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2nd Edition

Calculus II For Dummies®

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- Get ahead of the curve with easy-to-understand explanations of complicated subject matter
- Score high in your Calculus II class

Mark Zegarelli

Math tutor and writer

Calculus II
FOR
DUMMIES®
2ND EDITION

by Mark Zegarelli

WILEY

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Mark Zegarelli is the author of *Logic For Dummies* (Wiley), *Basic Math & Pre-Algebra For Dummies* (Wiley), and numerous books of puzzles. He holds degrees in both English and math from Rutgers University, and he lives in Long Branch, New Jersey, and San Francisco, California.

Dedication

For my brilliant and beautiful sister, Tami. You are an inspiration.

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Introduction

Calculus is the great Mount Everest of math. Most of the world is content to just gaze upward at it in awe. But only a few brave souls attempt the ascent.

Or maybe not.

In recent years, calculus has become a required course not only for math, engineering, and physics majors, but also for students of biology, economics, psychology, nursing, and business. Law schools and MBA programs welcome students who've taken calculus because it requires discipline and clarity of mind. Even more and more high schools are encouraging students to study calculus in preparation for the Advanced Placement (AP) exam.

So perhaps calculus is more like a well-traveled Vermont mountain, with lots of trails and camping spots, plus a big ski lodge on top. You may need some stamina to conquer it, but with the right guide (this book, for example!), you're not likely to find yourself swallowed up by a snowstorm half a mile from the summit.

About This Book

You *can* learn calculus. That's what this book is all about. In fact, as you read these words, you may well already be a winner, having passed a course in Calculus I. If so, then congratulations and a nice pat on the back are in order.

Having said that, I want to discuss a few rumors you may have heard about Calculus II:

- Calculus II is harder than Calculus I.
- Calculus II is harder, even, than either Calculus III or Differential Equations.
- Calculus II is more frightening than having your home invaded by zombies in the middle of the night and will result in emotional trauma requiring years of costly psychotherapy to heal.

Now, I admit that Calculus II is harder than Calculus I. Also, I may as well tell you that many — but not all — math students find it to be harder than the two semesters of math that follow. (Speaking personally, I found Calc II to be easier than Differential Equations.) But I'm holding my ground that the long-term psychological effects of a zombie attack far outweigh those awaiting you in any one-semester math course.

The two main topics of Calculus II are integration and infinite series. *Integration* is the inverse of differentiation, which you study in Calculus I. (For practical purposes, integration is a method for finding the area of unusual geometric shapes.) An *infinite series* is a sum of numbers that goes on forever, like 1 + 2 + 3 + . . . or + + + ... Roughly speaking, most teachers focus on integration for the first two-thirds of the semester and infinite series for the last third.

This book gives you a solid introduction to what's covered in a college course in Calculus II. You can use it either for self-study or while enrolled in a Calculus II course.

So feel free to jump around. Whenever I cover a topic that requires information from earlier in the book, I refer you to that section in case you want to refresh yourself on the basics.

Here are two pieces of advice for math students (remember them as you read the book):

- **Study a little every day.** I know that students face a great temptation to let a book sit on the shelf until the night before an assignment is due. This is a particularly poor approach for Calc II. Math, like water, tends to seep in slowly and swamp the unwary!

  So, when you receive a homework assignment, read over every problem as soon as you can and try to solve the easy ones. Go back to the harder problems every day, even if it's just to reread and think about them. You'll probably find that over time, even the most opaque problem starts to make sense.

- **Use practice problems for practice.** After you read through an example and think you understand it, copy the problem down on paper, close the book, and try to work it through. If you can get through it from beginning to end, you're ready to move on. If not, go ahead and peek, but then try solving the problem later without peeking. (Remember, on exams, no peeking is allowed!)

Conventions Used in This Book

Throughout the book, I use the following conventions:

- *Italicized* text highlights new words and defined terms.
- **Boldfaced** text indicates keywords in bulleted lists and the action parts of numbered steps.
- `Monofont` text highlights web addresses.
- Angles are measured in radians rather than degrees, unless I specifically state otherwise. (See Chapter 2 for a discussion about the advantages of using radians for measuring angles.)

What You're Not to Read

All authors believe that each word they write is pure gold, but you don't have to read every word in this book unless you really want to. You can skip over sidebars (those gray shaded boxes) where I go off on a tangent, unless you find that tangent interesting. Also feel free to pass by paragraphs labeled with the Technical Stuff icon.

If you're not taking a class where you'll be tested and graded, you can skip paragraphs labeled with the Tip icon and jump over extended step-by-step examples. However, if you're taking a class, read this material carefully and practice working through examples on your own.

Foolish Assumptions

Not surprisingly, a lot of Calculus II builds on topics introduced in Calculus I and Pre-Calculus. So here are the foolish assumptions I make about you as you begin to read this book:

- If you're a student in a Calculus II course, I assume that you passed Calculus I. (Even if you got a D-minus, your Calc I professor and I agree that you're good to go!)
- If you're studying on your own, I assume that you're at least passably familiar with some of the basics of Calculus I.

I expect that you know some things from Calculus I, but I don't throw you in the deep end of the pool and expect you to swim or drown. Chapter 2 contains a ton of useful math tidbits that you may have missed the first time around. And throughout the book, whenever I introduce a topic that calls for previous knowledge, I point you to an earlier chapter or section so you can get a refresher.

How This Book Is Organized

This book is organized into six parts, starting you off at the beginning of Calculus II, taking you all the way through the course, and ending with a look at some advanced topics that await you in your further math studies.

Part I: Introduction to Integration

In Part I, I give you an overview of Calculus II, plus a review of more foundational math concepts.

Chapter 1 introduces the definite integral, a mathematical statement that expresses area. I show you how to formulate and think about an area problem by using the notation of calculus. I also introduce you to the Riemann sum equation for the integral, which provides the definition of the definite integral as a limit. Beyond that, I give you an overview of the entire book.

Chapter 2 gives you a need-to-know refresher on Pre-Calculus and Calculus I.

Chapter 3 introduces the indefinite integral as a more general and often more useful way to think about the definite integral.

Part II: Indefinite Integrals

Part II focuses on a variety of ways to solve indefinite integrals.

Chapter 4 shows you how to solve a limited set of indefinite integrals by using anti-differentiation — that is, by reversing the differentiation process. I show you 17 basic integrals, which mirror the 17 basic derivatives from Calculus I. I also show you a set of important rules for integrating.

Chapter 5 covers variable substitution, which greatly extends the usefulness of anti-differentiation. You discover how to change the variable of a function that you're trying to integrate to make it more manageable by using the integration methods in Chapter 4.

Chapter 6 introduces integration by parts, which allows you to integrate functions by splitting them into two separate factors. I show you how to recognize functions that yield well to this approach. I also show you a handy method — the DI-agonal method — to integrate by parts quickly and easily.

In Chapter 7, I get you up to speed integrating a whole host of trig functions. I show you how to integrate powers of sines and cosines, and then tangents and secants, and finally cotangents and cosecants. Then you put these methods to use in trigonometric substitution.

In Chapter 8, I show you how to use partial fractions as a way to integrate complicated rational functions. As with the other methods in this part of the book, using partial fractions gives you a way to tweak functions that you don't know how to integrate into more manageable ones.

Part III: Intermediate Integration Topics

Part III discusses a variety of intermediate topics, after you have the basics of integration under your belt.

Chapter 9 gives you a variety of fine points to help you solve more complex area problems. You discover how to find unusual areas by piecing together one or more integrals. I show you how to evaluate improper integrals — that is, integrals extending infinitely in one direction. I discuss how the concept of signed area affects the solution to integrals. I show you how to find the average value of a function within an interval. And I give you a formula for finding arc-length, which is the length measured along a curve.

And Chapter 10 adds a dimension, showing you how to use integration to find the surface area and volume of solids. I discuss the meat-slicer method and the shell method for finding solids. I show you how to find both the volume and surface area of revolution. And I show you how to set up more than one integral to calculate more complicated volumes.

Part IV: Infinite Series

In Part IV, I introduce the infinite series — that is, the sum of an infinite number of terms.

Chapter 11 gets you started working with a few basic types of infinite series. I start off by discussing infinite sequences. Then I introduce infinite series, getting you up to speed on expressing a series by using both sigma notation and expanded notation. Then I show you how every series has two associated sequences. To finish up, I introduce you to two common types of series — the geometric series and the *p*-series — showing you how to recognize and, when possible, evaluate them.

In Chapter 12, I show you a bunch of tests for determining whether a series is convergent or divergent. To begin, I show you the simple but useful *n*th-term test for divergence. Then I show you two comparison tests — the direct comparison test and the limit comparison test. After that, I introduce you to the more complicated integral, ratio, and root tests. Finally, I discuss alternating series and show you how to test for both absolute and conditional convergence.

And in Chapter 13, the focus is on a particularly useful and expressive type of infinite series called the Taylor series. First, I introduce you to power series. Then I show you how a specific type of power series — the Maclaurin series — can be useful for expressing functions. Finally, I discuss how the Taylor series is a more general version of the Maclaurin series. To finish up, I show you how to calculate the error bounds for Taylor polynomials.

Part V: Advanced Topics

In Part V, I pull out my crystal ball, showing you what lies in the future if you continue your math studies.

In Chapter 14, I give you an overview of Calculus III, also known as multivariable calculus, the study of calculus in three or more dimensions. First, I discuss vectors and show you a few vector calculations. Next, I introduce you to three different three-dimensional (3-D) coordinate systems: 3-D Cartesian coordinates, cylindrical coordinates, and spherical coordinates. Then I discuss functions of several variables, and I show you how to calculate partial derivatives and multiple integrals of these functions.

Chapter 15 focuses on differential equations — that is, equations with derivatives mixed in as variables. I distinguish ordinary differential equations from partial differential equations, and I show you how to recognize the order of a differential equation. I discuss how differential equations arise in science. Finally, I show you how to solve separable differential equations and how to solve linear first-order differential equations.

Part VI: The Part of Tens

Just for fun, Part VI includes a few top-ten lists on a variety of calculus-related topics.

Chapter 16 provides you with ten insights from Calculus II. These insights provide an overview of the book and its most important concepts.

Chapter 17 gives you ten useful test-taking tips. Some of these tips are specific to Calculus II, but many are generally helpful for any test you may face.

Icons Used in This Book

Throughout the book, I use four icons to highlight what's hot and what's not:

This icon points out key ideas that you need to know. Make sure you understand the ideas before reading on!

Tips are helpful hints that show you the easy way to get things done. Try them out, especially if you're taking a math course.

Warnings flag common errors that you want to avoid. Get clear where these little traps are hiding so you don't fall in.

This icon points out interesting trivia that you can read or skip over as you like.

Where to Go from Here

You can use this book either for self-study or to help you survive and thrive in a course in Calculus II.

If you're taking a Calculus II course, you may be under pressure to complete a homework assignment or study for an exam. In that case, feel free to skip right to the topic that you need help with. Every section is self-contained, so you can jump right in and use the book as a handy reference. And when I refer to information that I discuss earlier in the book, I give you a brief review and a pointer to the chapter or section where you can get more information if you need it.

If you're studying on your own, I recommend that you begin with Chapter 1, where I give you an overview of the entire book, and read the chapters from beginning to end. Jump over Chapter 2 if you feel confident about your grounding in Calculus I and Pre-Calculus. And, of course, if you're dying to read about a topic that's later in the book, go for it! You can always drop back to an easier chapter if you get lost.

Part I

Introduction to Integration

The 5th Wave — By Rich Tennant

"We all know it's a pie, Helen. There's no need to pipe the number 3.141592653 on the top."

In this part . . .

I give you an overview of Calculus II, plus a review of Pre-Calculus and Calculus I. You discover how to measure the areas of weird shapes by using a new tool: the definite integral. I show you the connection between differentiation, which you know from Calculus I, and integration. And you see how this connection provides a useful way to solve area problems.

Chapter 1

An Aerial View of the Area Problem

In This Chapter

- Measuring the area of shapes by using classical and analytic geometry
- Understanding integration as a solution to the area problem
- Building a formula for calculating definite integrals using Riemann sums
- Applying integration to the real world
- Considering sequences and series
- Looking ahead at some advanced math

Humans have been measuring the area of shapes for thousands of years. One practical use for this skill is measuring the area of a parcel of land. Measuring the area of a square or a rectangle is simple, so land tends to get divided into these shapes.

Discovering the area of a triangle, circle, or polygon is also easy, but as shapes get more unusual, measuring them gets harder. Although the Greeks were familiar with the conic sections — parabolas, ellipses, and hyperbolas — they couldn't reliably measure shapes with edges based on these figures.

Descartes's invention of analytic geometry — studying lines and curves as equations plotted on a graph — brought great insight into the relationships among the conic sections. But even analytic geometry didn't answer the question of how to measure the area inside a shape that includes a curve.

In this chapter, I show you how *integral calculus* (*integration* for short) developed from attempts to answer this basic question, called the *area problem.* With this introduction to the definite integral, you're ready to look at the practicalities of measuring area. The key to approximating an area that you don't know how to measure is to slice it into shapes that you do know how to measure (for example, rectangles).

Slicing things up is the basis for the *Riemann sum,* which allows you to turn a sequence of closer and closer approximations of a given area into a limit that gives you the exact area that you're seeking. I walk you through a step-by-step process that shows you exactly how the formal definition for the definite integral arises intuitively as you start slicing unruly shapes into nice, crisp rectangles.

Checking Out the Area

Finding the area of certain basic shapes — squares, rectangles, triangles, and circles — is easy. But a reliable method for finding the area of shapes containing more esoteric curves eluded mathematicians for centuries. In this section, I give you the basics of how this problem, called the *area problem,* is formulated in terms of a new concept, the definite integral.

The *definite integral* represents the area of a region bounded by the graph of a function, the *x*-axis, and two vertical lines located at the *limits of integration.* Without getting too deep into the computational methods of integration, I give you the basics of how to state the area problem formally in terms of the definite integral.

Comparing classical and analytic geometry

In *classical geometry,* you discover a variety of simple formulas for finding the area of different shapes. For example, Figure 1-1 shows the formulas for the area of a rectangle, a triangle, and a circle.

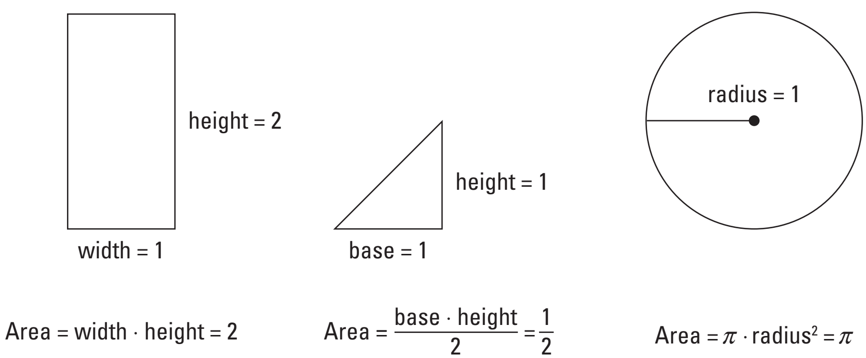

Figure 1-1: Formulas for the area of a rectangle, a triangle, and a circle.

Wisdom of the ancients

Long before calculus was invented, the ancient Greek mathematician Archimedes used his *method of exhaustion* to calculate the exact area of a segment of a parabola. Indian mathematicians also developed *quadrature* methods for some difficult shapes before Europeans began their investigations in the 17th century. These methods anticipated some of the methods of calculus. But before calculus, no single theory could measure the area under arbitrary curves.

When you move on to *analytic geometry* — geometry on the Cartesian graph — you gain new perspectives on classical geometry. Analytic geometry provides a connection between algebra and classical geometry. You find that circles, squares, and triangles — and many other figures — can be represented by equations or sets of equations, as shown in Figure 1-2.

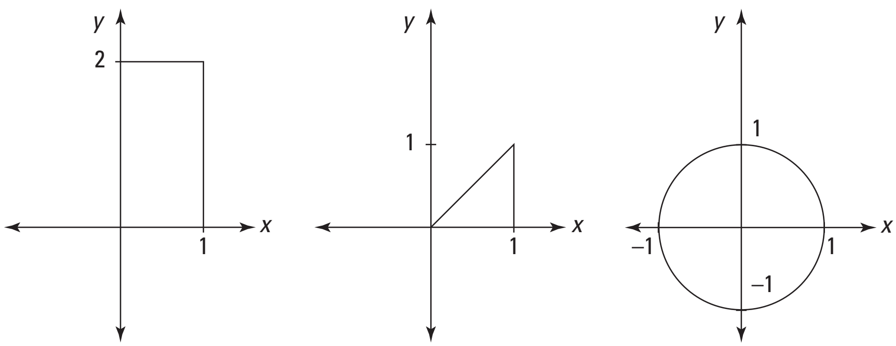

Figure 1-2: A rectangle, a triangle, and a circle embedded on the graph.

You can still use the trusty old methods of classical geometry to find the areas of these figures. But analytic geometry opens up more possibilities — and more problems.

Discovering a new area of study

Figure 1-3 illustrates three curves that are much easier to study with analytic geometry than with classical geometry: a parabola, an ellipse, and a hyperbola.

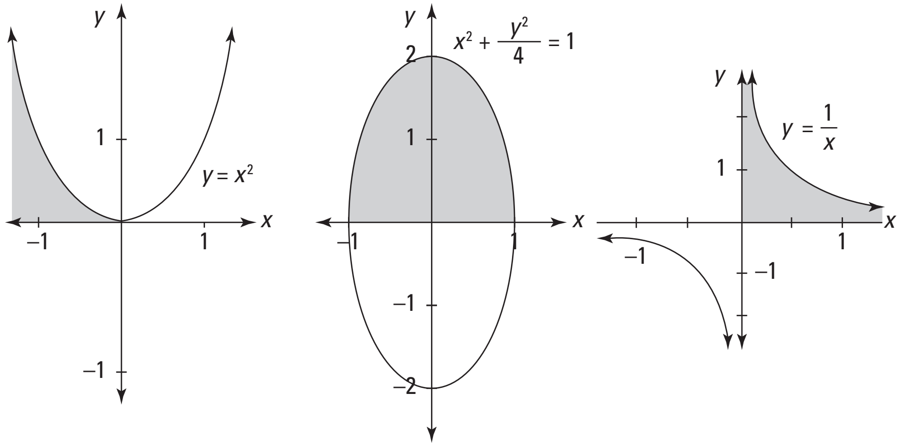

Figure 1-3: A parabola, an ellipse, and a hyperbola embedded on the graph.

Analytic geometry gives a very detailed account of the connection between algebraic equations and curves on a graph. But analytic geometry doesn't tell you how to find the shaded areas shown in Figure 1-3.

Figure 1-4 shows three more equations placed on the graph: a sine curve, an exponential curve, and a logarithmic curve.

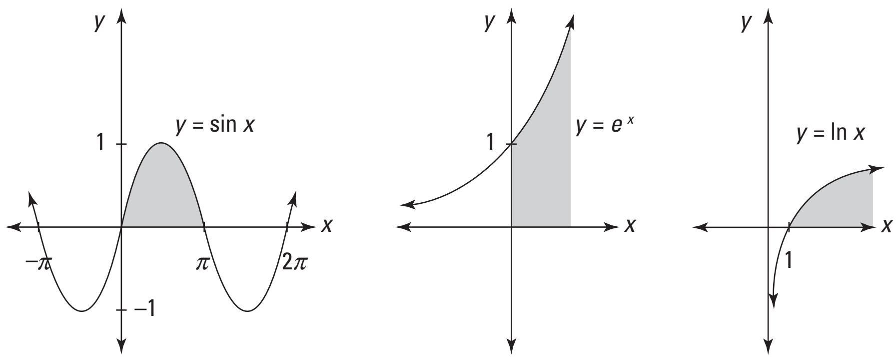

Figure 1-4: A sine curve, an exponential curve, and a logarithmic curve embedded on the graph.

Again, analytic geometry provides a connection between these equations and how they appear as curves on the graph. But it doesn't tell you how to find any of the shaded areas in Figure 1-4.