# Modelling Single-name and Multi-name Credit Derivatives

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*To Penny, Rory and Fergal.*

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Dominic O'Kane

April 2008.

# About the Author

Dominic O'Kane is an affiliated Professor of Finance at the French business school EDHEC which is based in Nice, France. Until May 2006, Dominic O'Kane was a managing director and ran the European Fixed Income Quantitative Research group at Lehman Brothers, the US investment bank. Dominic spent seven of his nine years at Lehman Brothers working as a quant for the credit derivatives trading desk. Dominic has a doctorate in theoretical physics from Oxford University.

# Introduction

The aim of this book is to present an up-to-date, comprehensive, accessible and practical guide to the models used to price and risk-manage credit derivatives. It is both a detailed introduction to credit derivative modelling and a reference for those who are already practitioners.

This book is up-to-date as it covers many of the important developments which have occurred in the credit derivatives market in the past 4-5 years. These include the arrival of the CDS portfolio indices and all of the products based on these indices. In terms of models, this book covers the challenge of modelling single-tranche CDOs in the presence of the correlation skew, as well as the pricing and risk of more recent products such as constant maturity CDS, portfolio swaptions, CDO squareds, credit CPPI and credit CPDOs.

For each model, the reader is taken through the underlying assumptions and then the mathematical derivation. The application of the model to pricing and risk-management is explained with the goal of trying to build intuition. There is also a focus on the efficient implementation of each model.

Product coverage is extensive and is split into two parts. Part I covers single-name credit derivatives and Part II covers multi-name credit derivatives. We begin the first part with the traditional credit products including the credit risky bond, the floating rate note and the asset swap. Although they are not credit derivatives, these are included for the sake of completeness and because they are a pricing reference for the credit default swap. We then move on to the credit default swap (CDS) which, reflecting its importance, is covered in considerable detail. We also include a discussion of digital CDS, forward CDS and loan CDS. We then cover other single-name products including the constant maturity default swap and the default swaption.

Part II covers products whose risk is linked to the credit performance of more than one credit. These are known as 'multi-name' products. We begin with the biggest growth product of the credit derivatives market, the CDS index. This then leads us to the many product innovations which have resulted from the arrival of these CDS indices. These include the tranched CDS indices and CDS index swaptions. We also cover advanced correlation products such as the CDO-squared and the leveraged super senior. Dynamically managed structures such as the credit CPPI and CPDO are also examined. Towards the end of the book we consider a number of the newer products which are beginning to be traded. These include forward starting tranches and tranche options.

In this book we set out in detail the models which have been developed to address the challenges posed by these products. Of these challenges, the most important has been the modelling of default correlation. We therefore cover in detail the Gaussian copula and the modelling of default dependency in general, especially within a copula framework. After establishing the arbitrage-free conditions for a correlation model, we devote an entire chapter to *base correlation*, which has become a widely used pricing and risk-management approach. We discuss its implementation and in doing so highlight the advantages and disadvantages of base correlation as a pricing and risk-management framework. We then discuss a range of specific copula models, highlighting the pros and cons of each. This takes us to the subject of much current research–the development of usable dynamic correlation models. In the final two chapters of this book we discuss the two main categories of dynamic models known as bottom-up and top-down. We also set out in detail some specific models which fall into these two categories.

The credit derivatives market has changed significantly in the past four to five years and most of these developments are contained within this book. However, the market continues to evolve. As a result, I would suggest that readers keen to keep abreast of the latest modelling and market developments periodically visit www.defaultrisk.com and the technical forums of www.wilmott.com.

# Notation

| Symbol | Description |
|---|---|
| $t$ | today (valuation date). |
| $t = 0$ | Contract initiation date. |
| $t_S$ | Contract settlement date. |
| $t_E$ | Option expiry date. |
| $T$ | Contract maturity date. |
| $t_n$ | $n$th cash flow date on the premium leg. Usually $t_0 = t$ and $t_N = T$. |
| $\Delta(t_{n-1}, t_n, b)$ | The year fraction from date $t_{n-1}$ to $t_n$ in a basis $b$. We typically drop the $b$. |
| $Z(t, T)$ | Libor zero coupon bond price from time $t$ to $T$. We sometimes use $Z(t) = Z(0, t)$. |
| $\hat{z}(t, T)$ | Zero recovery credit risky zero coupon bond price from today time $t$ to time $T$. |
| $L(T_1, T_2)$ | The observed value of the Libor rate which sets at time $T_1$ for a period $[T_1, T_2]$. |
| $L(t, T_1, T_2)$ | The value at time $t$ of a forward Libor rate which sets at time $T_1$ for a period $[T_1, T_2]$. |
| $D(t, T)$ | Present value of \$1 paid at the time of |

| | |
|---|---|
| | default as seen at time $t$. |
| $Q_i(t, T)$ | Survival probability from today time $t$ to time $T$ for issuer $i$. We sometimes use $Q_i(0, t)$. |
| $S(t, T)$ | $Q_i(t)$ = CDS contractual spread at time $t$ for a contract which matures at time $T$. |
| $S(t, t_F, T)$ | Forward CDS contractual spread at time $t$ for a contract with forward start $t_F$ which matures at time $T$. Note that $S(t, t, T) = S(t, T)$. |
| PV01($t$, $T$) | The time $t$ present value of a $1 Libor quality annuity which matures at time $T$. |
| RPV01($t$, $T$) | The time $t$ present value of a credit risky $1 annuity which matures at time $T$. |
| $r(t)$ | The risk-free short interest rate at time $t$. |
| $\beta(t)$ | Value of the rolled-up money-market account $\beta(t) = \exp(r(s)ds)$. |
| $\beta(t, T)$ | Value of $\beta(T)\beta(t)^{-1}$. |
| $\lambda(t)$ | The hazard rate or intensity process at time $t$. |
| $\Pr(x)$ | The probability that $x$ is true. |
| $C(u_1,\ldots,u_n)$ | The $n$-dimensional default copula. |
| $\hat{C}(u_1,\ldots,u_n)$ | The $n$-dimensional survival copula. |
| $\varphi(a)$ | The Gaussian density. |

| | |
|---|---|
| $\Phi(a)$ | The uni-variate Gaussian cumulative distribution function. |
| $\Phi_2(a, b, \rho)$ | The bi-variate Gaussian cumulative distribution function. |
| $\Phi_n(\mathbf{a}, \Sigma)$ | The $n$-variate Gaussian cumulative distribution function with correlation matrix $\Sigma$. |
| $t_\nu(x)$ | The uni-variate Student-$t$ cumulative distribution function with $\nu$ degrees of freedom. |
| $L(T)$ | The fractional portfolio loss at horizon time $T$. |
| $L(T, K_1, K_2)$ | The fractional tranche loss at horizon time $T$. |
| $F(x)$ | The cumulative portfolio loss distribution, i.e. $F(x) = \Pr(L \leq x)$. |
| $f(x)$ | The density of the portfolio loss distribution $f(x) = \partial F(x)/\partial x$. |
| $O(x)$ | 'Order of' $x$. If $f(x)$ is $O(x^n)$ then $n$ is the exponent of the dominant polynomial term in $f(x)$. |
| $\mathrm{int}(x)$ | Function which returns the integer part of a number without rounding, e.g. int (5.7322) = 5. |
| $\mathrm{ceil}(x)$ | Function which returns the smallest integer value greater than or equal to $x$, e.g. ceil (5.7322) = 6. |

# 1

# The Credit Derivatives Market

## 1.1 INTRODUCTION

Without a doubt, credit derivatives have revolutionised the trading and management of credit risk. They have made it easier for banks, who have historically been the warehouses of credit risk, to hedge and diversify their credit risk. Credit derivatives have also enabled the creation of products which can be customised to the risk-return profile of specific investors. As a result, credit derivatives have provided something new to both hedgers and investors and this has been a major factor in the growth of the credit derivatives market.

From its beginning in the mid-1990s, the size of the credit derivatives market has grown at an astonishing rate and it now exceeds the size of the credit bond market. According to a recent ISDA survey,[1] the notional amount outstanding of credit derivatives as of mid-year 2007 was estimated to be $45.46 trillion. This significantly exceeds the size of the US corporate bond market which is currently $5.7 trillion and the US Treasury market which is currently $4.3 trillion.[2] It also exceeds the size of the equity derivatives market which ISDA also estimated in mid-2007 to have a total notional amount outstanding of $10.01 trillion.

In addition to its size, what is also astonishing about the credit derivatives market is the breadth and liquidity it has attained. This has been due largely to the efforts of the dealer community which has sought to structure products in a way that maximises tradabililty and standardisation and hence liquidity. The CDS indices, introduced in 2002 and discussed extensively in this book, are a prime example of this. They cover over 600 of the most important corporate and sovereign credits. They typically trade with a bid-offer spread of less than 1 basis point and frequently as low as a quarter of a basis point.[3]

To understand the success of the credit derivatives market, we need to understand what it can do. In its early days, the credit derivatives market was dominated by banks who found credit derivatives to be a very useful way to hedge the credit risk of a bond or loan that was held on their balance sheet. Credit derivatives could also be used by banks to manage their regulatory capital more efficiently. More recently, the credit derivatives market has become much more of an investor driven market, with a focus on developing products which present an attractive risk-return profile. However, to really understand the appeal of the credit derivatives market, it is worth listing the many uses which credit derivatives present:

- Credit derivatives make it easier to go short credit risk either as a way to hedge an existing credit exposure or as a way to express a negative view on the credit market.
- Most credit derivatives are unfunded. This means that unlike a bond, a credit derivative contract requires no initial payment. As a consequence, the investor in a credit derivative does not have to fund any initial payment. This means that credit derivatives may present a cheaper alternative to buying cash bonds for investors who fund above Libor. It also makes it easier to leverage a credit exposure.

- Credit derivatives increase liquidity by taking illiquid assets and repackaging them into a form which better matches the risk-reward profiles of investors.
- Credit derivatives enable better diversification of credit risk as the breadth and liquidity of the credit derivatives market is greater than that of the corporate bond market.
- Credit derivatives add transparency to the pricing of credit risk by broadening the range of traded credits and their liquidity. We estimate that there are over 600 corporate and sovereign names which have good liquidity across the credit derivatives market.[4] The scope of the credits is global as it includes European, North American and Asian corporate credits plus Emerging Market sovereigns.
- Credit derivatives shift the credit risk which has historically resided on bank loan books into the capital markets and in doing so it has reduced the concentrations of credit risk in the banking sector. However, this does raise the concern of whether this credit risk is better managed in less regulated entities which sit outside the banking sector.
- Credit derivatives allow for the creation of new asset classes which are exposed to new risks such as credit volatility and credit correlation. These can be used to diversify investment portfolios.

The relatively short history of the credit derivatives market has not been uneventful. Even before the current credit crisis of 2007–2008, the credit derivative market has weathered the 1997 Asian Crisis, the 1998 Russian default, the events of 11 September 2001, the defaults of Conseco, Railtrack, Enron, WorldCom and others, and the downgrades of Ford and General Motors. What has been striking about all of these events is the ability of the credit derivatives market to work through these events and to emerge stronger. This has been largely due to the willingness of the market participants to resolve any problems which these events may have exposed in either the mechanics of the products or their legal documentation. Each of these events has also strengthened the market by demonstrating that it is often the only practical way to go short and therefore hedge these credit risks.

In this chapter, we discuss the growth in the credit derivatives market size. We present an overview of the different credit derivatives and discuss a market survey which shows how the importance of these products has evolved over time. We then discuss the structure of the credit derivatives market in terms of its participants.

## 1.2 MARKET GROWTH

The growth of the credit derivatives market has been phenomenal. Although there are different ways to measure this growth, each with its own particular approach, when plotted as a function of time, they all show the same exponential growth shown in Figure 1.1. Let us consider the three sources of market size data:

1. The British Bankers' Association (BBA) surveys the credit derivatives market via a questionnaire every two or so years. Their questionnaire is sent to about 30 of the largest investment banks who act as dealers in the credit derivatives market. Their latest report was published in 2006 and estimated the total market notional at the end of 2006 to be $20.207 trillion.
2. The International Swaps and Derivatives Association (ISDA) conducts a twice-yearly survey of the market. In the most recent, they surveyed 88 of their member firms including the main credit derivatives dealers about the size of their credit derivatives positions. The

collected numbers were adjusted to correct for double-counting.[5] The mid-2007 survey estimated the size of the credit derivatives market to be $45.25 trillion, an increase of 32% in the first six months of 2007.

3. The US Office for the Comptroller of the Currency conducts a quarterly survey of the credit derivatives market size. The survey covers just the US commercial bank sector. The June 2007 survey found that the total notional amount of credit derivatives held by US commercial banks was $10.2 trillion, an increase of 86% on the first quarter of 2006. This number is lower than the others partly because it excludes trades done by many non-US commercial banks and investment banks.

Although these numbers all differ because of the differing methodologies and timings, what is beyond doubt is the rapid growth that has been experienced by the credit derivatives market.

**Figure 1.1** Evolution of the credit derivatives market size using estimates calculated in the BBA Credit Derivatives Report 2006. Source: British Banker’s Association