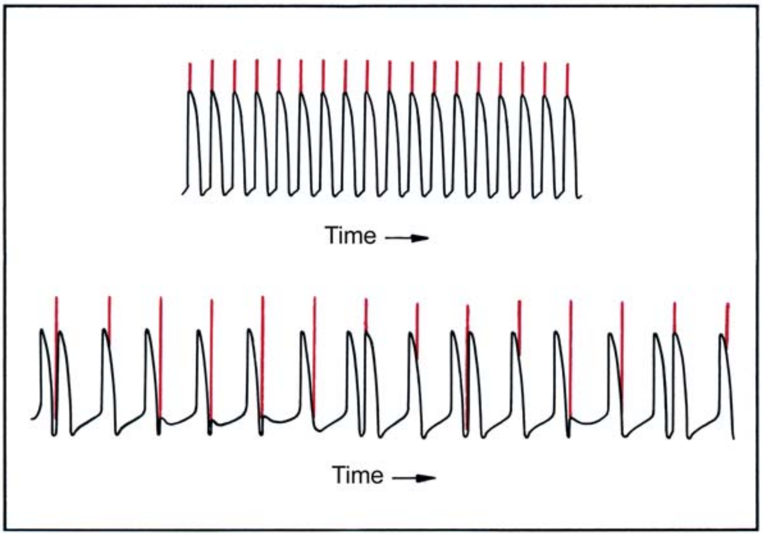


Plate II.

Plate III.



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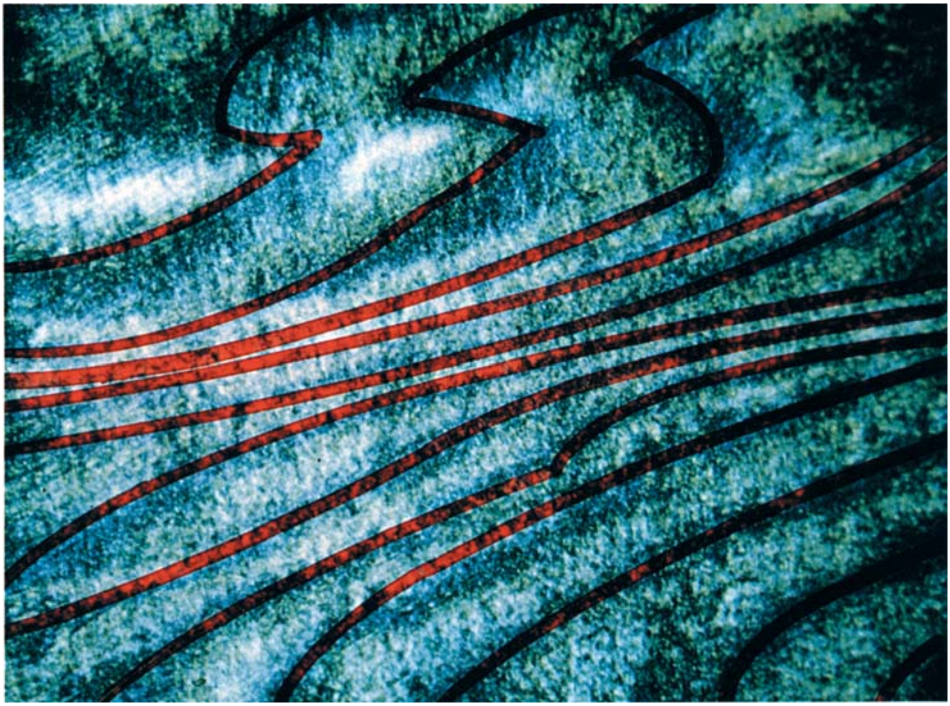

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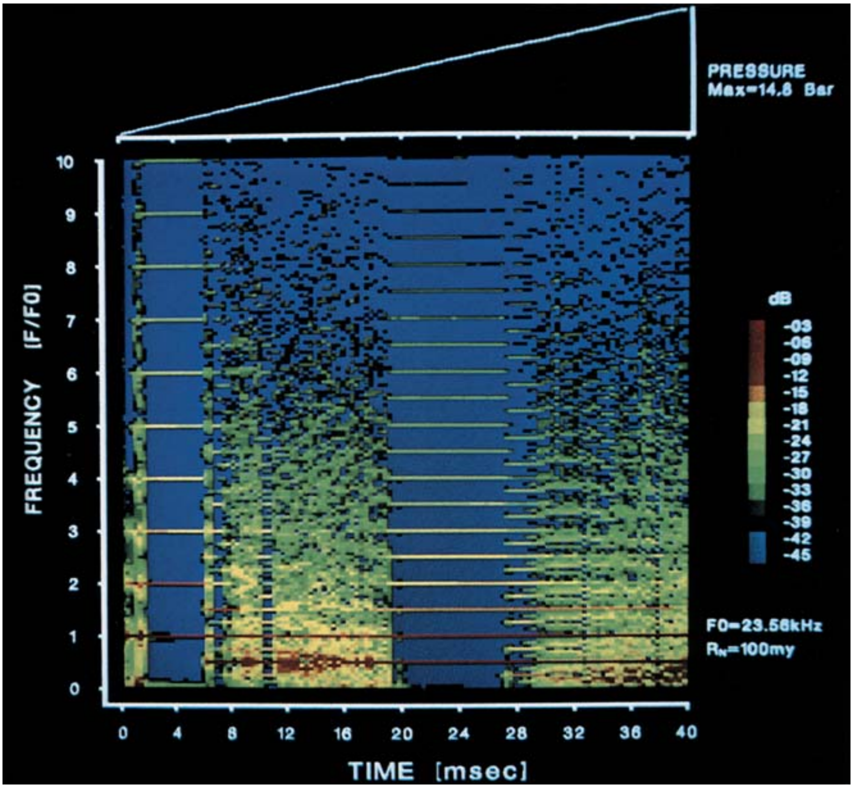

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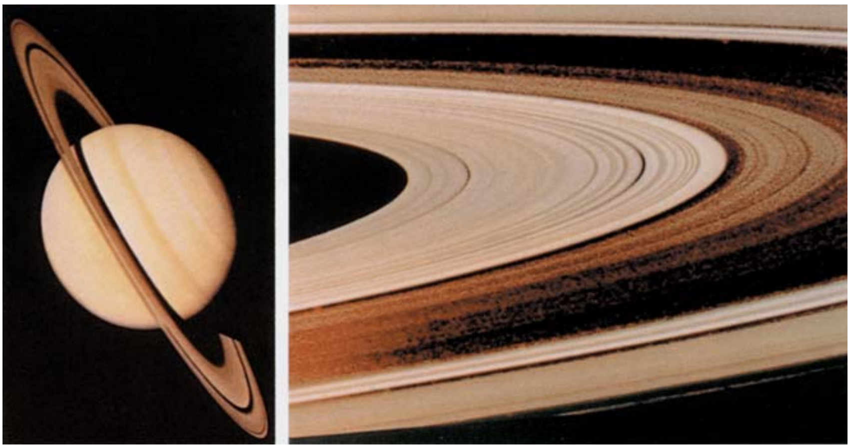

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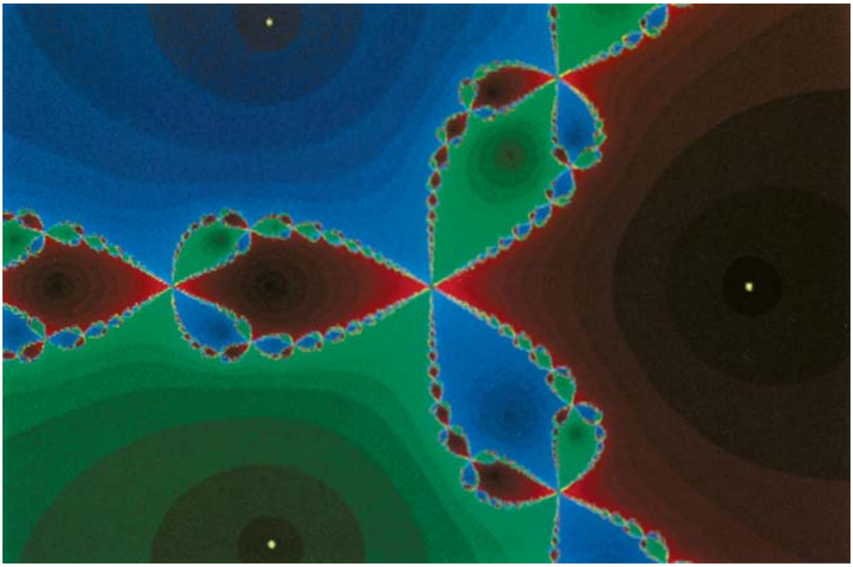

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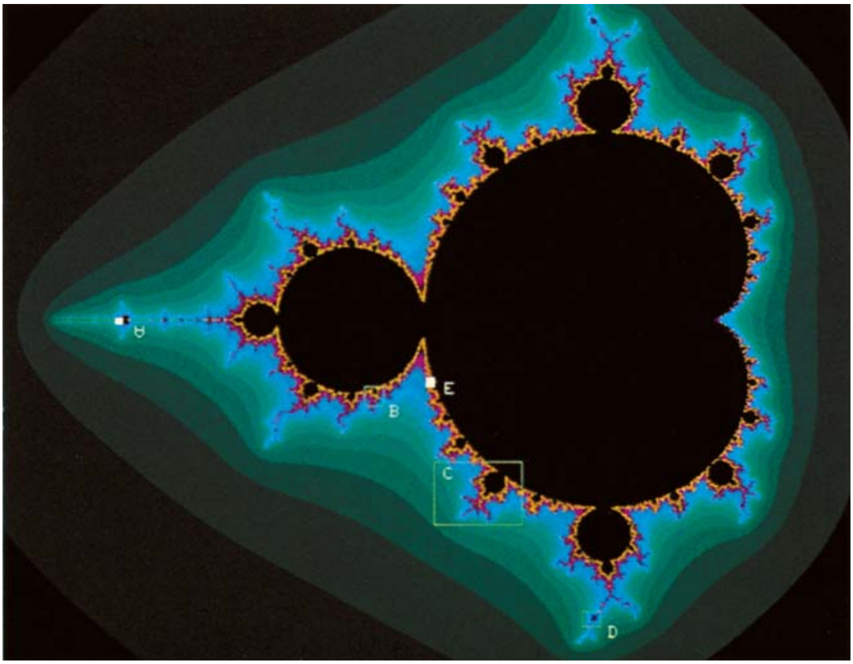

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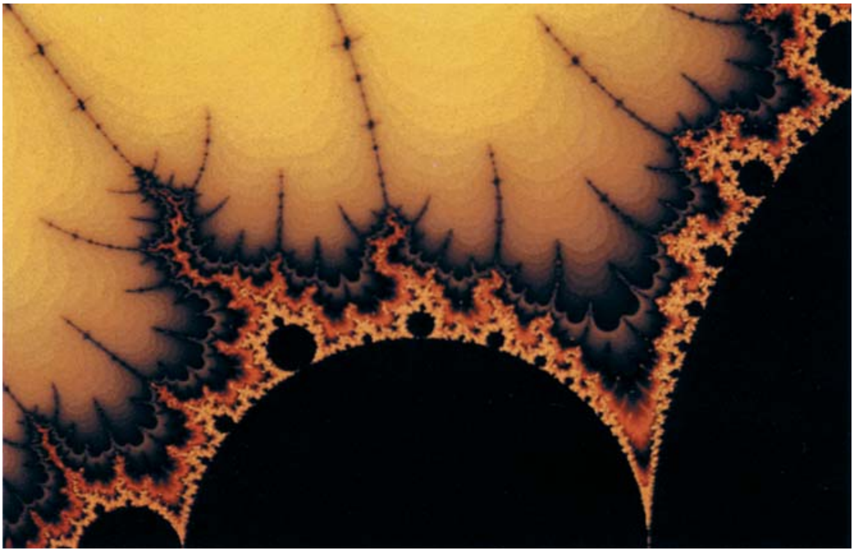

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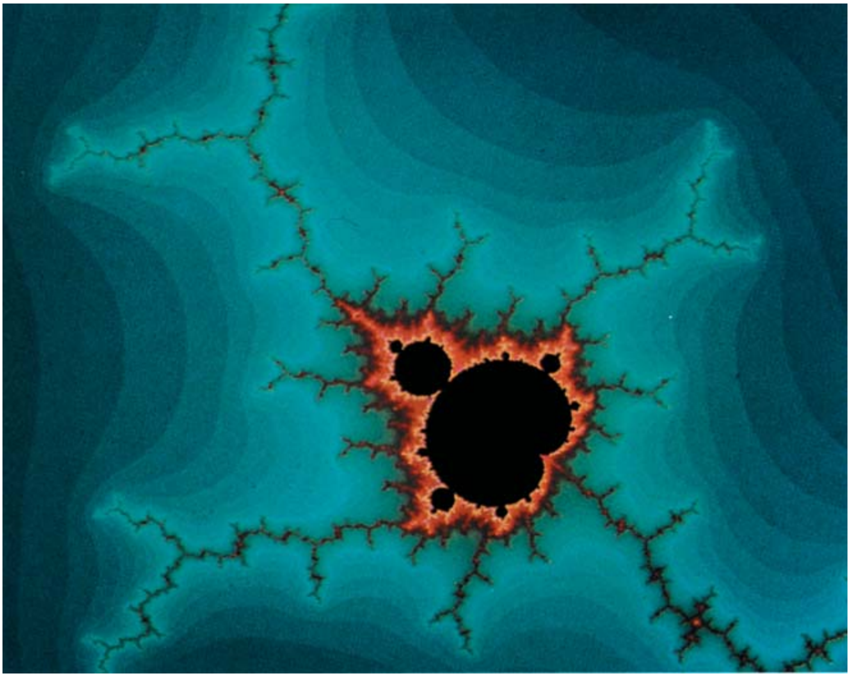

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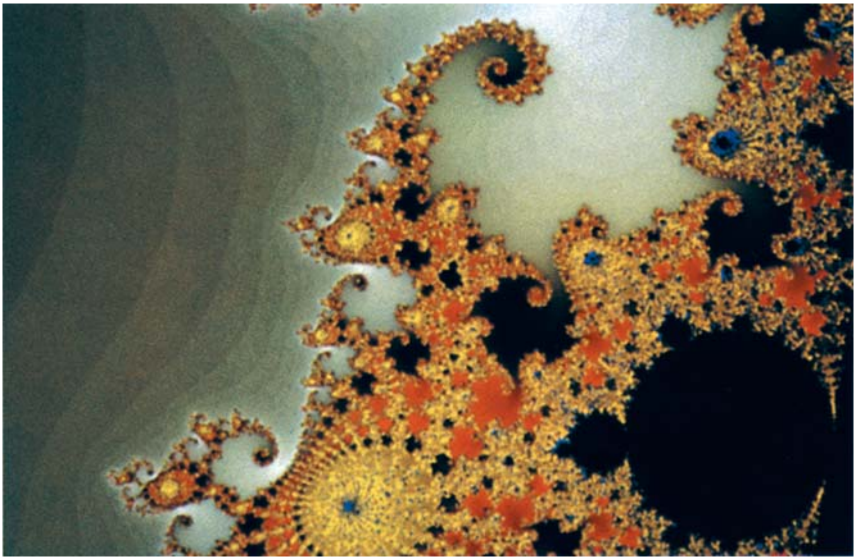

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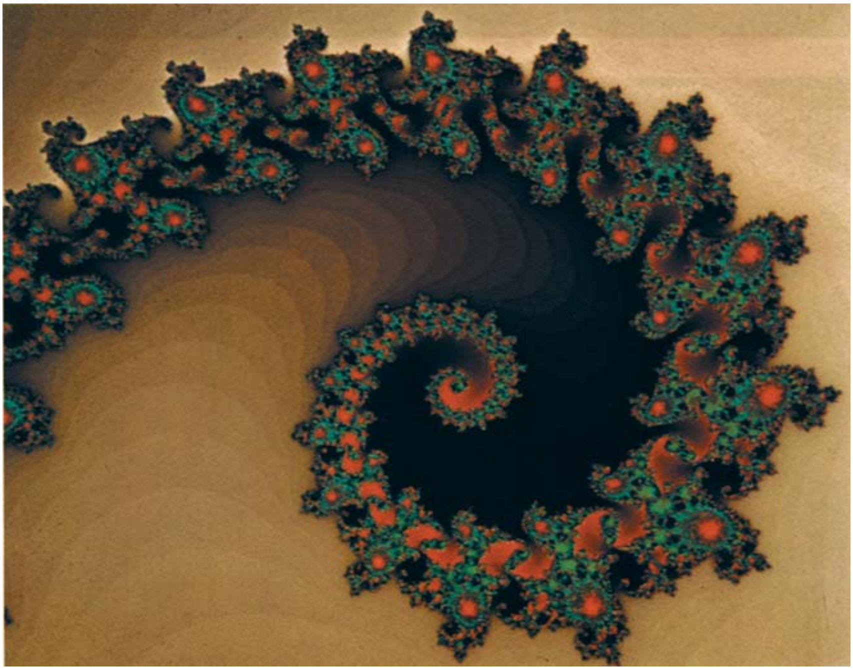

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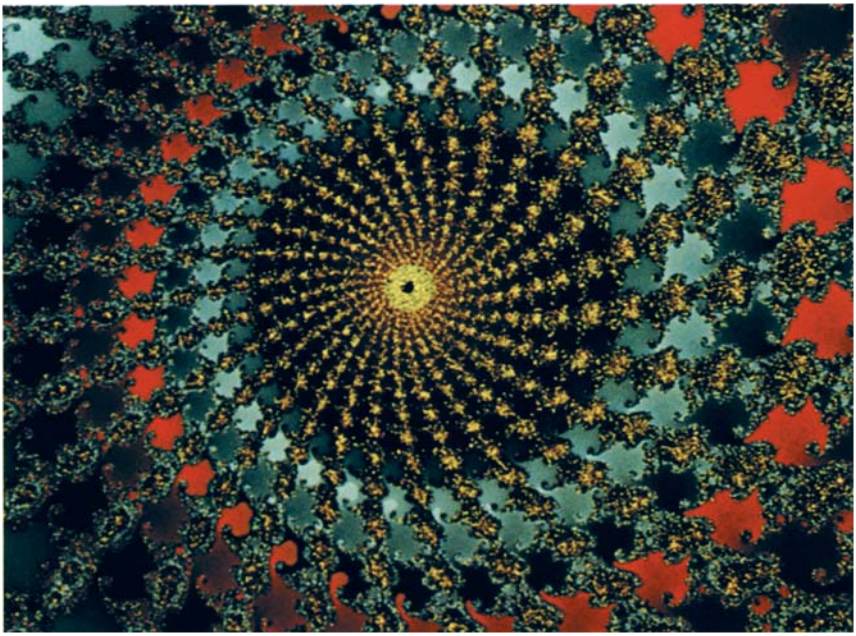
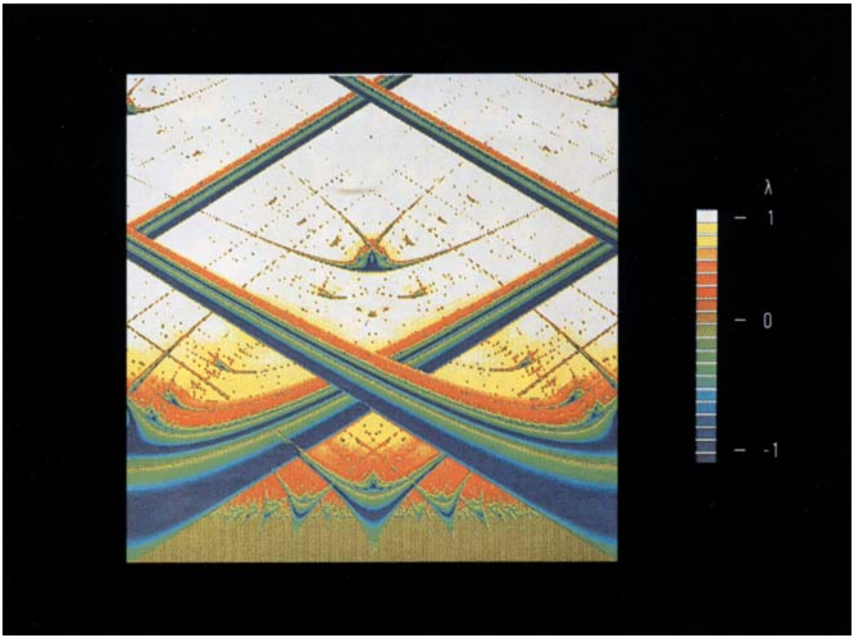


Plate XIV.

Plate XV.



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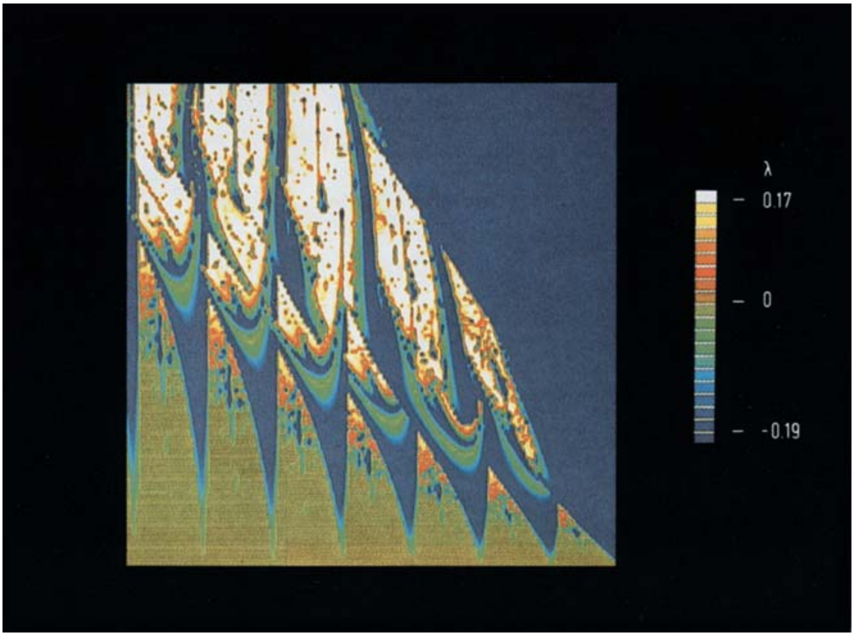

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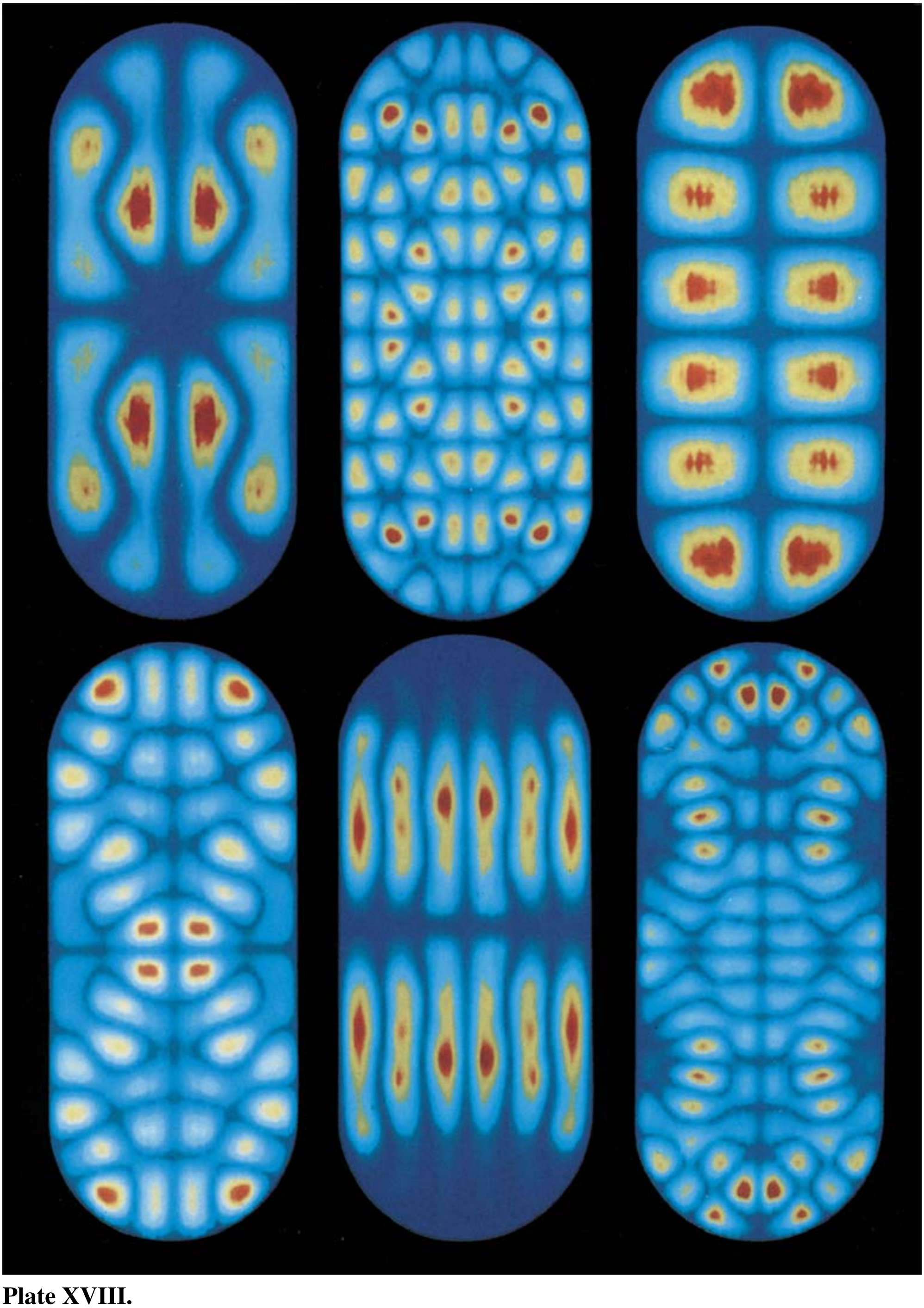

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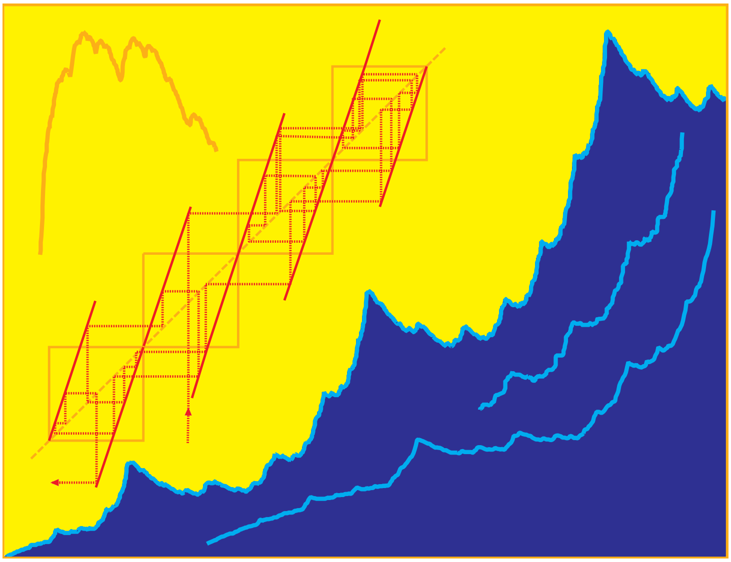

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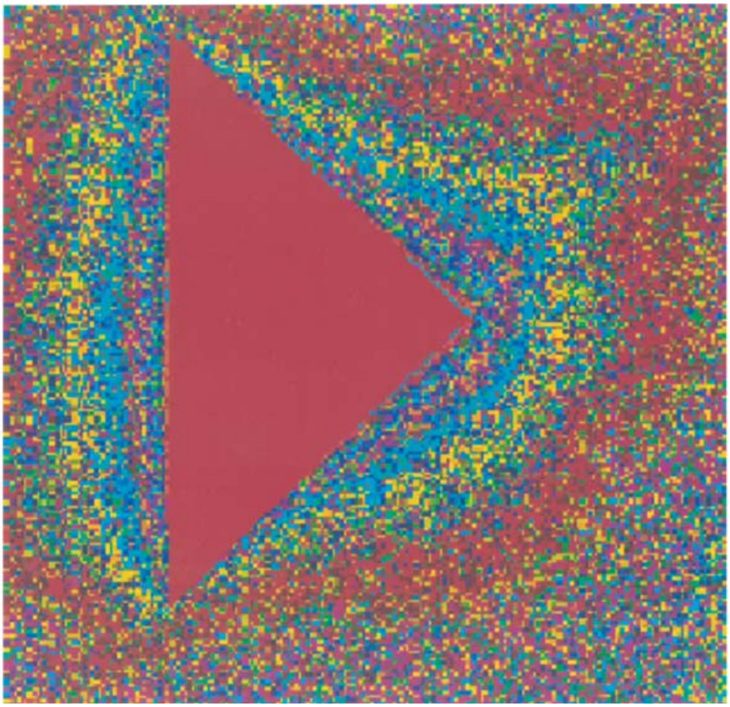

Plate XX.

1 Introduction

Ante mare et terras et, quod tegit omnia,
caelum Unus erat toto naturae vultus in
orbe, Quem dixere Chaos, rudis
indigestaque moles Nec quicquam nisi
pondus iners congestaque eodem Non
bene iunctarum discordia semina rerum.
Ovid

It seems appropriate to begin a book which is entitled "Deterministic Chaos" with an explanation of both terms. According to the Encyclopaedia Britannica the word "chaos" is derived from the Greek "$\chi\alpha o\sigma$" and originally meant the infinite empty space which existed before all things. The later Roman conception interpreted chaos as the original crude shapeless mass into which the Architect of the world introduces order and harmony. In modern usage which we will adopt here, chaos denotes a state of disorder and irregularity.

In the following, we shall consider physical systems whose time dependence is deterministic, i. e., there exists a prescription, either in terms of differential or difference equations, for calculating their future behavior from given initial conditions. One could assume naively that deterministic motion (which is, for example, generated by continuous differential equations) is rather regular and far from being chaotic because successive states evolve continuously from each other. But it was already discovered at the turn of the century by the mathematician H. Poincaré (1892) that certain mechanical systems, whose time evolution is governed by Hamilton's equations, could display chaotic motion. Unfortunately, this was considered by many physicists as a mere curiosity, and it took another 70 years until, in 1963, the meteorologist E. N. Lorenz found that even a simple set of three coupled, first-order, nonlinear differential equations can lead to completely chaotic trajectories. Lorenz's paper, the general importance of which is recognized today, was also not widely appreciated until many years after its publication. He discovered one of the first examples of deterministic chaos in dissipative systems.

In the following, deterministic chaos denotes the irregular or chaotic motion that is generated by nonlinear systems whose dynamical laws uniquely determine the time evolution of a state of the system from a knowledge of its previous history. In recent years – due to new theoretical results, the availability of high speed computers, and refined experimental techniques – it has become clear that this phenomenon is abundant in nature and has far-reaching consequences in many branches of science (see the long list in Table 1, which is far from complete).

Table 1: Some nonlinear systems which display deterministic chaos. (For numerals, see "References" on page 259.)

Forced pendulum [1]
Fluids near the onset of turbulence [2]
Lasers [3]
Nonlinear optical devices [4]
Josephson junctions [5]
Chemical reactions [6]
Classical many-body systems (three-body problem) [7]
Particle accelerators [8]
Plasmas with interacting nonlinear waves [9]
Biological models for population dynamics [10]
Stimulated heart cells (see Plate IV at the beginning of the book) [11]

We note that nonlinearity is a necessary, but not a sufficient condition for the generation of chaotic motion. (Linear differential or difference equations can be solved by Fourier transformation and do not lead to chaos.) The observed chaotic behavior in time is neither due to external sources of noise (there are none in the Lorenz equations) nor to an infinite number of degrees of freedom (in Lorenz's system there are only three degrees of freedom) nor to the uncertainty associated with quantum mechanics (the systems considered are purely classical). The actual source of irregularity is the property of the nonlinear system of separating initially close trajectories exponentially fast in a bounded region of phase space (which is, e. g., three-dimensional for Lorenz's system).

It becomes therefore practically impossible to predict the long-time behavior of these systems, because in practice one can only fix their initial conditions with finite accuracy, and errors increase exponentially fast. If one tries to solve such a nonlinear system on a computer, the result depends for longer and longer times on more and more digits in the (irrational) numbers which represent the initial conditions. Since the digits in irrational numbers (the rational numbers are of measure zero along the real axis) are irregularly distributed, the trajectory becomes chaotic.

Lorenz called this sensitive dependence on the initial conditions the butterfly effect, because the outcome of his equations (which describe also, in a crude sense, the flow of air in the earth's atmosphere, i. e., the problem of weather forecasting) could be changed by a butterfly flapping wings. This also seems to be confirmed sometimes by daily experience.

The results described above immediately raise a number of fundamental questions:

– Can one predict (e. g., from the form of the corresponding differential equations) whether or not a given system will display deterministic chaos?

– Can one specify the notion of chaotic motion more mathematically and develop quantitative measures for it?

– What is the impact of these findings on different branches of physics?

– Does the existence of deterministic chaos imply the end of long-time predictability in physics for some nonlinear systems, or can one still learn something from a chaotic signal?