

*Alexander Kukushkin*

# **Radio Wave Propagation in the Marine Boundary Layer**



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## Cover Picture

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## Preface

This book is about the parabolic approximation to a diffraction problem over a sea surface. While the parabolic equation method in radio wave propagation over the earth's surface was introduced by V.A. Fok almost fifty years ago, its popularity has grown recently due to the development of advanced computational methods based on the parabolic approximation. Numerous computational techniques have been evolved and used for analysis of radio- and acoustic wave propagation in either deterministic or random media.

This book is concerned with the analytical solution to a problem of wave propagation over the sea surface in the atmospheric boundary layer. Two basic mathematical methods have been used, depending on the ease of obtaining a closed analytical solution:

1. Expansion of the quantum-mechanical amplitude of the transition into a complete and orthogonal set of eigen functions of the continuous spectrum.
2. The Feynman path integral.

It is not intended to provide a full step by step mathematical background to the above methods but, rather, is dedicated to the application and analysis of the physical mechanisms associated with the combined effect of scattering, diffraction and refraction. The mathematical foundations for the above methods can be found in numerous monographs and handbooks dedicated to quantum mechanics and mathematical theory.

The book is arranged as follows: Chapter 1 presents the basic assumptions used to describe the propagation media, i.e. the atmospheric boundary layer. It provides a simplified description of the turbulent structure of the refractive index in the atmospheric boundary layer and summarises the model of the troposphere to be used in the analysis of the wave propagation. It introduces some foundation for the composition of the refractive index as two components: a deterministic layered structure and a relatively small-scale random component of turbulent refractive index. A basic classification of the propagation mechanisms, such as refraction, ducting, diffraction and scattering is briefly introduced according to the presence and value of the negative gradients of refractivity in the troposphere.

Chapter 2 commences with an overview of the mathematical methods developed for analysis of the problem of wave propagation and scattering in a stratified medi-

um with random fluctuations of the refractive index. It also positions the method introduced in this book as an extension of the well-known analogy between the quantum-mechanical problem of the quasi-stationary states of the Schrödinger equation and the problem of radio wave propagation in the earth's troposphere. The advantage of using this approach is that the Green function to the parabolic equation is expanded over the complete set of orthogonal eigen functions of the continuous spectrum. This representation is equivalent to a Feynman path integral which is used in Chapter 3 to investigate the higher order moments of the wave field over the surface with impedance boundary conditions.

Some new physical mechanisms associated with scattering are analysed and explained in Chapter 3.

Chapter 4 introduces a perturbation theory for normal waves in a stratified troposphere. The problem here is that the common perturbation theory does not work for equations with a potential unlimited at infinity. Such potentials appear in the problem of an electron in a magnetic field or in radiowave propagation over the earth's surface in the parabolic equation approximation. A modified perturbation theory is applied to the analysis of the spectrum of normal waves (propagation constants) for the boundary problem with a somewhat arbitrary profile of the refractive index. The analytical solution and numerical results are discussed for two practically important models of refractive index in the near-surface domain: the bilinear approximation and the logarithmic profile. Also in Chapter 4, we present a closed analytical solution for a second moment of the wave field (coherence function) in the presence of an evaporation duct filled with random inhomogeneities of refractive index. The mechanism of interaction between discrete and continuous modes due to scattering of the random irregularities in the refractive index is analysed in detail.

Chapter 5 deals with the elevated tropospheric duct. We start from a normal mode structure for the trilinear profile of the refractive index and analyse the wave field in geometric optic approximation thus introducing rays and modes. The specific case of the presence of two waveguides, elevated duct and evaporation duct, simultaneously, is analysed in detail by means of presenting the mechanism of exchange of the wave field energy in a two-channel system.

In Chapter 5 we also introduce the mechanism of excitation of the normal waves in an elevated duct by means of single scattering on turbulent irregularities of refractive index. This case may represent significant practical interest in the case of ground-air communication for two reasons: first, the elevated ducts are often detached from the surface and the near-surface antenna is ineffective in excitation of the trapped modes, and secondly, strong anisotropic irregularities are often present in the upper boundary of the elevated tropospheric duct due to the physics of its creation and, therefore, can produce a significant scattering effect of the incoming waves from a surface-based antenna.

Finally, in Chapter 6 we analyse some non-conventional mechanisms of the over-horizon propagation. First, the effect of a stochastic waveguide created by anisotropic irregularities in the refractive index. This mechanism is analysed in terms of the perturbation theory presented in Chapter 3. The second mechanism is a single scattering of diffracted field in the earth's troposphere. This mechanism is rather com-



plementary to a conventional single-scattering theory; it cannot explain the observed levels of the signal but, contrary to conventional theory, reveals a correct behaviour with regard to frequency.

The Appendix provides a brief theory of the Airy functions and some asymptotic representations.

The analytical solutions and results considered in this book are chiefly applicable to radio propagation in the UHF/SHF band, i.e., from 300 MHz to 20 GHz where refraction and scattering play a major role in anomalous propagation phenomena, such as a waveguide mechanism in tropospheric ducts.

I want to thank my former colleagues I. Fuks and V. Freilikher in cooperation with whom most of the theoretical studies have been performed. It was my privilege to work with my colleagues in such a productive and encouraging environment. I wish to thank V. Sinitin who introduced me to research activities in this area and supported me at the start of my career.

Most of all, I am gratefully obliged to my lovely wife, Galina, for making it possible.

Alexander Kukushkin  
Sydney, Australia, March 2004



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## 1

## Atmospheric Boundary Layer and Basics of the Propagation Mechanisms

The troposphere is the lowest region of the atmosphere, about 6 km high at the poles and about 18 km high at the equator. In this book we study radio wave propagation along the ocean and can reasonably assume that all processes of propagation occur in a lower region of the earth's troposphere. That lower region and the atmospheric conditions are of most importance for the subject under study here.

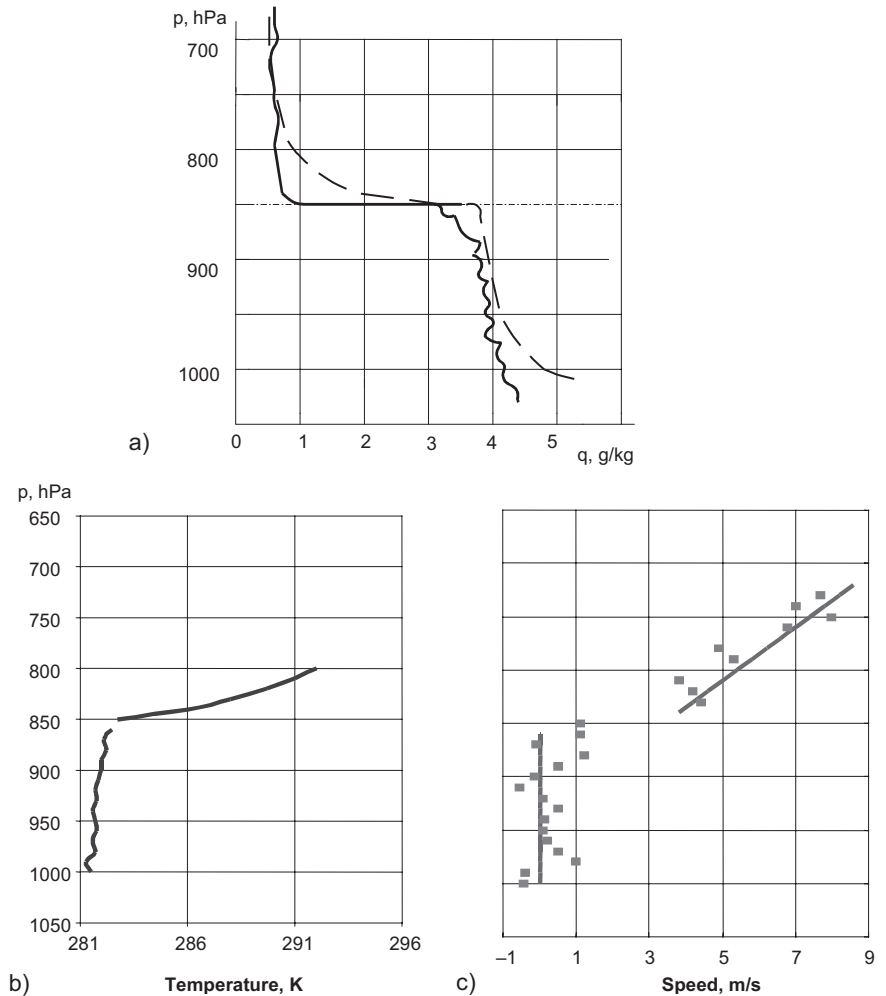
From the perspective of radio communications/propagation we limit our objective to an investigation of the impact of the atmospheric structure on the characteristics of the radio signal propagating through the atmospheric turbulence. All known methods of solutions to a similar problem are based on the separation of the space-time scales of the variations in both the refractive index  $n$  and electromagnetic field  $\vec{E}$ ,  $\vec{H}$  in two domains, described in terms of deterministic and stochastic methods. Intuition suggests that the spectrum of turbulent variations in the refractive index  $n$  will have the energy of its fluctuations confined to a limited space-time domain or, at least, have a clear minimum and, desirably, a gap spread over a significant interval in the time-space domain. It is apparent that the horizontal scales of variations in refractive index larger than the length of the radio propagation path have no immediate effect on the characteristics of the radio signal and rather affect its long term variations over the permanent path. This comes down to an upper boundary of the spatial variations of refractive index in a horizontal plane of about 100 km. The vertical irregularities are of most importance since they are responsible for the refraction and scattering of the radio waves in troposphere. However, there are some natural limitations on the region of the troposphere which might be of interest in its impact on radio propagation. The troposphere is naturally divided into two regions: the lower part of the troposphere, commonly called an atmospheric boundary layer, and the area above, called clear atmosphere.

The electric properties of the troposphere can be characterised by the dielectric permittivity  $\epsilon$  or the refractive index  $n = \sqrt{\epsilon}$ . The numerical value of the non-dimensional parameter  $n$  is pretty close to unity, however even a relatively small deviation of the refractive index from unity may have significant impact on radio wave propagation. Therefore, common practice is to use another definition of the refractive index  $N = (n-1) \times 10^6$  instead of  $n$ , measurable in so-called  $N$ -units. The refractive index  $N$ , also called the refractivity, has the following relationships with atmospheric pressure  $p$ , temperature  $T$  and humidity, the mass-fraction of the water vapor,  $q$ , in the air:

$$N = \frac{A_N \cdot p}{T} \left( 1 + B_N \frac{q}{T} \right) \quad (1.1)$$

where  $A_N = 77.6 \text{ N-units} \times \text{K hPa}^{-1}$ ,  $B_N = 7733 \text{ K}^{-1}$ . The components  $p$ ,  $T$  and  $q$  are random functions of the coordinates and time. The stochastic behavior of the meteorological parameters  $p$ ,  $T$ ,  $q$  and, therefore, the refractive index  $N$  is caused by atmospheric turbulence.

There are several reasons to separate the region of the first 1–2 km of atmosphere over the earth’s surface, called the atmospheric boundary layer, ABL. The upper boundary of the ABL is seen as the height at which the atmospheric wind changes direction due to a combined effect of the friction and Coriolis force. Among those reasons are:



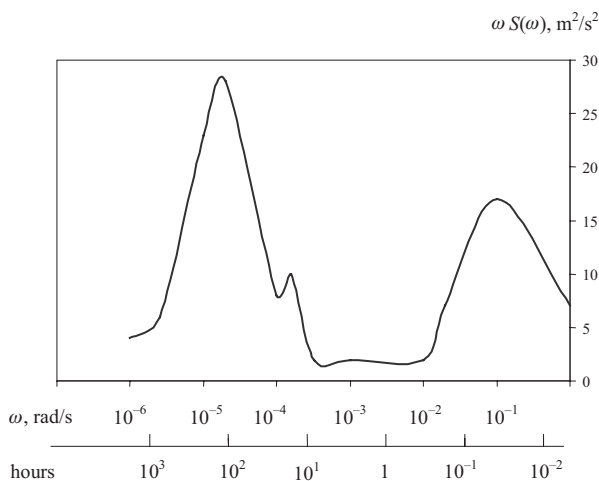
**Figure 1.1** Meteorological parameters in the atmospheric boundary layer as function of height (atmospheric pressure,  $p$ ): Humidity (a), temperature (b) and wind speed (c). All parameters experience sharp variations at the upper boundary of the atmospheric boundary layer.



- The interaction of the earth's surface and atmosphere is especially pronounced in this region.
- The meteorological parameters such as temperature, humidity and wind speed experience daily variations in this region due to apparent cyclic variations in the sun's radiation due to the earth's rotation.
- The ABL can be regarded as an area constantly filled with atmospheric turbulence. This is quite opposite to the atmospheric layer above the ABL, the so-called region of clear atmosphere, where turbulence is present only in isolated spots.
- The border between the clear atmosphere and the ABL is clearly pronounced with sharp variations in all meteorological parameters, as illustrated in Figure 1.1.

The spectrum of turbulent fluctuations in the atmospheric boundary layer is extremely wide: the linear scales of the variations range from a few millimetres to the size of the earth's equator, the time scales from tens of milliseconds to one year. Studies of the energy spectrum of the turbulent fluctuations of the meteorological parameters (temperature, humidity, pressure and wind speed) [1] have shown that the energy spectrum reveals three distinct regions: large scale quasi-two-dimensional fluctuations in a range of frequencies from  $10^{-6}$ – $10^{-4}$  Hz, the meso-meteorological minimum with low intensity of the fluctuations in the range  $10^{-3}$ – $10^{-4}$  Hz and a small scale three-dimensional fluctuation region with frequencies above  $10^{-3}$  Hz.

Figure 1.2 shows the energy spectrum of fluctuations of the horizontal component of the wind speed in the atmosphere, taken from Ref.[1], where the ordinate corresponds to the product of the spectrum density  $S(\omega)$  and the cyclic frequency  $\omega$  of the variations in one of the meteorological components, and the abscissa corre-



**Figure 1.2** Energy spectrum of the fluctuations in the wind speed in the atmospheric boundary layer.

sponds to the frequency  $\omega = 2\pi/T$ ,  $T$  being a period of variations. As observed in Figure 1.2, there are two major extremes of the function  $\omega \cdot S(\omega)$ : the high frequency maximum corresponds to a linear scale of turbulence of the order of tens and hundreds of meters, the low frequency maximum has a time scale of 5–10 days which is caused by synoptic variations (cyclones and anti-cyclones), the respective horizontal scale is thousands of kilometres and the vertical scale is of the order of 10 km. There is also an extended minimum in the spectrum  $\omega \cdot S(\omega)$  that corresponds to the fluctuations with respective horizontal scales from 1 to 500 km and is called the meso-pause. The region of low frequency variation is called the macro-range while the region to the left of the meso-pause (high frequency variations) is called the micro-range.

The nature of the atmospheric turbulence is different in these two regions: in the macro-range the synoptic processes can be regarded as two-dimensional variations, while in the micro-range, with scales up to a hundred meters, the turbulence is three-dimensional and locally uniform. The meso-pause is a transition region where a combined mechanism is observed. It is important to notice that by describing small-scale three-dimensional fluctuations in the micro-range region one can use Taylor's hypothesis of "frozen turbulence" which allows a transformation from time-to space-fluctuation scales by means of  $L = 2\pi v/f$ , where  $L$  is the spatial scale of the irregularities,  $f$  is the frequency of time variations in the refractive index, and  $v$  is the mean speed of the incident flow.

The basic conclusion that follows from the above observations is that, to some extent, the refractive index  $N$  and the dielectric permittivity  $\varepsilon$  can be presented as a sum of a slow varying component  $\varepsilon_0(\vec{r})$  regarded as a quasi-deterministic function of the coordinates  $\vec{r} = \{x, y, z\}$  and the random component  $\delta\varepsilon(\vec{r})$ . As observed from Figure 1.2, the quasi-deterministic component  $\varepsilon_0(\vec{r})$  still varies in the horizontal plane and the energy of variations in the meso-pause minimum is not negligible. However, these variations have less impact on radio wave propagation than either variations of  $\delta\varepsilon(\vec{r})$  in the micro-range or over-the-height variations in the "deterministic" component which may be responsible for a ducting in the troposphere.

Mathematically, the problem of radio wave propagation in a randomly inhomogeneous medium comes down to solving a stochastic wave equation with dielectric permittivity  $\varepsilon(\vec{r}, t)$  which is a random function of coordinates and time. In many cases the process of propagation of a monochromatic wave in the troposphere can be considered in a quasi-steady state approximation, i.e. "frozen" in time. It is then convenient to represent the dielectric permittivity in the form  $\varepsilon_0(\vec{r}) \equiv \varepsilon_0(z) + \delta\varepsilon(\vec{r})$ , where  $\varepsilon_0(z) = \langle \varepsilon_0(\vec{r}) \rangle$ . The angular brackets denote averaging over the ensemble of the realisations of  $\varepsilon(\vec{r})$ . In fact, the mean characteristic of the tropospheric dielectric permittivity  $\langle \varepsilon(\vec{r}) \rangle$  is commonly understood as a large-scale structure homogeneous in the horizontal plane and practically non-varying over the time over which the signal measurements have been performed and then, as a result of mathematical idealisation,  $\langle \varepsilon(\vec{r}) \rangle = \varepsilon_0(z)$ . In radio-meteorology mean characteristics of the meteorological parameters are usually understood to be the values obtained by averaging over a 30 min interval [2], i.e. averaging is performed over a frequency interval the lower limit of which is positioned within the limits of the meso-meteorological minimum.

The average characteristic obtained in this way is usually a function of the height  $z$  above the surface (sea, ground) and varies slowly with the horizontal coordinates and time. Assuming ergodicity and Taylor's hypothesis, such averaging over a time interval corresponds to the averaging over the ensemble of the realisations of  $\varepsilon(\vec{r}, t)$ .

As a compromise in analytical studies of radio wave propagation, a common approach is to neglect the residual variations in  $\langle \varepsilon(\vec{r}) \rangle$  over the horizontal coordinates, i.e. to regard the  $\varepsilon_0(\vec{r}) \equiv \varepsilon_0(z)$ . This assumption results in the introduction of the traditional model of a stratified atmosphere, in which the average values of refractive index  $N$  vary over the vertical coordinate  $z$ , the height above the ground. This traditional model provides some basis for a classification of the radio wave propagation mechanisms, in particular, a separation of the propagation into two classes: standard and non-standard. The following Sections 1.2 and 1.3 provide a brief analysis of the standard and non-standard models, while Section 1.4 deals with a statistical model for a random component of the refractive index.

## 1.1

### Standard Model of the Troposphere

The standard mechanism of radio wave propagation is classified under the condition where the average vertical gradients of the refractive index  $\gamma_N = dN/dz$  are close to the value  $\gamma_N^{st} = -39$  N-units  $\text{km}^{-1}$ . Such conditions of refractivity constitute a model of *standard linear atmosphere* defined as

$$N = 289 - 39 \cdot z \quad (1.2)$$

and are applicable for heights less than 2 km.

Let us consider this model in detail involving a geometrical optic presentation for wave propagation.

Let us define "ray" as a normal to a wavefront propagating through the medium with varying refractivity  $n(z)$ . As is known, the ray bends in such a medium and the bending is defined by Snell's law. Introducing the horizontally stratified medium in terms of the set of thin layers with value of refractivity  $n_i$ ,  $i = 0, 1, \dots$ , such as illustrated in Figure 1.3, Snell's law can be written as

$$n_i \cos \phi_i = \text{const} \quad (1.3)$$

where  $\phi_i$  is a sliding angle. Introducing the differentials of the ray direction  $dz$ ,  $dS$  in the  $i$ th layer and differentiating both sides of Eq. (1.3) with respect to  $S$ :

$$\cos \phi_i \frac{dn_i}{dS} - n_i \sin \phi_i \frac{d\phi_i}{dS} = 0. \quad (1.4)$$

Substituting  $dS = -dz/\sin \phi_i$ , we obtain

$$\frac{d\phi_i}{dS} = -\frac{\cos \phi_i (dn/dz)}{n_i}. \quad (1.5)$$

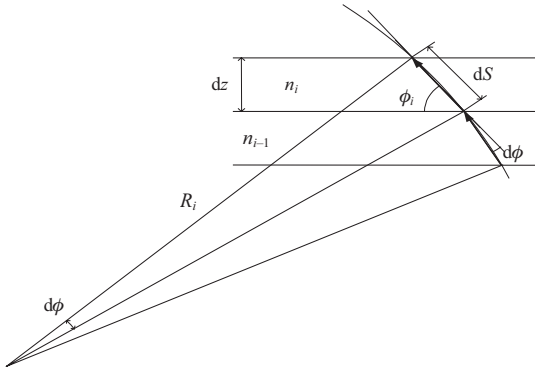


Figure 1.3 Illustration of Snell's law.

The radius of curvature at any point,  $R_i = dS/d\phi_i$ , and using Eq. (1.5) it results that

$$R_i = \frac{n_i}{\cos \phi_i} \frac{1}{(-dn/dz)}. \quad (1.6)$$

For the standard atmosphere with  $dn/dz = 39 \times 10^{-6} \text{ km}^{-1}$ , the radius of curvature is given by

$$R_i = 25,000 \frac{n_i}{\cos \phi_i}. \quad (1.7)$$

If the launch angle  $\phi_i$  is close to the horizontal, the ratio  $\frac{n_i}{\cos \phi_i} \approx 1$  and a propagation path can be described as a circle of radius  $R = 25,000 \text{ km}$ , Figure 1.4(a). By comparison, the radius of the earth's curvature is  $a = 6370 \text{ km}$ , and  $1/a = 157 \times 10^{-6} \text{ km}^{-1}$ . When the curvatures of both the propagation path and the earth are reduced by  $39 \times 10^{-6}$ , as in Figure 1.4(b), the propagation path has an effective curvature of zero (which is a straight line) and that hypothetical earth has an effective curvature of  $(157 - 39) \times 10^{-6} \text{ km}^{-1} = 118 \times 10^{-6} \text{ km}^{-1}$ . The equivalent radius of the sphere  $a_e$  can therefore be defined as

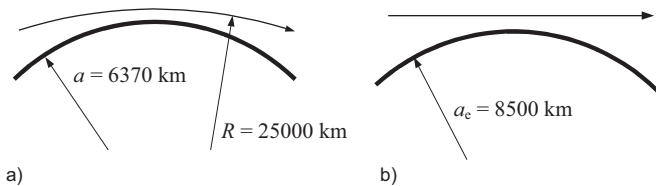


Figure 1.4 Introduction of the "effective" radius of the earth: a) Ray refraction in a "normal" atmosphere with "true" earth radius  $a = 6370 \text{ km}$ . b) Effective ray refraction in case when the difference in curvatures of both the ray and earth surface in (a) is compensated by introduction of the modified earth radius  $a_e = 8500 \text{ km}$ .