Solidification and Crystallization

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Preface

The Federation of European Materials Societies (FEMS) organized the biannual meeting of EUROMAT conference 2003 in Lausanne, Switzerland. This conference was essentially focussed on materials development and application. One of the conference topic was dedicated to *PHASE TRANSFORMATION* chaired by Professor Wilfried Kurz, Lausanne. It consisted of two symposia S1 on *SOLIDIFICATION* and S3 on *SOLID STATE TRANSFORMATION*.

Papers submitted originally to Symposium S2 *MODELLING* were integrated into Symposium S1. All together 96 abstracts not only from Europe but also from Asia and America were submitted to Symposium S1 with which S1 became the largest symposium of EUROMAT 2003 conference. The papers were distributed to three sessions on *Modelling*, two sessions on *Microstructures* and one session on *Properties of Melts, Nucleation, Dendritic Growth, Phase Selection, Multiphase Alloys* and *Processing Methods*, respectively. In addition there was a poster session.

The symposium was started with a plenary lecture by Professor Michel Rappaz on *Modelling of Solidification with Special Emphasis on the Last Stage Solidification*. The oral sessions and the poster session attracted many attendees and experienced vivid discussions showing that the research area of solidification is carried by a large and very active scientific community. In particular, remarkable progress was reported on new experimental methods and techniques for direct observation and analysis of solidification pathways both under near- and non-equilibrium conditions. Simultaneously, modelling of solidification proceeded to a big step forwards enabling now even the computer assisted description of solidification phenomena in multiphase and multicomponent alloys. Such progress is of fundamental interest for materials science but also very important in developing innovative techniques in materials production routes of improved efficiency in casting processes of foundry industry. Since by far most of the materials are produced from the liquid state as their parent phase even small but steady advances in solidification processing leads to correspondingly large consequences in progress of economy and living conditions of daily human life. It is therefore mandatory for a prosperous society.

The present Publications to EUROMAT 2003 contains selected papers of symposium S1 which have been invited for submission to publication in the present book. Each manuscript was peer reviewed by two independent experts before acceptance for publication. Certainly, the peer review process has essentially contributed to the

quality of the present volume. I appreciate the valuable and important assistance of all experts who delivered reports on submitted manuscripts.

I would like to extend my gratitude to Professor Wilfried Kurz for inviting me as an organizer of the symposium *Solidification* of the topic of *Phase Transformation*, and the colleagues who took over the chairmanship of the different sessions as Andreas Ludwig (Leoben), Michel Rappaz (Lausanne), Heike Emmerich (Dortmund), A. Lindsay Greer (Cambridge), Heiner Müller-Krumbhaar (Jülich), Dirk Holland-Moritz (Köln), Andy Mullis (Leeds), Lorenz Ratke (Köln), Peter Schuhmacher (Leoben) and Britta Nestler (Karlsruhe). I enjoyed the cooperation with Michel Rappaz in his help of combining Symposium S2 (Modelling) with Symposium S1 (Solidification). My thanks are to the local organisers in Lausanne for an excellent conference performance, Peter-Paul Schepp from the Deutsche Gesellschaft für Materialkunde in the preparation of the programme, the Deutsche Forschungsgemeinschaft for support within the priority program SPP1120 and Jörn Ritterbusch and Jörg Wrzesinski from WILEY-VCH for their great help in editing this book.

Köln, August 2004

Dieter M. Herlach
Organizer of Symposium
Solidification of EUROMAT 2003

Contents

1
Complex Structures: A Symbiosis of Experiments and Numerical Studies

J. H. Bilgram and H. M. Singer

Abstract

Dendrites are the best studied structures formed during solidification of undercooled melt. Additionally other complex structures have been found in experiments and numerical studies. We perform 2D/3D-numerical simulations of solidification processes and experiments which allow *in situ* investigations of three-dimensional growth of xenon crystals into undercooled pure melt. Dendrites, seaweed, doublons, triplons etc. can be produced in experiments and numerical studies depending on initial conditions. In an interplay of numerical studies and experiments we use simulations to interpret and to plan experiments. Experimental results are used as a basis of model calculations and the development of models of complex shapes. Three-dimensional growth shapes of crystals are reconstructed using sophisticated image processing combined with experimentally determined shape parameters.

1.1
Introduction

Applications in automotive industry call for aluminium castings to be mass-produced without casting defects. Predictive models are necessary to prevent such defects. During the growth of a solid from its undercooled parent melt microstructures are formed. The best-known cases of such structures are dendrites. Microstructural changes can be observed as undercooling is increased, namely the typical length scales like the tip radius decrease with increasing undercooling [1, 2]. These are quantitative changes. In addition to that, for some materials it has been observed that they also undergo an abrupt decrease in microscale with increasing undercooling. This effect is known as spontaneous grain refinement. It has been observed in solidification from undercooled pure metallic melts [3] and alloys [4, 5]. Evidence has been found that spontaneous grain refinement is initiated by changes in the morphology of the microscopic structures [3]. This is in difference to the above mentioned quantitative grain refinement with increasing undercooling. In order to obtain more information on shapes, developed during solidification, numerical modeling and analytical stu-

dies of the solidification process have been performed. Starting with a mathematical formulation of the dendrite problem, the goal is to find out which solutions exist and which morphologies are possible. Namely the stability of solutions has been studied [6, 7, 8]. A very astonishing result was the discovery of a doublon in 2D studies [9]. It is formed by two crystalline tips with a straight channel in between. This is a typical non-equilibrium shape. It was followed up by numerical studies which found a triplet in an 3D channel [8]. In a next step a morphology diagram has been developed for 2D systems [10, 11]. Regions for stable fractal and compact growth of dendrites and seaweed have been determined. Seaweed has been considered to be built up from doublons. In this study the stability regions of morphologies have been determined in a field of supercooling of the melt vs. anisotropy of the surface free energy. Supercooling is given in the range from 0 to 1 in dimensionless units and the anisotropy in the range from 0 to 1. High anisotropies lead to dendritic growth, at low anisotropy and sufficient undercooling seaweed is growing. The amount of noise in the system under consideration may influence the position of the phase boundaries. Therefore the phase diagram has to be considered to be a qualitative one.

To verify predictions, numerical and experimental studies have to be combined. We hope that such combined studies will stimulate the application of theoretical results in the design casting processes.

1.2 Experimental Studies

Existing models for dendritic growth do not predict morphological parameters with sufficient accuracy for reliable calculations to be applied in industrial casting processes. Metallographic techniques show microstructures which have been transformed during solidification and subsequent cooling. The study of decanted structures leads to an overestimation of the fraction of solid and does not provide information of solidification kinetics. Time resolved synchrotron radiation imaging techniques do no not provide a lateral resolution suficient to determine typical length scales of the fine structure and it is not possible to study three-dimensional structures with this technique up to now [12]. Thus the use of transparent materials is the only way to determine growth parameters of morphologies during the solidification process. Two types of experiments have been performed: quasi two dimensional studies on organic alloys [13] and studies of pure materials during free growth in three dimensions [14, 15, 16].

First experimental evidence of doublons has been achieved by Akamatsu et al. [13]. A thin cuvette was used and the material was an mixture of two organic liquids CBr_4 and C_2Cl_6. This system has two properties, which are important for the experiment:
i) An alloy is used. It is well known that in alloys much higher constitutional undercoolings are possible than for bulk undercoolings in a pure melt.
ii) Interface kinetics and surface free energy of the solid-liquid interface are anisotropic. Therefore this system has the great advantage that it is possible to change anisotropy of the surface free energy by changing the orientation of the seed crystal.

In these experiments the growth of seaweed has been observed and phase diagrams of the type as developed by Brener et al. [11] have been verified.

The calculations of Brener et al. have been performed for a 2D system. The experiments of Akamatsu et al. have been performed in a thin cuvette and thus it is a quasi-two-dimensional system. A two-dimensional doublon is topologically different from a doublon in three dimensions, because in 2D there is only one way out of the channel, namely at the tips. In the 3D case it is possible to move out of the channel in a direction perpendicular to the plane defined by the two branches limiting the channel. This fact changes topology significantly and also the possibility for the latent heat to diffuse away.

Experiments in three dimensions to detect doublons and seaweed have been performed in a growth vessel with a volume of 100 cm^3 of pure liquid xenon. We use xenon as a transparent model substance for metals, because it forms a simple liquid, it crystallizes in an fcc-lattice, and it forms a solid-liquid interface which is rough at atomic scale. Details of the experimental setup are described in [16, 17]. At the beginning of the experiment the melt is brought to a homogeneous temperature T_∞ which is below the melting temperature, i.e. the melt is homogeneously undercooled. Then a crystal is nucleated in a capillary and grows through the capillary into the undercooled melt. The dimensions of the growth vessel are sufficiently large to ensure free growth of the crystal. According to the properties of the solid-liquid interface and the undercooling in the range of 10^{-3} to 10^{-2} in dimensionless units, a dendrite grows to a stationary state after several minutes and the temperature distribution around the dendrite reaches a steady state. An important feature of the experimental setup [17] is the possibility to shift this dendrite up and down inside of the growth vessel. Shifting the dendrite in the growth vessel means to disturb the temperature distribution around the crystal and to wash off the isotherms. Immediately after such a shift the crystal is surrounded by melt with a spatial homogeneous temperature. Any surface element of the dendrite tip is in contact with liquid at a temperature close to T_∞. The thermal gradient at the crystal surface is everywhere the same. (Neglecting that the surface has been non isothermal before shifting the crystal.) Therefore any surface element of the crystal is growing with the same growth rate. This leads to an increase in the tip radius of the dendrite (Figure 1.) With increasing tip radius the dendrite reaches a state where the tip becomes unstable and tip splitting is observed. After tip splitting two tips continue to grow and form a doublon with a straight channel between the two tips. A sequence of contours is shown in Figure 1. By this procedure it is possible to grow various morphologies. Figure 2 shows a dendrite, a doublon and a seaweed morphology.

In difference to Brener et al. [11] we distinguish between doublon and seaweed morphology. This difference may originate from the three-dimensional geometry. The three morphologies can be distinguished from symmetry considerations: A dendrite shows a high degree of symmetry. For the seaweed no growth direction can distinguished in the plane of projection. The doublon has an axis of symmetry in the center of the channel.

In three dimensions the morphological transitions are not limited to the three cases shown in Figure 2. In addition to that we find triplons and quadruplons and

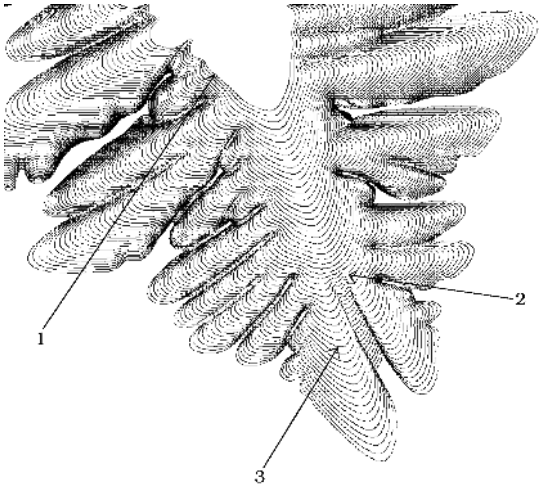

Fig. 1. A sequence of contours taken after the crystal has been shifted in the growth vessel. First we have the shape of the dendrite (arrow 1). During the temporal development the tip radius increases and the tip radius becomes unstable (arrow 2). Finally two tips are formed with a straight channel of liquid in between forming a doublon (arrow 3).

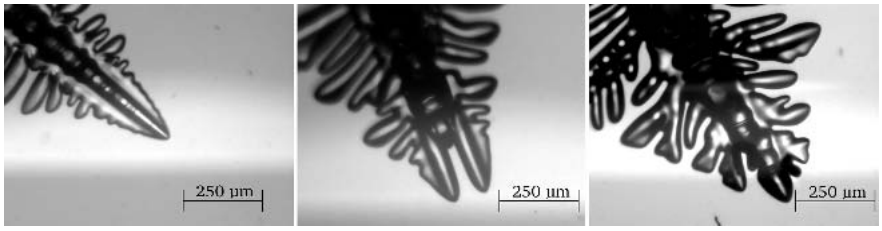

Fig. 2. Three morphologies: A dendrite, a doublon, and a seaweed.

Fig. 3. A crystal with three tips is formed in a similar process as observed in Figure 1.

even higher multiples of tips. Up to now we do not know the precise conditions for the occurrence of a special morphology. Figure 3 shows the development of a triplon out of a dendrite.

1.3 Numerical Studies

In order to verify and refine the qualitative analytical predictions by Brener et al. [11] we have performed phase field simulations with two different phase field models in 2D and 3D. Phase field models are an elegant approach to solve the analytical equations numerically. Instead of calculating a sharp phase boundary between solid and liquid in the Stefan- or sharp interface-problem every grid point holds in addition to the temperature also its phase state. The phase may have arbitrary values between 0 (solid) and 1 (liquid). A thin, but finite region of steep changes between solid and liquid models the interface. This width can be chosen and determines how well the approximation is in accordance with the sharp interface equations.

For our investigations we have chosen the models of Wheeler et al. [18] and Karma et al. [19]. The main difference between the two models is that in the Wheeler model the ratio between kinetic and capillary terms is fixed, where as in the Karma model the kinetic term can be cancelled by appropriately chosen terms for low undercoolings. We have performed simulations on a 2000×2000 grid with different undercoolings and anisotropies. The anisotropy axis goes from 0 to 6.5% as this is the maximal value where a crystal still grows in rough growth. For higher anisotropies the crystal becomes facetted as there occur forbidden growth directions. An initial coarse scan of 100 simulations was used to partition the plane. Subsequently a binary search for the morphology boundary was performed. By this refinement we were able to tie up the uncertainty region for the morphology boundary to up to 10^{-2}%. For lower undercoolings the simulation time increases significantly, therefore the uncertainty becomes bigger. In Figure 4 the results for the 2D simulations can be seen for both models. We have identified 3 regions of different morphologies: for low anisotropies seaweed structures (sw) and for high anisotropies dendrites are observed (den). For intermediate ansiotropies and high enough undercoolings doublons are found (db). Both models show a similar behavior as the analytical predictions for the morphology boundary (seaweed-dendrite): The shape is concave for increasing anisotropy. However there is a big difference to the predictions for low undercoolings: while the predictions state that the whole shape of the boundary is concave we find in our simulations for low undercoolings and anisotropies a convex boundary. Comparing the two phase field models with each other we find qualitative correspondence. However we state that the boundary of the Karma model is slightly more on the left than the one of the Wheeler model. A detailed discussion of this behavior can be found in [20].

Three-dimensional simulations are very time-consuming. Even though we use an adaptive mesh code which runs parallel on 32 processors, it was not possible to obtain the same fine resolutions as for 2D simulations. As we have found qualitative agreement between the two models we have preformed 3D simulations in a domain 400^3 only for the Karma model. The results are qualitatively similar to the 2D case however with two exceptions: i) The morphology boundary for the same parameters as in the 2D case is shifted along the anisotropy axis to the right by an amount of 0.32%. This result can be explained by the topological difference between 2D and

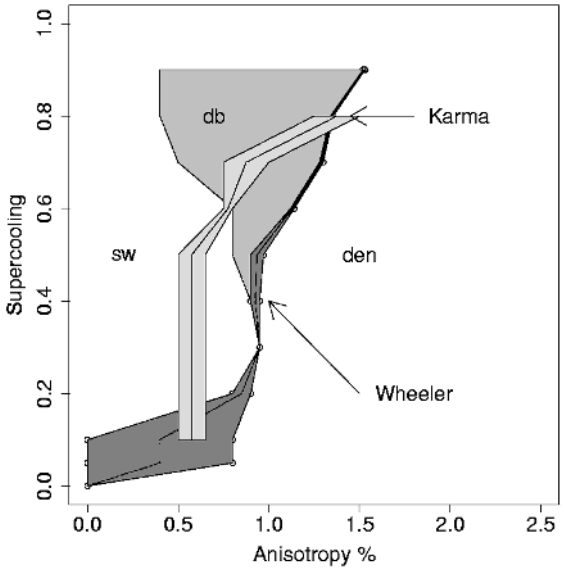

Fig. 4. Morphology diagram in 2 dimensions of the Wheeler model and the Karma model [20]. The zones marked with Wheeler and Karma correspond to the uncertainty regimes given by these models. The dendritic domain is denoted as "den". Seaweed as "sw" and doublon as "db".

3D: in 3D there is one additional dimension where heat can be transported away, therefore more anisotropy is needed to stabilize the structure to be dendritic. ii) No doublon region at all was found in 3D as opposed to 2D where we also started all simulations with the same initial conditions (a small spherical seed) and found a distinct doublon region. When starting with a spherical seed in 3D only seaweeds or dendrites can be simulated. It was however possible to simulate 3D doublons nevertheless by using special initial conditions [20]: by placing two identical seeds at a large enough distance, both start growing as a dendrite. However very soon they start interacting with each other and instead of developing four fins each they grow in parallel and develop only 3 fins each as observed in our experiments (Figure 3).

1.4 Conclusions

Quenching of metals during solidification provides some information on microstructures but not on the dynamics of morphology transitions during the solidification process. Transparent substances have been used to simulate solidification of metals. Rare gases have low melting entropy and form a „simple liquid" similar to metals and can therefore be used as model substances to simulate transparent metals. In these systems the solidification process can be studied *in situ*. Up to now the in-

formation from model experiments with transparent model substances has been limited because no 3D information on the microstructure formed during solidification was available.

Two new features have been introduced into the study of solidification by model experiments: 1.) the study of various morphologies and morphological transitions, and 2.) the possibility to reconstruct the shapes of transparent 3D objects [21]. Morphology changes seem to lead to an explanation of grain refinement in metals, a feature that remained unexplained since a long time. The determination of the 3D shape of objects formed during freezing provides for the first time the possibility to determine 3D shape parameters necessary to apply theories in industrial processes. We have started to determine such parameters and to characterize various morphologies formed during solidification and their transitions. We have the unique possibility to combine theoretical studies with experiments.

The interaction of experiment and theory is crucial. Theoretical studies are stimulated by unexpected experimental observations, and using an expression coined by L. Pasteur, the eye of the experimentalist is trained by theoretical results to see new structures.

Acknowledgements

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2
Thermal Roughening of a Solid-on-Solid Model with Elastic Interaction

Frank Gutheim, Heiner Müller-Krumbhaar, Efim Brener and
Christoph Pütter

2.1
Introduction

At low temperatures crystal surfaces are known to assume the shape of a plane facet. With increasing temperature, fluctuations gradually contribute a nonzero thickness to the initially flat facet. This surface thickness finally diverges at a finite temperature, the roughening temperature, where the order of the facet is lost completely. For kinetics of crystal growth and solidification, the roughening transition plays a decisive role. This transition can be described by a set of renormalization group equations first analyzed by Kosterlitz and Thouless [1]. Because of its unusual properties and the relation to the two-dimensional Coulomb gas, this roughening transition has attracted substantial attention.

Various discrete solid-on-solid (SOS) models have been shown to undergo this type of transition. Most of these models incorporate local interactions, at most next-nearest neighbor interactions. Within some of these models a transition involving in-plane disorder is possible, usually referred to as preroughening.

Interaction of surface defects by means of elastic deformation of the crystal, however, is of a long-ranged nature and has apparently not been previously studied in the context of roughening. Leaving the matter of preroughening aside, we will try to elucidate the effects of long-range elastic interactions on the roughening process.

2.2
Step Interaction

Elastic step interaction on the surface of a semi-infinite crystal can be described in terms of elastic force dipoles located at the step edges. Using symmetry arguments one can determine two types of force dipoles that are considered to be present at a step. One type involves in-plane forces perpendicular to the step, the other arises from forces orthogonal to the crystal surface. The former leads to attractive or repulsive interaction depending on the signs of the steps, the latter produces a sign-independent behavior,

which is strictly repulsive. There are materials where the sign-dependent contributions are small compared to the step repulsion caused by in-plane forces. Thus we restrict our model to the case, where we can neglect sign-dependence of the steps [2]. We can make another simplification of the step-step interaction by assuming a scalar $w \sim 1/r^3$ interaction associated with isotropic dipoles at the step. From this, the energy per unit length of the line for a configuration with two parallel steps at distance d is just

$$\tilde{W}_{\text{scalar}} = 4\,\gamma\,\frac{1}{d^2} + 2\,\gamma\,\frac{1}{\varepsilon^2}, \tag{1}$$

where the interaction is limited to distances greater than ε. The factor γ is given by

$$\gamma = \frac{1-\sigma^2}{\pi E}\,Q^2, \tag{2}$$

where Q is the dipole moment per unit length of the step, σ the Poisson ratio and E Young's modulus.

2.3 Model Description

Within the framework of a solid-on-solid model we describe the crystal surface by a simple height field h_i of integer multiples of the lattice constant a. Like in a common SOS model, overhangs are forbidden. We define an elastic step interaction by introducing a field of elastic dipole charges q. To every lattice site k a dipole charge q_k proportional to the number of height differences to the four neighboring sites is assigned. The elastic dipole charges interact, in consequence of Eq. (1), via a modified r^{-3} interaction potential $\Psi_{p_{\max},l}(r)$,

$$\Psi_{p_{\max},l}(r) = \begin{cases} \min\left((a/r)^3, p_{\max}\right) & \text{if } r \le l \\ 0 & \text{if } r > l \end{cases} \tag{3}$$

where r is the in-plane distance between two lattice sites and $p_{\max}$ is a number limiting the interaction potential in vicinity of $r = 0$. We also introduce the cutoff length l, i. e., the potential vanishes for distances greater than l. This gives rise to the elastic energy

$$E_{\text{el}} = \frac{w_{\text{el}}}{2}\sum_{i,j} q_i q_j \Psi_{p_{\max},l}(r_{ij}), \tag{4}$$

where r_{ij} is the distance between lattice sites i and j and w_{el} can be adjusted to give the desired interaction strength. Note that the case $i = j$ is not excluded from the summation. For two straight steps of length L with distance d and without cutoff this elastic energy contribution consists of the self energies of the steps and the expected $\sim d^{-2}$ step interaction term

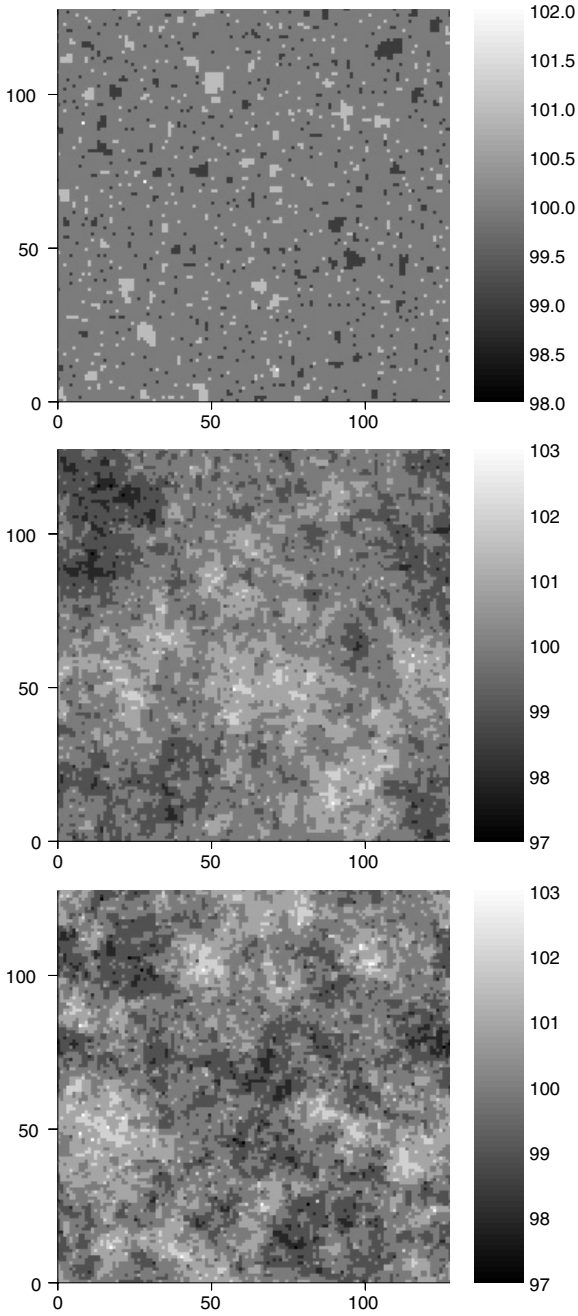

Fig. 1 Roughening transition of a surface under elastic interaction without cutoff. The height field of our model for three different configurations, $T \approx 4.5\ w_{el}$, $T \approx 13\ w_{el}$ (top to bottom), are shown as topviews. The heights are coded in grayscales. The average height corresponds to 100 atomic units.

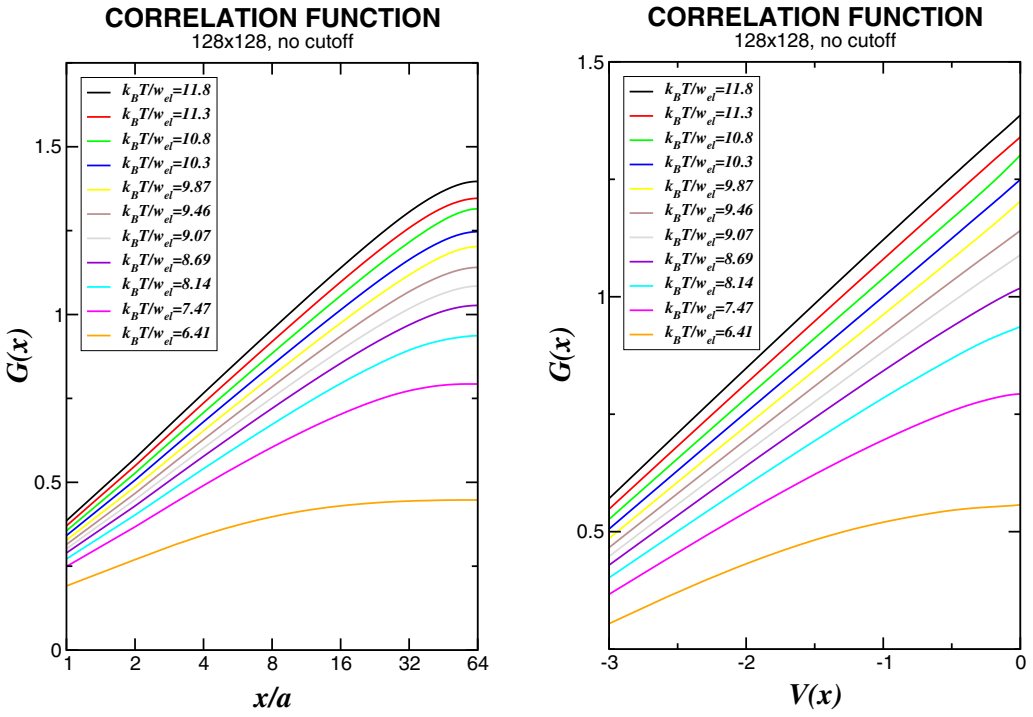

Fig. 2 Height-height correlation function without cutoff along the main directions of the lattice. Left: The correlation function saturates for all temperatures due to the finite size of the system. Right: Same plot, but with finite size correction [2]. The correlation function saturates for small temperatures and shows logaritzmic behavior for $T \geq T_R$. The first straight line gives an estimate of $k_n T_R / w_{el} \approx 9.0$.

$$E_{\text{int}} \approx 8\, w_{\text{el}}\, \frac{L}{d^2}\,, \tag{5}$$

for large distances $d \gg a$.

Modifying $p_{\max}$ the self energy contribution of a straight step can be adjusted to the desired line energy, independently of the step-step interaction amplitude.

For given $p_{\max}$ the relative amplitude of line energy and step-step interaction is fixed and we can concentrate on the crossover from a local to a long-range model depending on the cutoff length l, which is studied using the interaction potentials $\Psi_{1,l}$ where $p_{\max} = 1$. Later, however, other relative amplitudes are studied for the potential $\Psi_{p_{\max},\infty}$ without cutoff.

The simulation is carried out on a square lattice of size $(L/a)^2 = 64 \times 64$ to 128×128. In order to calculate the difference in energy for every metropolis Monte Carlo trial, we apply a multigrid scheme based on Ref. [3], which reduces computational cost and which has already been applied successfully to submonolayer epitaxy [4].