

# Helicopter Test and Evaluation



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# HELICOPTER TEST AND EVALUATION

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# Preface

The authors of this book have been employed at the Empire Test Pilots' School (ETPS), part of QinetiQ, for the past ten years instructing student test pilots and flight test engineers on helicopter testing. Alastair Cooke has a masters degree in flight dynamics from Cranfield University and graduated as a flight test engineer from ETPS in 1989. Eric Fitzpatrick is a former military helicopter pilot and instructor who graduated as a test pilot from ETPS in 1986.

This book has been produced using the experience of the authors in flight test and flight test training gained over a combined period of 25 years in the field. Much of the material has its origin in the training notes produced by ETPS. These have been developed over a period of over forty years since the start of specific training for helicopter test pilots. The book has been designed to appeal to professionals working in the area of rotorcraft test and evaluation but it is hoped that it will also prove useful to a wider audience. In our experience, we have found that helicopter pilots are generally not well informed about the process that has led to their aircraft entering service, nor about why it has certain limitations imposed on it. We hope this book will provide pilots with this information as well as being a useful text for practising engineers and technologists. The rotor theory presented is more extensive than is found in most aeronautical degree courses and so the book should prove useful to graduates specializing in rotorcraft technology. Perhaps uniquely, this work approaches this important subject from both the theoretical and practical viewpoints.

For each topic the theory is explained briefly and is then followed by details of the practical aspects of testing a helicopter. These details include the safety considerations related to the anticipated tests, planning the tests themselves, and, where appropriate, the most efficient way to conduct individual flights. Following a description of each type of test, typical results are examined and an explanation given as to why they would be important to the clearance process. Whenever possible examples of actual test results have been presented and used in the subsequent discussion. The book is split into four main sections:

- Introduction: covering a methodology for testing and general aspects of test programmes.
- Performance: in this section level flight, vertical and climb/descent performance is addressed. The planning of performance trials is covered together with the methods for airborne data gathering and analysis of results.
- Handling qualities: this is a major section and covers the basics of helicopter stability and control testing. Also included are frequency domain methods and the use of mission task elements.
- Systems: in this section the major systems required to enable a helicopter to fly are

covered. This includes assessment of the cockpit, air data systems, engine control systems, and automatic flight control systems. Also addressed in this section is the testing of system failures.

Although highly specialized, the topic of helicopter testing is still vast and no single text could hope to cover everything. The authors have attempted, therefore, to concentrate on the most important aspects using their own knowledge of the subject as a guide. Inevitably, a number of important areas have not been covered; for example, it has not been possible to include information on specialist areas such as underslung load trials, deck operation trials or armament testing. The amount of information on systems testing to include in the book was a difficult decision due to the plethora of systems that can be fitted to a modern rotorcraft. Consequently, it was decided to detail the general methodology used in this type of testing and then to concentrate on systems which are intrinsic to the operation of all helicopters. Thus, it has not been possible to include the testing of hydraulics, electrics and lubrication systems. Similarly the testing of a number of cockpit systems such as piloting vision aids, navigation systems, weapons, and mission displays were considered to be beyond the scope of this book. Finally, it was not possible to include some types of testing which often play a large part in the life of a test pilot such as environmental trials, notably cold weather and icing. Despite these omissions, it is believed that the book covers all the essential areas of rotorcraft testing and will prove useful to a large part of the helicopter community. As the authors' background has been in military test and evaluation this has clearly influenced the subject matter that has been presented. However, most of the information given in the book can be applied, with only minor modification, to the testing of civil rotorcraft.

Before closing we would like to acknowledge the help of Mr Mike Cook of Cranfield University, Mr Mark Roots of QinetiQ Ltd and Miss Julia Burden of Blackwell Publishing in helping us develop our manuscript for publication.

Alastair Cooke and Eric Fitzpatrick  
ETPS, QinetiQ Ltd  
MOD Boscombe Down

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### **The example helicopter**

On several occasions throughout this book quantitative calculations have been made to support the theoretical trends being discussed. Where possible the calculations have been made using the same baseline data, referred to as the *example helicopter*. The



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details of this helicopter, which is loosely based on the Westland Lynx, are summarized below:

*Main rotor*

Radius	6.5 m	Blade chord	0.4 m
Standard rotor speed	35 rad/s	Number of blades	4
Lift curve slope	0.1/°	Twist	10°
Profile drag coefficient	0.008 or 0.01		

*Tail rotor*

Tail arm	7.5 m	Radius	1.1 m
Blade chord	0.2 m	Standard rotor speed	165 rad/s
Number of blades	4	Profile drag coefficient	0.010
Fuselage frontal drag area	2 m <sup>2</sup>		
Fuselage vertical drag area	8 m <sup>2</sup>		
Mass (unless otherwise indicated)	5000 kg		

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# Abbreviations

AA	aircraft allowance
ACAH	attitude command/attitude hold
ACT	active control technology
ADI	attitude direction indicator
ADS	Aeronautical Design Standard
AFCS	automatic flight control system
AGL	above ground level
AI	attitude indicator
AltR	altimeter reading
AOA, AoA	angle of attack
AOB, AoB	angle of bank
APU	auxiliary power unit
ASE	automatic stabilization equipment
ASI	airspeed indicator
ASIR	airspeed indicator reading
ASW	anti-submarine warfare
AUM	all-up-mass
BCV	bleed control valve
CAA	Civil Aviation Authority
CAS	calibrated airspeed
CBEM	combined blade element and momentum (theory)
CFSS	collective fixed static stability
CG	centre of gravity
CP	collective pitch
CR Point	control reference point
CR Posn	control reference position
CWP	centralized warning panel
DA	density altitude
DEP	design eye position
DH	decision height
DVE	degraded visual environment
EAS	equivalent airspeed
ECS	engine control system
EFIS	electronic flight information system
EGT	exhaust gas temperature
EOL	engine-off landing
ERGA	engine and rotor governing assessment
ETPS	Empire Test Pilots' School
FADEC	full authority digital engine control
FAR	Federal Aviation Regulations
FBW	fly-by-wire
FCMC	flight control mechanical characteristics
FCS	flight control system
FFR	fuel flow rate
FIG	flight idle glide
FLIR	forward looking infra-red
FMECA	failure modes effect and criticality analysis

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FOD	foreign object debris
FOR	field of regard
FOV	field of view
FPT	free power turbine
FTE	flight test engineer
FW	fixed wing
GCA	ground controlled approach
GPS	global positioning system
GSDI	groundspeed and drift indicator
GVE	good visual environment
GW	gross weight
HDD	head down display
HIGE	inside ground effect hover
HMD	helmet mounted display
HOGE	outside ground effect hover
HQR	handling qualities rating
HSI	horizontal situation indicator
HTA	helicopter type allowance
HUD	head-up display
IAS	indicated airspeed
IAT	indicated air temperature
ICAO	International Civil Aviation Organization
IFR	instrument flight rules
IGE	inside ground effect
IGV	inlet guide vanes
ILS	instrument landing system
IMC	instrument meteorological conditions
INS	inertial navigation system
IRE	identified risk element
ISA	ICAO standard atmosphere
JAR	Joint Airworthiness Regulations
KCAS	knots calibrated airspeed
KEAS	knots equivalent airspeed
KIAS	knots indicated airspeed
KTAS	knots true airspeed
LCD	liquid crystal display
LDO	lateral/directional oscillation
LDP	landing decision point
LED	light-emitting diode
M	Mach number
MAR	military aircraft release
MAUM	maximum all-up-mass
MAUW	maximum all-up-weight
MFD	multi-function display
MPOG	minimum pitch on ground
MPV	maximum power vertical
MSL	mean sea level
MTBF	mean time between failures
NDB	non-directional beacon
NOE	nap of the earth
NTPS	National Test Pilots' School

NVG	night vision goggles
OAT	outside air temperature
ODM	operating data manual
OEI	one engine inoperative
OFE	operational flight envelope
OGE	outside ground effect
PE	pressure error
PEC	pressure error correction
PFL	practice forced landing
PFLF	power for level flight
PH	(hover) position hold
PIO	pilot-induced oscillation
PTIT	power turbine inlet temperature
PUPO	pull-up/push-over
RAD ALT	radar altimeter
RCAH	rate command/attitude hold
RCDH	rate command/direction hold
RCDI	rate of climb and descent indicator
RCHH	rate command/height hold
REP	reference eye position
RH	relative humidity
ROC, RoC	rate of climb
ROD, RoD	rate of descent
RPM	revolutions per minute
RPV	reduced power vertical
RRM	risk reduction measure
RRPM	rotor revolutions per minute
RVR	runway visual range
RW	rotary wing
SAR	search and rescue; specific air range
SAS	stability augmentation system
SFC	specific fuel consumption
SHSS	steady heading sideslip
SSC	side stick controller
TAS	true airspeed
TCDB	trim control displacement band
TCWP	trim changes with power
TDP	take-off decision point
TE	tip effects
TFCP	trimmed flight control position
TGT	turbine gas temperature
TI	trials instruction
TO1C	turn on one control
TPS	trailing pitot-static
VFR	visual flight rules
VMC	visual meteorological conditions
VNE	maximum never to exceed speed
VNO	maximum normal operational speed
VOR	VHF omnidirectional range
VSI	vertical speed indicator
VSS	variable stability system

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# Notation

## Arabic symbols

$A$	disk area
$a$	lift curve slope
$A_1$	lateral cyclic pitch
$a_1$	longitudinal cyclic flap
$A_R$	aspect ratio
$B$	tip loss factor
$b$	number of blades in main rotor or tail rotor
$B_1$	longitudinal cyclic pitch
$b_1$	lateral cyclic flap
$c$	rotor blade chord
$C_D$	drag coefficient
$C_L$	lift coefficient
$\bar{C}_{Lmax}$	maximum mean lift coefficient
$\bar{C}_{Lnom}$	nominal mean lift coefficient
$C_P$	power coefficient
$C_Q$	torque coefficient
$C_T$	thrust coefficient
$D$	drag; rotor diameter
$f$	drag area
$g$	acceleration due to gravity
$H_D$	density altitude
$H_p, h_p$	pressure altitude
$L$	lift
$l_{TR}$	tail rotor moment arm
$m$	mass
$N$	engine RPM
$N_c$	compressor RPM
$N_F$	free turbine RPM
$N_G$	gas generator RPM
$N_h$	high speed spool RPM
$N_l$	low speed spool RPM
$N_R$	rotor speed RPM
$P, P_s$	static pressure
$p$	change in roll rate from trim
$P_i$	induced power
$P_p$	total or pitot pressure
$P_{par}$	parasite power
$P_{pr}$	profile power
$P_\infty$	ambient pressure
$q$	change in pitch rate from trim
$Q_{MR}$	main rotor torque
$R$	rotor radius
$r$	change in yaw rate from trim

$s$	solidity
$S, S_f, S_v$	blade area, frontal drag area, vertical drag area
$T$	thrust; engine temperature
$T_2, T_{1/2}$	time to double or half bank angle
$u$	change in speed along $X$ -axis
$V$	true airspeed
$v$	change in speed along $Y$ -axis
$V_c$	calibrated airspeed; climb speed
$V_e$	equivalent airspeed
$V_f$	forward (horizontal) velocity component
$V_H$	maximum speed in level flight
$V_i, v_i$	indicated airspeed corrected for instrument error, induced velocity
$V_{imp}$	indicated minimum power speed
$V_{ME}$	maximum endurance airspeed
$V_{MP}$	minimum power airspeed
$V_{MR}$	maximum range airspeed
$V_{toss}$	take-off safety speed
$V_T$	tip speed
$V_v$	vertical velocity component
$V_Y, v_y$	speed for maximum rate of climb
$W$	weight
$w$	change in speed along $Z$ -axis
$Z$	tapeline height

### Greek symbols

$\alpha$	angle of attack
$\beta$	sideslip angle; flap angle
$\gamma$	Lock number; flight path angle; disk tilt
$\delta$	relative pressure; angle between rotor blade hinge lines
$\varepsilon$	rotor frequency parameter
$\zeta$	relative damping
$\eta$	vertical velocity ratio ( $V_v/v_{ih}$ ); efficiency
$\theta$	relative temperature; pitch attitude; rotor feather angle
$\theta_0, \theta_1$	collective pitch and twist
$\theta_{TR}$	tail rotor pitch
$\kappa$	non-uniform inflow factor
$\lambda$	inflow ratio
$\mu$	advance ratio ( $V/V_T$ ); forward velocity ratio ( $V_f/v_{ih}$ )
$\nu$	induced velocity ratio ( $v_i/v_{ih}$ )
$\xi$	lead/lag angle
$\rho$	air density
$\sigma$	relative density
$\tau$	time constant
$\phi$	bank angle; inflow angle
$\chi$	rotor wake skew angle
$\psi$	azimuth angle; yaw angle
$\Omega, \omega$	rotor speed; angular velocity; frequency

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## Chapter 1

# The Flight Test Process

### 1.1 INTRODUCTION

Flight test is an expensive activity, which by its very nature attracts levels of risk higher than normal operations. To ensure that all trials are conducted as efficiently and safely as possible, a flight test process has been developed over the years. This process is used, with only minor variations, in nearly all test organisations throughout the world whether they be military, civilian or based at a manufacturing facility. Many of the steps in this process have evolved as the result of painful lessons and, therefore, the authors consider them vital to the overall test activity.

It is perhaps true to say that at the beginning of their careers, flight test personnel show greater interest in the test methods that they need to apply than in the overall system used to approve, authorize and regulate test flying. Consequently it was decided to make the test process the first chapter of this book to demonstrate the importance that the authors attach to this subject. It is our belief that without an understanding of the process the knowledge contained within the rest of the book cannot be applied effectively.

Brief details of the flight test process are given below. For more details the reader is referred to the AGARD flight test techniques and instrumentation series [1.1 and 1.2]. Generally the flight test process can be broken down into three major areas:

- Flight test planning.
- Conducting the test.
- Post-test actions.

### 1.2 FLIGHT TEST PLANNING

Thorough planning is vital for all flight trials to ensure that they are conducted safely, efficiently and that the trials objective is met. When a trial is first proposed to a test establishment a management plan is produced which includes a work breakdown structure defining the individual elements of the trial. For each element the department or departments which are to undertake the work are determined together with costs and timescales. Part of the management plan is to define the exact project technical objectives and it is at this level that this book will examine the planning process.

#### 1.2.1 Technical objectives

To define the project technical objectives the exact requirements of the customer calling for the trial have to be understood clearly. Once this has been done then the

next step is to decide on the assessment philosophy, in other words ‘What are we going to do? – How much of it are we going to do?’ Deciding on what approach to take depends on a number of factors such as the level of expertise within the test organization, the resources available, and if the trial results need to be compared with earlier results using a particular test method. The approach to the trial and the scope of the work both depend on the requirement to collect evidence and to identify where that evidence is located.

Determining what evidence is needed to be able to make well-supported recommendations relies upon the professional knowledge of the test team, although previous trials may provide a guide. The evidence itself does not necessarily need to come from a flight conducted as part of the trial, indeed because flight testing is such an expensive activity other sources of evidence are invariably considered first. These sources may be the aircraft manufacturer, who may have conducted the required test, earlier test results held at the test establishment, read across from other similar aircraft, another test organization, or possibly an operator who already has experience of the test article. Modelling and simulation are also used extensively. In each case the evidence and the source are considered carefully to determine to what extent it can be relied upon. If the existing evidence is determined to be reliable then it may be used without further testing or, more likely, a limited number of tests would be planned to ‘spot check’ the evidence to increase confidence in its validity.

Once the existing evidence has been established then the difference between this and the evidence required is what the flight trial must address, this defines the scope of the test. The team then decides how much testing is required to gather the missing evidence to allow the test objective to be met. From this is defined the detailed work breakdown structure including test techniques, required environmental conditions, trials location, the order and interdependency of tests, and the allocation of tasks to parts of the organization. Having defined the tests to be made, the facilities required to conduct them have to be identified: these usually include the aircraft build standard, ranges, test instrumentation and equipment, the test crew and their training standard, and data replay and analysis equipment.

### 1.2.2 Assessing and managing risk

Test flying by its very nature involves a degree of risk above that which is routinely accepted during normal operations. To ensure that tests are conducted as safely as possible a process of identifying and reducing risk is employed. The aim of this process is to determine what the risks are, decide on the best way to reduce them and then to decide if the residual risk is acceptable.

#### 1.2.2.1 *Conducting a hazard analysis*

To identify risk areas the programme or flight is divided into sections and the potential dangers considered in each one: this will involve asking the question at each stage ‘What could possibly present a hazard?’ and ‘What would be the possible consequences?’ This procedure relies to a large extent on the experience of the test pilot or FTE, however, a thorough and logical assessment of each phase of flight should lead to the identification of individual risks. It is normal to involve a number of people by calling a review meeting where the group examine each aspect of the flight. Any databases



available are also examined to ensure that the lessons learned on previous trials of a similar type are incorporated. On completion of this process a list of Identified Risk Elements (IRE) can be produced for each section of the programme or flight.

#### 1.2.2.2 Risk reduction measures

Having identified the risks the next stage is to decide how they might be mitigated as much as possible. The measures which can be taken are known as risk reduction measures (RRM) and are typically divided into five main categories:

- (1) *Training*: Correct and comprehensive training of the test team is vital to reducing the risk on any test programme. This training will be required on the aircraft itself and in the case of initial trials on a new aircraft encompasses the ground instruction, simulator training, conversion course and continuation training. On other aircraft it may only involve currency training on type. The training may also involve the practise and perfection of the test techniques to be used or the flight skills to be employed during the programme, such as night flying, NVG instrument flying. Safety and survival training for all crew members is also conducted.
- (2) *Supervision*: The close supervision of the test programme from the initial inception of the requirement, through the raising of the trials instruction (TI), the test plan and the individual flight authorization is possibly the single most important RRM. It is often the greater experience and the objectivity of those in a supervisory position which is able to identify possible risks that may not be apparent to those who are intimately involved in the programme itself. The individuals with supervisory responsibility may also be able to identify any inadequacies with the performance of the trials personnel and take action accordingly.
- (3) *Procedures*: The application of logical procedures enables any potential risks to be reduced. These include the method to be used for setting up test points, gathering data and recovering to a normal flight condition. In addition procedures are developed to cover emergencies and other unexpected events. These procedures are laid out in detail in the test plan for the flight(s) and are reviewed during the pre-flight briefing prior to authorization. Procedures are also developed for the dissemination of results and information within the test team and within the test establishment; this is particularly important where changes may be made to the test aircraft during the test programme.
- (4) *Limitations*: Every flight and test programme is subject to a variety of limitations some of which are imposed by the manufacturer; the others are imposed by the test establishment. The limitations may be imposed for structural considerations, to prevent damage to systems, for handling reasons, or to enhance flight safety. In every case it is essential that all participants are aware of every limitation and that they are respected during all flights. The limitations are detailed in the test plan and revised in the pre-flight brief.
- (5) *Monitoring*: The monitoring of a test programme or flight can take a variety of forms all of which may be employed if appropriate. This monitoring usually involves the comparison of the actual results with the anticipated results to ensure that the programme is progressing in the expected manner: a lack of correlation between these results may indicate that not all factors have been

addressed correctly during the planning stage and therefore the plan should be reassessed. The comparison of actual with anticipated results is also conducted for individual flights. In addition the actual results in flight are assessed for each test point prior to moving on to the next increment; this may require the use of telemetry to a ground monitor or the use of in-flight plots to predict the likely results from the next point. External monitoring of the flight may also be employed through the use of chase aircraft and ground observers.

### 1.2.2.3 *Quantifying residual risk*

Once the trials risks have been identified and the RRM's have been employed there will still be a certain amount of residual risk which is then quantified. This will then lead to the trial being categorized as high, medium, or low risk. The main reason for allocating a category of risk is to alert the responsible senior officer to those trials which require particularly careful supervision. One definition of risk categories used at the UK test establishment at Boscombe Down is as follows:

- *Low risk*: the residual risk is no higher than normal peacetime flying.
- *Medium risk*: there is an increased risk of injury to crews or damage to the aircraft over normal operations.
- *High risk*: there is an increased chance of loss of life or loss of an aircraft.

### 1.2.3 **Trials documentation**

Part of the planning process which is often not given sufficient attention, is the technical documentation and reporting requirement. All the documents needed to conduct the trial safely require definition. These include TIs, instrumentation schedules, limitations documents, and risk assessments. In most test establishments the management procedures call for these documents and define their content. The eventual trials report or reports also require identification early in the planning stage.

#### 1.2.3.1 *The trials instruction*

The TI is the main output from the planning phase and is the document used to authorize and control the trial. In the TI is contained all the information that a senior person in the trials organization needs to be able to understand what is being planned and allows him or her to approve the trial. Thus the document needs to define the test methods, test procedures, and test conditions that will be used. The test methods and procedures are often standard and only require a reference to other documents which define them. The test conditions, on the other hand, always require careful thought as these vary with the trial being made. It is not simply a case of defining the target conditions for the trial but rather a range of permissible values of variables is defined so that the trial can proceed even if the exact desired conditions are not present. The TI also details the trials programme including such aspects as the size of increments in control parameters between successive tests and the criteria for progressing to the next test or phase. A key element of the TI is a section which details the safety precautions that will be taken. This safety section comes mainly from the risk assessment which is also included in the document so that the authorizing officer can ensure that the process has been conducted correctly.

### **1.2.3.2 *Sortie planning***

Each sortie in a test programme requires careful planning to ensure that the flight follows a safe progression and that the maximum amount of high quality data is gathered in the flight time available. Where possible tests are conducted concurrently, for example, information on flight control positions and engine bay temperatures can be gathered during level flight performance tests. This clearly requires careful liaison between the different parts of a flight test organization if they are all involved in the same trial. The individual test points are organized such that successive points can be achieved quickly. The principle is that no flight time is wasted. It is better to have too many test points in a sortie plan than too few but the plan still needs to be reasonable and achievable. The possible effects of weather are considered as well so that tests are organized into high level and low level sorties to ensure that time is not wasted if the cloud base is low. For each sortie the go/no go criteria is decided upon at the planning stage, for example, this might be a minimum light level or a maximum wind strength. Part of the sortie planning procedure is to produce a written flight brief, which will be used during the pre-flight briefing to ensure that all trials participants are fully prepared. Included in this brief will be the flight safety points, priorities, limitations, go/no go criteria, possible alternate tests, criteria for stopping the tests, and responsibilities of individual participants.

### **1.2.3.3 *Test cards***

A set of test cards is normally produced for each test flight. These cards serve two purposes: firstly they act as an aide-mémoire of the tests to be conducted and limitations that apply, and secondly they provide a means of recording manual data to back up flight test instrumentation data. It would be hard to overemphasize the importance of having carefully produced test cards. Each card defines the test being made with the conditions required and has sufficient space to record data if required. The limits for that particular test are written clearly together with any reminders. On each card the planned start and finish time for the test are included with the expected fuel state. It is often helpful to colour code the cards to indicate the priority of tests, which allows easy airborne re-planning in the event that the flight falls behind time. When producing the cards it is best to sit in the cockpit and note the most efficient order to gather information from the aircraft or test instruments. Once the cards have been made the crew again sit in the aircraft and go through a sortie rehearsal to ensure that the cards are satisfactory.

## **1.3 CONDUCTING THE TRIAL**

### **1.3.1 Establishing the test conditions**

For the results of a trial to have any meaning the conditions which pertained at the time of the test will be of critical importance. Thus, for example, if a performance trial is being conducted the test pilot will have to fly the aircraft at exactly the right speed, with the right value of sideslip, and at the correct altitude. The environmental conditions such as temperature, winds, turbulence levels, etc. will also have to be appropriate. In many trials a variety of conditions will be required and often at opposite extremes. For example, very still air conditions may be needed to obtain

accurate results but high levels of turbulence may also be required to give the test pilot conditions representative of operational flying to allow him or her to make qualitative judgements of the handling qualities. In all cases the test conditions need to be recorded for inclusion in the flight report.

### 1.3.2 Moving into the unknown

Nearly all test flying involves going from a known condition into an unknown one and it is how this is done which is important. Although there will always be an element of risk in this situation, provided the correct methodology is applied nasty surprises can mostly be avoided. The main part of this methodology is to adopt the incremental approach, i.e. only small changes are made in a single parameter when moving from a known condition to the next test point. As an example if control response tests are being conducted by making step cyclic inputs into a control fixture the size of the inputs would be increased very gradually. After each input the change in aircraft response is compared with the previous results with the aim of determining the relationship between changes in the input parameter and the aircraft output. Once a relationship has been established this can be used to predict the output for the next test point to determine if the test should continue. Any change in the relationship can also serve as a warning that extra caution is needed. To establish this input/output relationship a 'how goes it plot' is often constructed which in its simplest form consists of a graph where the two parameters are cross-plotted. Also marked on the graph will be the pre-determined test limits. Each test point adds to the trend line and the line is extrapolated prior to each subsequent point to ensure that the predicted result lies inside the test limits. For this system to work it is vital that only one parameter is changed at a time. Thus an input size change should not be made at the same time as the aircraft speed is changed. In addition to establishing trend lines for the test results the team also need to have a sound idea of the expected results prior to the trial taking place. This information can come from a variety of sources, such as results of previous trials under similar circumstances or as a result of computer modelling. Again the predicted trend information is compared with the actual results and major differences serve as a warning that all factors have not been accounted for.

## 1.4 POST-TRIALS ACTIONS

### 1.4.1 Trials reporting

The deliverable from most trials is a written report which addresses the trials objective and records the test data. Careful planning of the report takes place before the trial is conducted to ensure that the final product is of good quality. This planning will scope the document, produce a skeleton structure, and define responsibilities. One of the most important planning stage actions, particularly for large, multi-discipline reports, is to nominate a single person to be in overall charge of the report. He or she can then ensure that the style and content are consistent across the authoring departments and eliminate duplication, or worse, contradictions. Each report has an introductory section that contains all the information relating to how the trial was conducted. This

includes who performed the trial, when it was performed, the state of the aircraft or equipment tested, the instrumentation used, and the tests that were performed. The next section documents the results of the trial, including the conclusion of the test team and any recommendations made. In many test establishments this is written in a set layout known as the seven-part format. The aim of this format is to ensure that a person reading the report understands exactly what was done, what the results were, what the data mean, what implications a deficiency would have in operations, what the conclusion is, what the recommendations are, and whether or not the article under test meets the relevant specification or not. The seven-part format is shown below:

- Establish the test conditions;
- Present the data;
- Analyze and discuss the data;
- Relate to the operational role;
- Conclude;
- Recommend;
- Specification compliance.

Most reports have a summary and a section that groups together the recommendations. In all cases a successful report will make it clear what was done during the trial and will provide a convincing argument to implement the test team's recommendations.

#### **1.4.2 Learning from the past**

Human nature being what it is, there is a tendency to believe that a trial is complete once the report has been delivered and accepted by the customer. However it is important to have a trial's closure procedure which ensures that all the data gathered is retained. This data may be required to provide baseline information for comparison purposes when the aircraft is modified in the future. Given the long periods that rotorcraft are kept in service the data may have to be retained for a considerable number of years. As well as keeping copies of the customer reports all the other trials documentation such as trials instructions, risk assessments, etc. are retained to serve as templates for the future. A 'lessons learned' system is also maintained where information concerning mistakes, problems, solutions and good ideas can be retained even though the trials participants may move on.

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## Chapter 2

# Performance Theory

### 2.1 INTRODUCTION

Unlike a conventional fixed wing aircraft, which has separate means of generating forces for lift and forward propulsion, the helicopter uses only the thrust from a rotor to meet these two essential requirements for sustained flight. This limitation does, however, afford the helicopter a unique advantage over most fixed wing aircraft: the ability to generate a lift force even when the vehicle is stationary. Understanding the basic manner in which a rotor system develops thrust is fundamental to any study of the measurement of rotorcraft performance. It is also required in any introduction to the stability and control attributes of a typical helicopter. There are two basic theoretical approaches to understanding the generation of thrust from a rotor system: momentum theory; and blade element theory. Other more complex theories tend to build on the fundamentals introduced by these two approaches. Since many standard texts on helicopter performance cover momentum theory and blade element theory [2.1 to 2.7] it is only necessary for us to review the key points here. Forward flight produces asymmetric flow across a rotor disk and thus it is desirable for us to start by restricting our discussion to purely axial flight.

### 2.2 AXIAL FLIGHT: MOMENTUM THEORY

Momentum theory was initially developed by Rankine and Froude from their study of ship propellers or water screws. It later found application in the design of airscrews [2.3] and is used here to represent a climbing rotor. The theory makes certain assumptions:

- Air is an inviscid and incompressible fluid.
- The rotor acts as a uniformly loaded or ‘actuator’ disk with a infinite number of blades so that there is no periodicity in the wake.
- The flow both upstream and downstream of the disk is uniform, occurs at constant energy and is contained within a streamtube.
- No rotation is imparted on the fluid by the action of the rotor.

These assumptions necessarily limit the accuracy of the Froude theory. The momentum approach has, however, been extended to cover the more realistic case of a compressible fluid [2.7]. Other real effects such as swirl and unsteady flow can be accommodated by appropriate empirical factors [2.4, 2.6 and 2.8].

### 2.2.1 Froude theory

The Froude theory postulates that the flow above and below a climbing rotor can be considered as constant energy with energy being imparted only by the actuator disk. This energy is added to the airflow in the form of an increase in static pressure. The theory then draws some conclusions about the streamtube that the disk influences. Far above the rotor, the velocity of flow in the streamtube is equal to the freestream that is dependent on the rate of climb of the rotor itself. As the rotor draws air through its disk the velocity just above the disk is greater than the freestream and as a consequence of the continuity equation the streamtube will have contracted; also by virtue of Bernoulli's relationship the pressure just above the rotor will be less than ambient. Immediately below the disk the pressure will be greater than ambient because of the energy added by the rotor however, the velocity and streamtube area will be the same as just above the rotor. Far below the disk in what is termed the *ultimate wake*, the flow will achieve a pressure equal to ambient but the velocity will exceed the freestream again because of the energy imparted by the rotor. The continuity equation and Bernoulli relationship require that the cross-sectional area of the ultimate wake be less than the disk area, see Fig. 2.1. Adoption of the concept of an actuator disk leads to a very simple relationship for the thrust developed by a rotor:

$$T = A(P_2 - P_1)$$

where  $A$  is the disk area.

The pressure difference  $(P_2 - P_1)$  generated by the disk can be related to the vertical velocity by considering the changes in pressure and velocity occurring in the streamtube. Consider a helicopter climbing vertically at speed  $V_c$  and assume that the

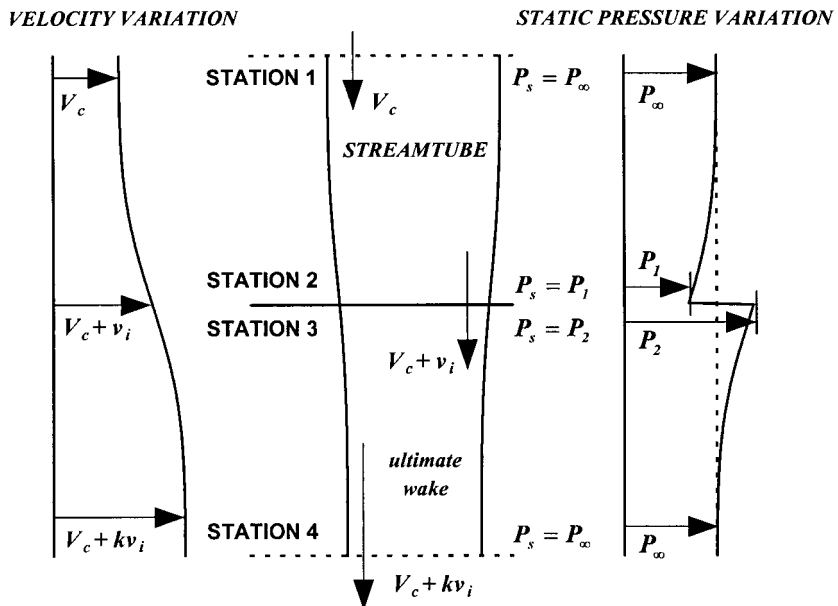


Fig. 2.1 Flow through the actuator disk.

acceleration of flow caused by the action of the rotor results in an increase in the flow velocity of  $v_i$  (Fig. 2.1). Likewise, assume that the continued acceleration of flow below the rotor leads to a total velocity rise of  $kv_i$  at the ultimate wake. Now Bernoulli states that:

$$P_\infty + \frac{1}{2} \rho V_c^2 = P_1 + \frac{1}{2} \rho (V_c + v_i)^2$$

$$P_2 + \frac{1}{2} \rho (V_c + v_i)^2 = P_\infty + \frac{1}{2} \rho (V_c + kv_i)^2$$

Thus:

$$T = A(P_2 - P_1) = \frac{1}{2} \rho A (2V_c + kv_i) kv_i$$

This relationship requires a value for  $k$  before it can be used to estimate the thrust required for a given rate of climb. If the momentum change caused by the disk is considered, an alternative expression for thrust can be developed. Recalling that force is equal to rate of change of momentum or massflow multiplied by a change in velocity gives [2.2]:

$$T = \rho A (V_c + v_i) kv_i$$

Hence  $k = 2$  and:

$$T = 2\rho A (V_c + v_i) v_i \tag{2.1}$$

This fundamental equation predicts that the induced velocity at the rotor disk is equal to half the total increase in flow velocity required to match the thrust requirement of the rotor. The maximum increase in velocity occurs at the ultimate wake, which is usually taken as one rotor diameter downstream of the disk. This *momentum disk* model can be applied to any working state of the rotor in which a continuous streamtube is formed.

### 2.2.2 Power considerations

The total change in energy per unit mass ( $\Delta E$ ) along the streamtube is given by:

$$\Delta E = 2(V_c + v_i)v_i$$

As power can be defined as massflow multiplied by change in energy:

$$P = \dot{m}\Delta E = \rho A (V_c + v_i) \times 2(V_c + v_i)v_i$$

$$P = T(V_c + v_i) = TV_c + Tv_i \tag{2.2}$$

So the power required to drive a climbing rotor can be seen to come from two sources: the power required to generate the rate of climb (the *useful* power =  $TV_c$ ) and the power required to generate the thrust (the *induced* power =  $Tv_i$ ). The power calculated using Equation (2.2) represents an ideal minimum value because this simple theory neglects all forms of losses.

It has been seen that power estimations require a knowledge of thrust, rate of climb and induced velocity. Whilst the thrust can be related to the weight of the helicopter



and the rate of climb is an easily specified variable the induced velocity is more difficult to determine. However, Equation (2.1) can be re-written as:

$$v_i^2 + v_i V_c - \frac{T}{2\rho A} = 0$$

Only the positive root of this quadratic has any meaning in this case, so:

$$v_i = -\frac{V_c}{2} + \sqrt{\frac{V_c^2}{4} + \frac{T}{2\rho A}} \quad (2.3)$$

Note that Equation (2.3) can be used to predict the induced velocity in the hover ( $V_c = 0$ ):

$$v_{ih} = \sqrt{\frac{T}{2\rho A}} \quad (2.4)$$

## 2.3 AXIAL FLIGHT: BLADE ELEMENT THEORY

Simple momentum theory treats the rotor as an actuator disk through which a uniform flow passes. Unfortunately this theory tells us little about the flow around the individual blades that make up the rotor system. Momentum theory cannot, therefore, be used to predict the magnitude of any losses associated with realistic flow around rotor blades. Blade element theory overcomes some of the restrictions inherent in momentum theory as it is based upon the idea that the rotor blades function as high aspect ratio wings constrained to rotate around a central mast as the rotor system advances through the air. As before our study of blade element theory begins with purely axial flight.

### 2.3.1 Elemental forces, thrust, torque and power

Consider a rotor consisting of  $b$  blades climbing at speed  $V_c$ . The blades are each of length  $R$ , and turn at a rotational speed of  $\Omega$  rad/s. If we now examine the forces generated on a small element,  $\delta r$ , of a blade located at  $r$  from the hub we can gain insight into how a complete rotor system generates thrust and drag. Figure 2.2 shows a blade element and Fig. 2.3 depicts the forces acting on such an element. Each element of the blade is assumed to develop the full aerodynamic forces and moments as it would in two-dimensional flow at the same conditions that occur at its radial station. No allowance is made at this stage for finite span or blade wake effects.

Firstly, we must determine the total flow through the disk. As we have seen for a rotor in a climb this is composed of the climb velocity,  $V_c$ , and the induced velocity  $v_i$ . The resultant velocity at a blade element therefore has a vertical component,  $V_c + v_i$ , and a horizontal component  $\Omega r$ . From Fig. 2.3 it can be seen that the resultant velocity,  $V$ , is given by:

$$V = \sqrt{(V_c + v_i)^2 + \Omega^2 r^2}$$

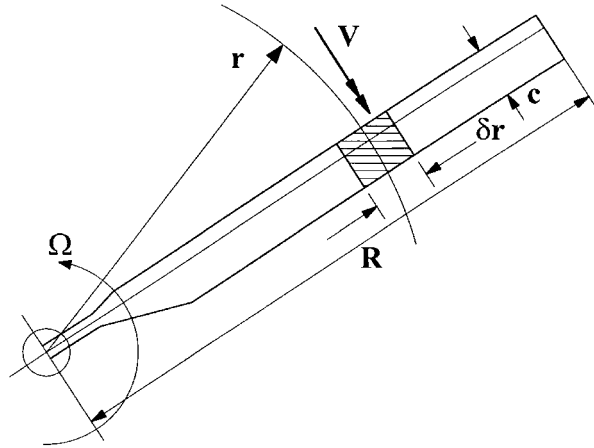


Fig. 2.2 Definition of the blade element.

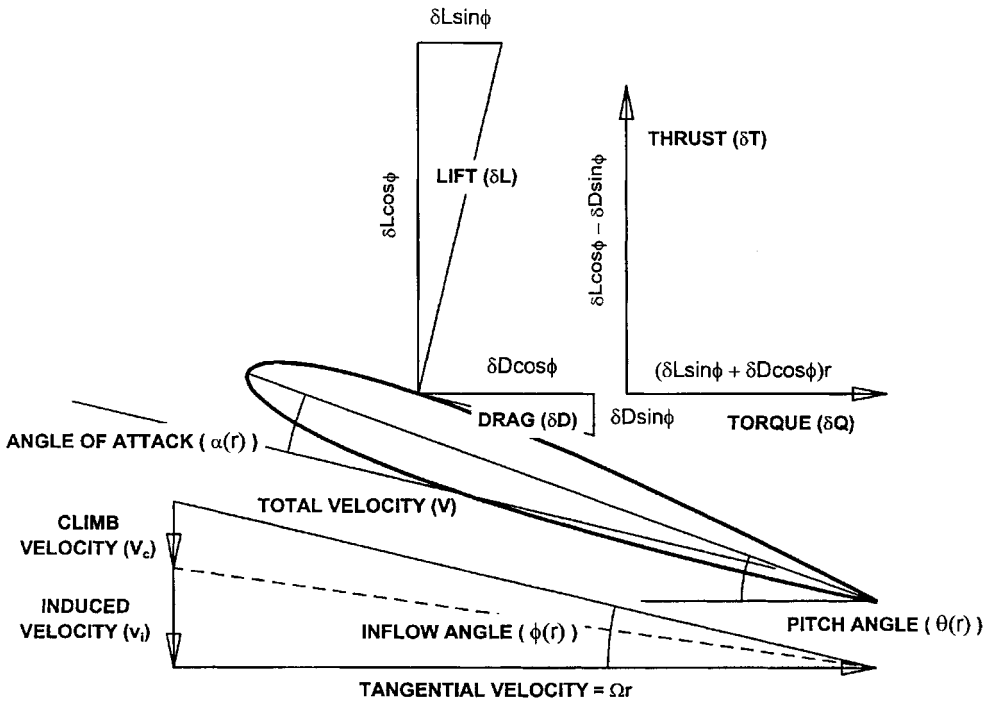


Fig. 2.3 Conditions at the blade element.

This is commonly approximated to  $V = \Omega r$ . Justification for this approximation can be seen by considering the example helicopter in a rapid vertical climb (2000 ft/min or 10.2 m/s). Using Equation (2.4),  $v_{ih} = 12.3$  m/s and later work will show that under these conditions  $v_i = 0.7v_{ih} = 8.6$  m/s. If the blade tip is considered then  $V = \Omega R = 227.5$  m/s whereas including the vertical component gives  $V = 228.3$  m/s. Thus in this case

the standard approximation underestimates the total velocity by only 0.3%. Now the blade section incidence,  $\alpha$ , will depend on its radial position,  $r$ , and from Fig. 2.3 for any spanwise position  $r$ :

$$\alpha(r) = \theta(r) - \phi(r)$$

Using small angle approximations:

$$\alpha(r) = \theta(r) - \left( \frac{V_c + v_i}{\Omega r} \right) = \theta(r) - \lambda(r)$$

This local incidence will lead to elemental lift and drag:

$$\delta L = \frac{1}{2} \rho V^2 S C_L = \frac{1}{2} \rho \Omega^2 r^2 c(r) a \alpha(r) \delta r$$

$$\delta D = \frac{1}{2} \rho V^2 S C_D = \frac{1}{2} \rho \Omega^2 r^2 c(r) C_D(r) \delta r$$

which, from Fig. 2.3, can be combined to give elemental thrust and torque. Assuming the inflow angle is small and the lift/drag ratio is large leads to:

$$\delta T = b \delta L$$

$$\delta Q = b(\phi \delta L + \delta D)r$$

The elementary power,  $\delta P$ , required to produce the elementary thrust can be found from the elementary torque:

$$\delta P = \Omega \delta Q = b \Omega (\phi \delta L + \delta D)r = \Omega r (\phi \delta T + b \delta D) = (V_c + v_i) \delta T + \Omega r b \delta D$$

If this elemental equation is integrated along the rotor blade:

$$P = (V_c + v_i)T + \frac{1}{2} \rho \Omega^3 b \int c(r) r^3 C_D dr \quad (2.5)$$

Equation (2.5) is similar to the result obtained by adopting momentum theory except that we now have an extra term in the expression that represents the *profile power* required to keep the rotor turning against the torque produced by the profile drag. The total rotor thrust, torque and power can be obtained by integrating analytically the expressions for  $\delta T$ ,  $\delta Q$  and  $\delta P$  over the span of the blade. For a rectangular blade at constant pitch, where  $C_D$  is now the mean profile drag coefficient for whole rotor, then:

$$\left. \begin{aligned} T &= \frac{1}{2} \rho a b c V_T^2 R \left( \frac{\theta}{3} - \frac{\lambda}{2} \right) \\ Q &= T \frac{(V_c - v_i)}{\Omega} + \frac{1}{8} \rho b c V_T^2 R^2 C_D \\ P &= T(V_c - v_i) + \frac{1}{8} \rho b c V_T^3 R C_D \end{aligned} \right\} \quad (2.6)$$

### 2.3.2 Constant chord blades with linear twist

Most modern blades feature a degree of negative twist, decreasing the pitch angle towards the tip, as means of optimizing the blade loading distribution. This can be expressed using the following equation:

$$\theta(r) = \theta_0 = \frac{r}{R} \theta_1$$

where  $\theta_0$  is the collective pitch applied at the hub;  $\theta_1$  is the total twist (normally negative) and  $r/R$  factors the total twist according to radial position. Now:

$$\alpha(r) = \theta(r) - \left( \frac{V_c + v_i}{\Omega r} \right) = \theta_0 + \frac{r}{R} \theta_1 - \left( \frac{V_c + v_i}{\Omega r} \right)$$

Therefore, for a rectangular blade, the elemental thrust equation becomes:

$$\delta T = \frac{1}{2} \rho abc \Omega^2 \left[ \theta_0 r^2 + \frac{r^3}{R} \theta_1 - \left( \frac{V_c + v_i}{\Omega r} \right) r \right] \delta r$$

So:

$$T = \frac{1}{2} \rho abc \Omega^2 \left[ \frac{\theta_0}{3} R^3 + \frac{\theta_1}{4} R^3 - \frac{1}{2} \left( \frac{V_c + v_i}{\Omega} \right) R^2 \right] \delta r$$

$$T = \frac{1}{2} \rho abc V_T^2 R \left( \frac{\theta_0}{3} + \frac{\theta_1}{4} - \frac{\lambda}{2} \right) \quad (2.7)$$

## 2.4 NON-DIMENSIONAL COEFFICIENTS

Before examining how the realism of the theories introduced above might be improved, it is necessary to discuss the concept of non-dimensional coefficients. These coefficients are analogous to the lift and drag coefficients that are a common feature of aerodynamics. Rather than lift and drag, in the case of rotorcraft, coefficients of thrust ( $C_T$ ), torque ( $C_Q$ ) and power ( $C_P$ ) are used. These coefficients are defined as:

$$C_T = \frac{T}{\rho A \Omega^2 R^2} = \frac{T}{\rho A V_T^2}$$

$$C_Q = \frac{Q}{\rho A \Omega^2 R^3} = \frac{Q}{\rho A V_T^2 R}$$

$$C_P = \frac{P}{\rho A \Omega^3 R^3} = \frac{Q \Omega}{\rho A V_T^2 \Omega R} = C_Q$$

Note that power coefficient is numerically equal to torque coefficient (though, of course, they are different physically). Now converting Equation (2.7) into coefficient terms gives:

$$C_T = \frac{1}{2} a \frac{bcR}{A} \left( \frac{1}{3} \theta_0 + \frac{1}{4} \theta_1 - \frac{1}{2} \lambda \right)$$

$$C_T = \frac{1}{2} as \left( \frac{1}{3} \theta_0 + \frac{1}{4} \theta_1 - \frac{1}{2} \lambda \right)$$

where  $s$  is defined as the *solidity* of the disk that is the ratio of total blade area to disk area. Hence for a rotor with rectangular blades  $s = bcR/A$ . The relationship for thrust

coefficient can be simplified by altering the definition of blade pitch from the blade root to a position at three-quarter radius ( $\theta_{0.75}$ ), since:

$$\begin{aligned} C_T &= \frac{1}{2}sa \left( \frac{1}{3}\theta_0 + \frac{1}{4}\theta_1 - \frac{1}{2}\lambda \right) \\ &= \frac{1}{2}sa \left[ \frac{1}{3} \left( \theta_0 + \frac{3}{4}\theta_1 \right) - \frac{1}{2}\lambda \right] \\ C_T &= \frac{1}{2}sa \left( \frac{1}{3}\theta_{0.75} - \frac{1}{2}\lambda \right) \end{aligned}$$

Hence a rotor with zero twist will generate the same thrust coefficient as one with linear twist, provided the pitch setting of the untwisted blade is equal to that at the three-quarter radius on the twisted blade. Also from Equation (2.6):

$$P = T(V_c - v_i) + \frac{1}{8}\rho bc V_T^3 RC_D$$

Thus:

$$\begin{aligned} C_P &= \frac{T}{\rho AV_T^3} (V_c + v_i) + \frac{1}{8} \frac{\rho bc R V_T^3}{\rho AV_T^3} C_D \\ &= C_T \frac{V_c + v_i}{V_T} + \frac{1}{8} s C_D \\ C_P &= C_T \frac{V_c}{V_T} + C_T \frac{v_i}{V_T} + \frac{1}{8} s C_D \end{aligned}$$

This equation is frequently used in performance analysis. It shows that for a climbing rotor the power required can be sub-divided into three parts: the first term represents the useful power; the second the induced power; and the third the profile power. If the particular case of *hovering* flight is considered ( $V_c = 0$ ) then from momentum theory the induced velocity can be related to the thrust coefficient:

$$v_{ih} = \sqrt{\frac{T}{2\rho A}} = \sqrt{\frac{TV_T^2}{2\rho AV_T^2}} = \sqrt{\frac{C_T V_T^2}{2}} = V_T \sqrt{\frac{C_T}{2}}$$

In addition, the power coefficient becomes:

$$C_P = C_T \sqrt{\frac{C_T}{2}} + \frac{1}{8} s C_D = \frac{1}{\sqrt{2}} C_T^{3/2} + \frac{1}{8} s C_D$$

Now for a given rotor with fixed solidity, provided the profile drag coefficient remains constant, then:

$$C_P = k_1 C_T^{3/2} + k_2 \quad (2.8)$$

Equation (2.8) is a very important result since it suggests that for a hovering rotor the power coefficient is directly proportional to the thrust coefficient (or aircraft weight) *provided* the drag coefficient remains unchanged. Likewise if the helicopter mass is

fixed and the rotor profile drag coefficient is constant then the power required to drive the rotor will vary as  $\Omega^3$  so that  $C_p$  will remain unchanged. This simple rule forms the basis of hover performance testing and is illustrated in Fig. 2.4.

## 2.5 AXIAL FLIGHT: IMPROVED THEORETICAL ESTIMATES

Although the blade element theory introduced above has allowed the real effect of rotor driving torque arising from profile drag to be included, it has still relied upon the assumption that the inflow is constant along the rotor radius. Likewise, no allowance has been made for three-dimensional flow effects such as tip vortices. Minor modifications to the blade element theory are used to account for these important effects.

### 2.5.1 Variations in induced velocity

So far, it has been assumed that the induced velocity is constant across the disk. In addition, no description has been given of the precise relationship between blade pitch and induced velocity. In fact flow is induced downwards through the disk as a *consequence* of the aerofoil's *inclination* to the direction of rotation. As such it is often necessary to develop a relationship between blade pitch and induced velocity for any radial station. Combining the momentum and blade element theories introduced earlier leads to:

$$\delta T = \frac{1}{2} \rho abc \Omega^2 r^2 \left[ \theta(r) - \left( \frac{V_c + v_i}{\Omega r} \right) \right] \delta r = 2\rho(V_c + v_i)v_i 2\pi r \delta r$$

Therefore:

$$\frac{1}{2} \rho abc \Omega r [\Omega r \theta(r) - (V_c + v_i)] = 4\rho(V_c v_i + v_i^2) \pi r$$

or:

$$\left[ \frac{v_{ih}}{V_T} \right]_r = -\frac{as}{16} + \sqrt{\left( \frac{as}{16} \right)^2 + \left( \frac{as}{8} \right) \frac{r}{R} \theta(r)} \quad (2.9)$$

where  $[v_{ih}/V_T]_r$  represents the ratio of the induced velocity in the hover at radius  $r$  to the tip speed, and  $\theta(r)$  is the pitch at radius  $r$ . Figure 2.5 shows the variation of induced velocity for a hovering rotor calculated using Equation (2.9).

### 2.5.2 Fuselage download

The slipstream from the rotor exerts a downward force on the helicopter fuselage, which is in addition to any vertical drag associated with axial flight. This means that in the hover the main rotor must generate sufficient thrust to support not only the weight of the helicopter but balance this download. The rotor wake contracts from the diameter of the rotor to its ultimate wake size in about a quarter of rotor radius