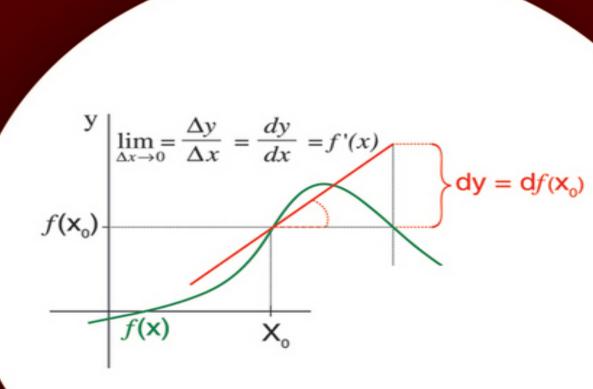
DIFFERENTIAL CALCULUS

Systematic Studies with Engineering Applications for Beginners





Contents

<u>Cover</u>
<u>Title Page</u>
<u>Copyright</u>
<u>Foreword</u>
<u>Preface</u>
<u>Biographies</u>
<u>Introduction</u>
<u>Acknowledgments</u>
<u>Chapter 1: From Arithmetic to</u> <u>Algebra</u>
1.1 Introduction
1.2 The Set of Whole Numbers
1.3 The Set of Integers
1.4 The Set of Rational Numbers
1.5 The Set of Irrational Numbers
1.6 The Set of Real Numbers
1.7 Even and Odd Numbers
<u>1.8 Factors</u>
1.9 Prime and Composite Numbers

1.10 Coprime Numbers
1.11 Highest Common Factor (H.C.F.)
1.12 Least Common Multiple (L.C.M.)
1.13 The Language of Algebra
1.14 Algebra as a Language for Thinking
1.15 Induction
1.16 An Important Result: The Number of
Primes is Infinite
1.17 Algebra as the Shorthand of
Mathematics
1.18 Notations in Algebra
1.19 Expressions and Identities in Algebra
1.20 Operations Involving Negative
Numbers
1.21 Division by Zero

<u>Chapter 2: The Concept of a Function</u>

- 2.1 Introduction
- 2.2 Equality of Ordered Pairs
- 2.3 Relations and Functions
- 2.4 Definition
- 2.5 Domain, Codomain, Image, and Range of a Function
- 2.6 Distinction Between "f" and "f(x)"
- 2.7 Dependent and Independent Variables
- 2.8 Functions at a Glance
- 2.9 Modes of Expressing a Function
- **2.10 Types of Functions**
- 2.11 Inverse Function f-1

- 2.12 Comparing Sets without Counting their Elements
- 2.13 The Cardinal Number of a Set
- 2.14 Equivalent Sets (Definition)
- 2.15 Finite Set (Definition)
- 2.16 Infinite Set (Definition)
- 2.17 Countable and Uncountable Sets
- 2.18 Cardinality of Countable and Uncountable Sets
- 2.19 Second Definition of an Infinite Set
- **2.20 The Notion of Infinity**
- 2.21 An Important Note About the Size of Infinity
- 2.22 Algebra of Infinity (∞)

<u>Chapter 3: Discovery of Real</u> <u>Numbers: Through Traditional</u> <u>Algebra</u>

- 3.1 Introduction
- 3.2 Prime and Composite Numbers
- 3.3 The Set of Rational Numbers
- 3.4 The Set of Irrational Numbers
- 3.5 The Set of Real Numbers
- 3.6 Definition of a Real Number
- 3.7 Geometrical Picture of Real Numbers
- 3.8 Algebraic Properties of Real Numbers
- 3.9 Inequalities (Order Properties in Real Numbers)
- 3.10 Intervals

3.11	Pro	perties	of A	bsol	ute	Val	ues
					u L C	- ui	ucs

- 3.12 Neighborhood of a Point
- 3.13 Property of Denseness
- 3.14 Completeness Property of Real Numbers
- 3.15 (Modified) Definition II (I.u.b.)
- 3.16 (Modified) Definition II (g.l.b.)

<u>Chapter 4: From Geometry to</u> <u>Coordinate Geometry</u>

- 4.1 Introduction
- <u>4.2 Coordinate Geometry (or Analytic Geometry)</u>
- 4.3 The Distance Formula
- 4.4 Section Formula
- 4.5 The Angle of Inclination of a Line
- 4.6 Solution(s) of an Equation and its Graph
- 4.7 Equations of a Line
- 4.8 Parallel Lines
- 4.9 Relation Between the Slopes of
- (Nonvertical) Lines that are Perpendicular
- to One Another
- 4.10 Angle Between Two Lines
- 4.11 Polar Coordinate System

<u>Chapter 5: Trigonometry and</u> <u>Trigonometric Functions</u>

- **5.1 Introduction**
- 5.2 (Directed) Angles

<u>Chapter 7a: The Concept of Limit of a Function</u>

7a.1 Introduction

<i>7a.2</i>	Useful	Notations

7a.3 The Concept of Limit of A Function: Informal Discussion

7a.4 Intuitive Meaning of Limit of A Function

<u>7a.5 Testing The Definition [Applications of The ε, δ Definition of Limit]</u>

7a.6 Theorem (B): Substitution Theorem

<u>7a.7 Theorem (C): Squeeze Theorem Or</u> Sandwich Theorem

7a.8 One-Sided Limits (Extension to the Concept of Limit)

<u>Chapter 7b: Methods for Computing</u> <u>Limits of Algebraic Functions</u>

7b.1 Introduction

7b.2 Methods for Evaluating Limits of

Various Algebraic Functions

<u>7b.3 Limit At Infinity</u>

7b.4 Infinite Limits

7b.5 Asymptotes

<u>Chapter 8: The Concept of Continuity</u> <u>of a Function, and Points of</u> <u>Discontinuity</u>

8.1 Introduction

8.2 Developing the Definition of Continuity
"At a Point"

- 8.3 Classification of the Points of
 Discontinuity: Types of Discontinuities
 8.4 Checking Continuity of Functions
 Involving Trigonometric, Exponential, and
 Logarithmic Functions
- 8.5 From One-Sided Limit to One-Sided Continuity and its Applications
- 8.6 Continuity on an Interval
- **8.7 Properties of Continuous Functions**

<u>Chapter 9: The Idea of a Derivative of a Function</u>

- 9.1 Introduction
- <u>9.2 Definition of the Derivative as a Rate</u> <u>Function</u>
- 9.3 Instantaneous Rate of Change of y [=f(x)] at $x = x_1$ and the Slope of its Graph at $x = x_1$
- 9.4 A Notation for Increment(s)
- 9.5 The Problem of Instantaneous Velocity
- 9.6 Derivative of Simple Algebraic Functions
- 9.7 Derivatives of Trigonometric Functions
- 9.8 Derivatives of Exponential and Logarithmic Functions
- 9.9 Differentiability and Continuity
- 9.10 Physical Meaning of Derivatives
- 9.11 Some Interesting Observations
- 9.12 Historical Notes

Chapter 10: Algebra of Derivatives: Rules for Computing Derivatives of Various Combinations of Differentiable Functions

10.1 Introduction

10.2 Recalling the Operator of

Differentiation

10.3 THE Derivative of a Composite

Function

10.4 Usefulness of Trigonometric Identities

in Computing Derivatives

10.5 Derivatives of Inverse Functions

Chapter 11a: Basic Trigonometric Limits and Their Applications in Computing Derivatives of Trigonometric Functions

11a.1 Introduction

11a.2 Basic Trigonometric Limits

11a.3 Derivatives of Trigonometric

Functions

<u>Chapter 11b: Methods of Computing</u> <u>Limits of Trigonometric Functions</u>

11b.1 Introduction

11b.2 Limits of Type (I)

11b.3 Limits of the Type (II) [lim f(x), where

<u>a ≠ 0]</u>

11b.4 Limits of Exponential and Logarithmic Functions

Chapter 12: Exponential Form(s) of a Positive Real Number and its Logarithm(s): Pre-Requisite for Understanding Exponential and Logarithmic Functions

12.1 Introduction

12.2 Concept of Logarithm

12.3 The Laws of Exponent

12.4 Laws of Exponents (or Laws of Indices)

12.5 Two Important Bases: "10" and "e"

12.6 Definition: Logarithm

12.7 Advantages of Common Logarithms

12.8 Change of Base

12.9 Why Were Logarithms Invented?

12.10 Finding A Common Logarithm of A

(Positive) Number

12.11 Antilogarithm

12.12 Method of Calculation USING

LOGARITHM

<u>Chapter 13a: Exponential and Logarithmic Functions and Their Derivatives</u>

13a.1 Introduction

13a.2 Origin of e

13a.3 Distinction Between Exponential and
Power Functions
13a.4 The Value of e
13a.5 The Exponential Series
13a.6 Properties of e and Those of Related
Functions
13a.7 Comparison of Properties of
Logarithm(s) to the Bases 10 and e
13a.8 A Little More About e
13a.9 Graphs of Exponential Function(s)
13a.10 General Logarithmic Function
13a.11 Derivatives of Exponential and
Logarithmic Functions
13a.12 Exponential Rate of Growth
13a.13 Higher Exponential Rates of Growth
13a.14 An Important Standard Limit
13a.15 Applications of the Function ex:
Exponential Growth and Decay
<u></u>

<u>Chapter 13b: Methods for Computing</u> <u>Limits of Exponential and Logarithmic</u> <u>Functions</u>

13b.1 Introduction

13b.2 Review of Logarithms

13b.3 Some Basic Limits

13b 4 Evaluation of Limits Based or

13b.4 Evaluation of Limits Based on the Standard Limit

<u>Chapter 14: Inverse Trigonometric</u> <u>Functions and Their Derivatives</u>

THIT IIIII OUUCUUI	<i>14.1</i>	Introd	luction
--------------------	-------------	--------	---------

14.2 Trigonometric Functions (With

Restricted Domains) and Their Inverses

14.3 The Inverse Cosine Function

14.4 The Inverse Tangent Function

14.5 Definition of the Inverse Cotangent Function

14.6 Formula for the Derivative of Inverse Secant Function

14.7 Formula for the Derivative of Inverse Cosecant Function

14.8 Important Sets of Results and their Applications

14.9 Application of Trigonometric Identities in Simplification of Functions and Evaluation of Derivatives of Functions
Involving Inverse Trigonometric Functions

<u>Chapter 15a: Implicit Functions and</u> Their Differentiation

15a.1 Introduction

<u>15a.2 Closer Look AT the Difficulties</u> Involved

<u>15a.3 The Method of Logarithmic</u> Differentiation

<u>15a.4 Procedure of Logarithmic</u> Differentiation

<u>Chapter 15b: Parametric Functions</u> <u>and Their Differentiation</u>

15b.1 Introduction

15b.2 The Derivative of a Function

Represented Parametrically

15b.3 Line of Approach for Computing the

Speed of a Moving Particle

15b.4 Meaning of dy/dx with Reference to

the Cartesian Form y = f(x) and Parametric

Forms x = f(t), y = g(t) of the Function

15b.5 Derivative of One Function with

Respect to the Other

<u>Chapter 16: Differentials "dy" and "dx": Meanings and Applications</u>

16.1 Introduction

16.2 Applying Differentials to Approximate
Calculations

16.3 Differentials of Basic Elementary

Functions

16.4 Two Interpretations of the Notation dy/dx

16.5 Integrals in Differential Notation

16.6 To Compute (Approximate) Small

<u>Changes and Small Errors Caused in Various</u> Situations

<u>Chapter 17: Derivatives and Differentials of Higher Order</u>

17.1 Introduction
17.2 Derivatives of Higher Orders: Implicit
Functions
17.3 Derivatives of Higher Orders:
Parametric Functions
17.4 Derivatives of Higher Orders: Product
of Two Functions (Leibniz Formula)
17.5 Differentials of Higher Orders
17.6 Rate of Change of a Function and

<u>Chapter 18: Applications of</u> <u>Derivatives in Studying Motion in a</u> <u>Straight Line</u>

18.1 Introduction

Related Rates

18.2 Motion in a Straight Line

18.3 Angular Velocity

18.4 Applications of Differentiation in Geometry

18.5 Slope of a Curve in Polar Coordinates

<u>Chapter 19a: Increasing and</u> <u>Decreasing Functions and the Sign of</u> <u>the First Derivative</u>

19a.1 Introduction

19a.2 The First Derivative Test for Rise and Fall

19a.3 Intervals of Increase and Decrease (Intervals of Monotonicity)

19a.4 Horizontal Tangents with a Local

Maximum/Minimum

19a.5 Concavity, Points of Inflection, and
the Sign of the Second Derivative

<u>Chapter 19b: Maximum and Minimum</u> <u>Values of a Function</u>

19b.1 Introduction

19b.2 Relative Extreme Values of a Function

19b.3 Theorem A

19b.4 Theorem B: Sufficient Conditions for

the Existence of a Relative Extrema—In

<u>Terms of the First Derivative</u>

19b.5 Sufficient Condition for Relative

Extremum (In Terms of the Second

<u>Derivative)</u>

19b.6 Maximum and Minimum of a Function on the Whole Interval (Absolute Maximum and Absolute Minimum Values)

19b.7 Applications of Maxima and Minima
Techniques in Solving Certain Problems
Involving the Determination of the Greatest
and the Least Values

<u>Chapter 20: Rolle's Theorem and the Mean Value Theorem (MVT)</u>

20.1 Introduction

20.2 Rolle's Theorem (A Theorem on the Roots of a Derivative)

20.3 Introduction to the Mean Value
Theorem
20.4 Some Applications of the Mean Value
Theorem

Chapter 21: The Generalized Mean Value Theorem (Cauchy's MVT), L' Hospital's Rule, and their Applications

- **21.1 Introduction**
- 21.2 Generalized Mean Value Theorem (Cauchy's MVT)
- 21.3 Indeterminate Forms and L'Hospital's Rule
- 21.4 L'Hospital's Rule (First Form)
- 21.5 L'Hospital's Theorem (For Evaluating Limits(s) of the Indeterminate Form 0/0.)
- 21.6 Evaluating Indeterminate Form of the Type ∞/∞
- 21.7 Most General Statement of L'Hospital's Theorem
- 21.8 Meaning of Indeterminate Forms
 21.9 Finding Limits Involving Various
 Indeterminate Forms (by Expressing them in the Form 0/0 or ∞/∞)

<u>Chapter 22: Extending the Mean</u> <u>Value Theorem to Taylor's Formula:</u>

<u>Taylor Polynomials for Certain</u> <u>Functions</u>

- 22.1 Introduction
- 22.2 The Mean Value Theorem For Second
- **Derivatives: The First Extended MVT**
- 22.3 Taylor's Theorem
- 22.4 Polynomial Approximations and
- <u>Taylor's Formula</u>
- **22.5 From Maclaurin Series To Taylor Series**
- **22.6 Taylor's Formula for Polynomials**
- 22.7 Taylor's Formula for Arbitrary
- **Functions**

<u>Chapter 23: Hyperbolic Functions and Their Properties</u>

- 23.1 Introduction
- 23.2 Relation Between Exponential and Trigonometric Functions
- 23.3 Similarities and Differences in the Behavior of Hyperbolic and Circular Functions
- 23.4 Derivatives of Hyperbolic Functions
- 23.5 Curves of Hyperbolic Functions
- 23.6 The Indefinite Integral Formulas for Hyperbolic Functions
- 23.7 Inverse Hyperbolic Functions
- 23.8 Justification for Calling sinh and cosh
- as Hyperbolic Functions Just as sine and

<u>cosine are Called Trigonometric Circular</u> Functions

<u>Appendix A: (Related To Chapter-2)</u> <u>Elementary Set Theory</u>

A.1 Introduction

A.2 Elements of a Set

A.3 Set Notations

<u>A.4 Specifying Sets</u>

A.5 Singleton Set (or Unit Set)

A.6 The Null Set or the Empty Set

A.7 The Cardinal Number of a Set

A.8 Subset of a Set

A.9 Equality of Sets

A.10 Proper Subset

A.11 Comparability of Sets

A.12 Set of Sets

A.13 Power Set

A.14 Universal Set U

A.15 Operations on Sets

A.16 The Union (Logical Sum) of Two Sets A and B

A.17 The Intersection (Logical Product) of

Two Sets A and B

<u>A.18 Disjoint Sets</u>

A.19 Difference of Two Sets A and B

A.20 Complement of a Set

<u>Appendix B: (Related To Chapter-4)</u>

B.1 Introduction
<u>B.2</u>
B.3 The Idea of a Double Napped Right
Circular Cone and Conics
B.4 Conic Section: Definitions
B.5 Conics
<u>B.6</u>
B.7 Translation of Axes (or Shift of Origin
<u>Appendix C: (Related To Chapter-20</u>
<u>Index</u>

INTRODUCTION TO DIFFERENTIAL CALCULUS

Systematic Studies with Engineering Applications for Beginners

Ulrich L. Rohde

Prof. Dr.-Ing. Dr. h. c. mult. BTU Cottbus, Germany Synergy Microwave Corporation Paterson, NJ, USA

G. C. Jain

(Retd. Scientist) Defense Research and Development Organization Maharashtra, India

Ajay K. Poddar

Chief Scientist, Synergy Microwave Corporation, Paterson, NJ, USA

A. K. Ghosh

Professor, Department of Aerospace Engineering Indian Institute of Technology – Kanpur Kanpur, India



A JOHN WILEY & SONS, INC., PUBLICATION

Copyright © 2012 by John Wiley & Sons. All rights reserved Published by John Wiley & Sons, Inc., Hoboken, New Jersey Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at http://www.wiley.com/go/permission.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974,

outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data:

Introduction to differential calculus: systematic studies with engineering applications for beginners / Ulrich L. Rohde... [et al.]. – 1st ed.

p. cm.

Includes bibliographical references and index. ISBN 978-1-118-11775-0 (hardback)

Differential calculus-Textbooks. I. Rohde, Ulrich L. QA304.I59 2012
 513'.33-dc232011018421

Foreword

"What is Calculus?" is a classic deep question. Calculus is the most powerful branch of mathematics, which revolves around calculations involving varying quantities. It provides a system of rules to calculate quantities, which cannot be calculated by applying any other branch of mathematics. Schools or colleges find it difficult to motivate students to learn this subject, while those who do take the course find it very mechanical. Many a times, it has been observed that students incorrectly solve real-life problems by applying Calculus. They may not be capable to understand or admit their shortcomings in terms of basic understanding of fundamental concepts! The study of Calculus is one of the most powerful intellectual achievements of the human brain. One important goal of this manuscript is to give beginner-level students an appreciation of the beauty of Calculus. Whether taught in a traditional lecture format or in the lab with individual or group learning, Calculus needs focusing on numerical and graphical experimentation. This means that the ideas and techniques have to be presented clearly and accurately in an articulated manner.

The ideas related with the development of Calculus appear throughout mathematical history, spanning over more than 2000 years. However, the credit of its invention goes to the mathematicians of the seventeenth century (in particular, to Newton and Leibniz) and continues up to the nineteenth century, when French mathematician Augustin-Louis Cauchy (1789–1857) gave the definition of the limit, a concept which removed doubts about the soundness of Calculus, and made it free from all confusion. The history of controversy about Calculus is most illuminating as to the growth of mathematics. The soundness of Calculus was doubted by the greatest mathematicians of the eighteenth

century, yet, it was not only applied freely but great developments like differential equations, differential geometry, and so on were achieved. Calculus, which is the outcome of an intellectual struggle for such a long period of time, has proved to be the most beautiful intellectual achievement of the human mind.

There are certain problems in mathematics, mechanics, physics, and many other branches of science, which cannot be solved by ordinary methods of geometry or algebra alone. To solve these problems, we have to use a new branch of mathematics, known as Calculus. It uses not only the ideas and methods from arithmetic, geometry, algebra, coordinate geometry, trigonometry, and so on, but also the notion of limit, which is a new idea which, lies at the foundation of Calculus. Using this notion as a tool, the derivative of a function (which is a variable quantity) is defined as the limit of a particular kind.

In general, Differential Calculus provides a method for calculating "the rate of change" of the value of the variable quantity. On the other hand, Integral Calculus provides methods for calculating the total effect of such changes, under the given conditions. The phrase rate of change mentioned above stands for the actual rate of change of a variable, and *not its average rate of change*. The phrase "rate of change" might look like a foreign language to beginners, but concepts like rate of change, stationary point, and root, and so on, have precise mathematical meaning, agreed-upon all over the world. Understanding such words helps a lot in understanding the mathematics they convey. At this stage, it must also be made clear that whereas algebra, geometry, and trigonometry are the tools which are used in the study of Calculus, they should not be confused with the subject of Calculus.

This manuscript is the result of joint efforts by Prof. Ulrich L. Rohde, Mr. G. C. Jain, Dr. Ajay K. Poddar, and myself. All of

us are aware of the practical difficulties of the students face while learning Calculus. I am of the opinion that with the availability of these notes, students should be able to learn the subject easily and enjoy its beauty and power. In fact, for want of such simple and systematic work, most students are learning the subject as a set of rules and formulas, which is really unfortunate. I wish to discourage this trend.

Professor Ulrich L. Rohde, Faculty of Mechanical, Electrical, and Industrial Engineering (RF and Microwave Circuit Design & Techniques) Brandenburg University of Technology, Cottbus, Germany has optimized this book by expanding it, adding useful applications, and adapting it for today's needs. Parts of the mathematical approach from the Rohde, Poddar, and Böeck textbook on wireless oscillators (The Design of Modern Microwave Oscillators for Wireless Applications: Theory and Optimization, John Wiley & Sons, ISBN 0-471-72342-8, 2005) were used as they combine differentiation and integration to calculate the damped and using oscillation condition simple starting equations. This is a good transition for more challenging tasks for scientific studies with engineering applications for beginners who find difficulties in understanding the problem-solving power of Calculus.

Mr. Jain is not an educator by profession, but his curiosity to go to the roots of the subject to prepare the so-called concept-oriented notes for systematic studies in Calculus is his contribution toward creating interest among students for learning mathematics in general, and Calculus in particular. This book started with these concept-oriented notes prepared for teaching students to face real-life engineering problems. Most of the material pertaining to this manuscript on calculus was prepared by Mr. G. C. Jain in the process of teaching his kids and helping other students who needed help in learning the subject. Later on, his friends (including

me) realized the beauty of his compilation and we wanted to see his useful work published.

I am also aware that Mr. Jain got his notes examined from some professors at the Department of Mathematics, Pune University, India. I know Mr. Jain right from his scientific Research and at Armament Development career Establishment (ARDE) at Pashan, Pune, India, where I was a Senior Scientist (1982–1998) and headed the Aerodynamic Group ARDE, Pune in DRDO (Defense Research and Development Organization), India. Coincidently, Dr. Ajay K. Poddar, Chief Scientist at Synergy Microwave Corp., NJ 07504, USA was also a Senior Scientist (1990-2001) in a very responsible position in the Fuze Division of ARDE and was aware of the aptitude of Mr. Jain.

Dr. Ajay K. Poddar has been the main driving force towards the realization of the conceptualized notes prepared by Mr. Jain in manuscript form and his sincere efforts made timely publications possible. Dr. Poddar has made tireless effort by extending all possible help to ensure that Mr. Jain's notes published for the benefit of the students. contributions include (but are not limited to) valuable inputs suggestions throughout the preparation manuscript for its improvement, as well as many relevant literature acquisitions. I am sure, as a leading scientist, Dr. Poddar will have realized how important it is for the younger generation to avoid shortcomings in terms of basic understanding of the fundamental concepts of Calculus.

I have had a long time association with Mr. Jain and Dr. Poddar at ARDE, Pune. My objective has been to proofread the manuscript and highlight its salient features. However, only a personal examination of the book will convey to the reader the broad scope of its coverage and its contribution in addressing the proper way of learning Calculus. I hope this book will prove to be very useful to the students of Junior Colleges and to those in higher classes (of science

and engineering streams) who might need it to get rid of confusions, if any.

My special thanks goes to Dr. Poddar, who is not only a gifted scientist but has also been a mentor. It was his suggestion to publish the manuscript in two parts (Part I: Introduction to Differential Calculus: Systematic Studies with Engineering Applications for Beginners and Part Introduction to Integral Calculus: Systematic Studies with Engineering Applications for Beginners) so that beginners could digest the concepts of Differential and Integral Calculus without confusion and misunderstanding. It is the purpose of this book to provide a clear understanding of the concepts needed by beginners and engineers who are interested in the application of Calculus of their field of study. This book has been designed as a supplement to all current standard textbooks on Calculus and each chapter begins with a clear statement of pertinent definitions, principles, and theorems together with illustrative and other descriptive material. Considerably more material has been included here than can be covered in most high schools and undergraduate study courses. This has been done to make the book more flexible; to provide concept-oriented notes and stimulate interest in the relevant topics. I believe that students learn best when procedural techniques are laid out as clearly and simply as possible. Consistent with the reader's needs and for completeness, there are a large number of examples for self-practice.

The authors are to be commended for their efforts in this endeavor, and I am sure that both Part I and Part II will be an asset to the beginner's handbook on the bookshelf. I hope that after reading this book, the students will begin to share the enthusiasm of the authors in understanding and applying the principles of Calculus and its usefulness. With all these changes, the authors have not compromised our belief that the fundamental goal of Calculus is to help

prepare beginners enter the world of mathematics, science, and engineering.

Finally, I would like to thank Susanne Steitz-Filler, Editor (Mathematics and Statistics) at John Wiley & Sons, Inc., Danielle Lacourciere, Senior Production Editor at John Wiley & Sons, Inc., and Sanchari Sil at Thomson Digital for her patience and splendid cooperation throughout the journey of this publication.

Ajoy Kanti Ghosh Professor & Faculty Incharge (Flight Laboratory) Department of Aerospace Engineering IIT Kanpur, India

Preface

In general, there is a perception that Calculus is an extremely difficult subject, probably because the required number of good teachers and good books are not available. We know that books cannot replace teachers, but we are of opinion that good books can definitely reduce dependence on teachers, and students can gain more confidence by learning most of the concepts on their own. In the process of helping students to learn Calculus, we have gone through many books on the subject and realized that whereas a large number of good books are available at the graduate level, there is hardly any book available for introducing the subject to beginners. The reason for such a situation can be easily understood by anyone who knows the subject of Calculus and hence the practical difficulties associated with the process of learning the subject. In the market hundreds of books are available on Calculus. All these books contain a large number of important solved problems. Besides, the rules for solving the problems and the list of necessary formulae are given in the books, without discussing anything about the basic concepts involved. Of course, such books are useful for passing the examination(s), but Calculus is hardly learnt from these books. Initially, the coauthors had compiled conceptoriented notes for systematic studies in differential and integral Calculus, intended for beginners. These notes were used by students in school- and undergraduate-level courses. The response and the appreciation experienced from the students and their parents encouraged us to make these notes available to the beginners. It is due to the efforts of our friends and well-wishers that our dream has now materialized in the form of two independent books: Part I for Differential Calculus and Part II for Integral Calculus. Of

course there are some world class authors who have written useful books on the subject at introductory level, presuming that the reader has the necessary knowledge prerequisites. Some such books are: What is calculus about? (By Professor WW Sawyer), Teach yourself calculus (By P. Abbott, B.A), Calculus Made Easy (By S.P. Thomson) and *Calculus Explained* (By W.J. Reichmann). Any person with some knowledge of Calculus will definitely appreciate the contents and the approach of the authors. However, a reader will be easily convinced that most of the beginners may not be able to get (from these books) the desired benefit, for various reasons. From this point of view, both Parts (Part-I & Part-II) of our book would prove to be unique since this provide comprehensive material on Calculus for the beginners. The first six chapters of Part-I would help the beginner to come up to the level, so that one can easily learn the concept of limit, which is in the foundation of calculus. The purpose of these works is to provide the basic (but solid) foundation of Calculus to beginners. The books aim to show them the enjoyment in the beauty and power of Calculus and develop the ability to select proper material needed for their studies in any technical and scientific field, involving Calculus.

One reason for such a high dropout rate is that at beginner levels, Calculus is so poorly taught. Classes tend to be so boring that students sometimes fall asleep. Calculus textbooks get fatter and fatter every year, with more multicolor overlays, computer graphics, and photographs of eminent mathematicians (starting with Newton and Leibniz), yet they never seem easier to comprehend. We look through them in vain for simple, clear exposition, and for problems that will hook a student's interest. Recent years have seen a great hue and cry in mathematical circle over ways to improve teaching Calculus to beginner and high-school students. Endless conferences have been held, many