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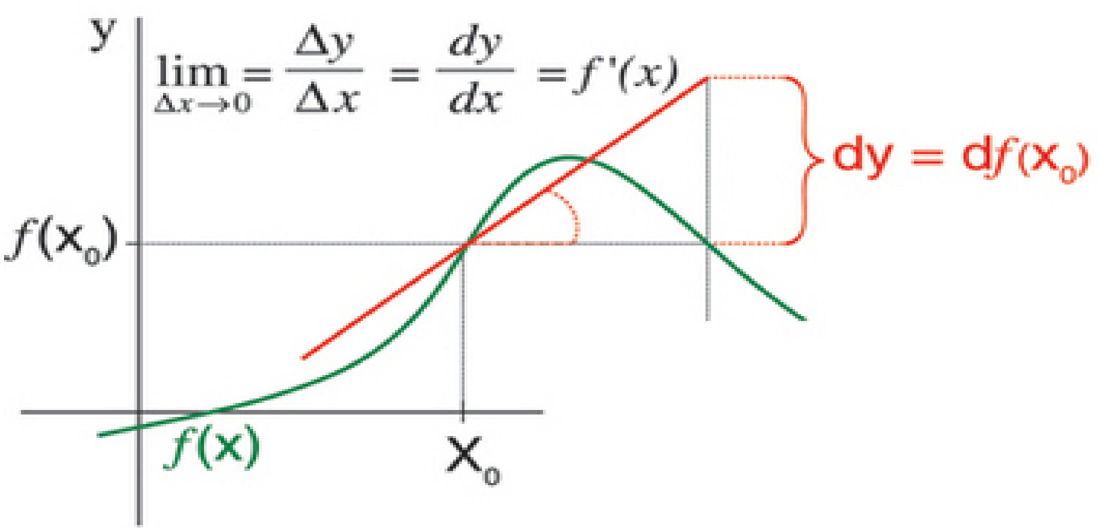
INTRODUCTION TO DIFFERENTIAL CALCULUS

Systematic Studies with Engineering Applications for Beginners



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Systematic Studies with Engineering Applications for Beginners

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Foreword

“What is Calculus?” is a classic deep question. Calculus is the most powerful branch of mathematics, which revolves around calculations involving varying quantities. It provides a system of rules to calculate quantities, which cannot be calculated by applying any other branch of mathematics. Schools or colleges find it difficult to motivate students to learn this subject, while those who do take the course find it very mechanical. Many a times, it has been observed that students incorrectly solve real-life problems by applying Calculus. They may not be capable to understand or admit their shortcomings in terms of basic understanding of fundamental concepts! The study of Calculus is one of the most powerful intellectual achievements of the human brain. One important goal of this manuscript is to give beginner-level students an appreciation of the beauty of Calculus. Whether taught in a traditional lecture format or in the lab with individual or group learning, Calculus needs focusing on numerical and graphical experimentation. This means that the ideas and techniques have to be presented clearly and accurately in an articulated manner.

The ideas related with the development of Calculus appear throughout mathematical history, spanning over more than 2000 years. However, the credit of its invention goes to the mathematicians of the seventeenth century (in particular, to Newton and Leibniz) and continues up to the nineteenth century, when French mathematician Augustin-Louis Cauchy (1789–1857) gave the definition of the limit, a concept which removed doubts about the soundness of Calculus, and made it free from all confusion. The history of controversy about Calculus is most illuminating as to the growth of mathematics. The soundness of Calculus was doubted by the greatest mathematicians of the eighteenth century, yet, it was not only applied freely but great developments like differential equations, differential geometry, and so on were achieved. Calculus, which is the outcome of an intellectual struggle for such a long period of time, has proved to be the most beautiful intellectual achievement of the human mind.

There are certain problems in mathematics, mechanics, physics, and many other branches of science, *which cannot be solved by ordinary methods of geometry or algebra alone*. To solve these problems, we have to use a new branch of mathematics, known as *Calculus*. It uses not only the ideas and methods from arithmetic, geometry, algebra, coordinate geometry, trigonometry, and so on, but also *the notion of limit*, which is a *new idea* which, lies at the *foundation of Calculus*. Using this notion as a tool, *the derivative* of a function (which is a variable quantity) is defined as the limit of a particular kind.

In general, *Differential Calculus* provides a method for calculating "*the rate of change*" of the value of the variable quantity. On the other hand, *Integral Calculus* provides methods for calculating the total effect of such changes, under the given conditions. The phrase *rate of change* mentioned above stands for the actual rate of change of a variable, and *not its average rate of change*. The phrase "rate of change" might look like a foreign language to beginners, but concepts like *rate of change, stationary point*, and *root*, and so on, have precise mathematical meaning, agreed-upon all over the world. Understanding such words helps a lot in understanding the mathematics they convey. At this stage, it must also be made clear that whereas algebra, geometry, and trigonometry are the tools which are used in the study of Calculus, they should not be confused with the subject of Calculus.

This manuscript is the result of joint efforts by Prof. Ulrich L. Rohde, Mr. G. C. Jain, Dr. Ajay K. Poddar, and myself. All of us are aware of the practical difficulties of the students face while learning Calculus. I am of the opinion that with the availability of these notes, students should be able to learn the subject easily and enjoy its beauty and power. In fact, for want of such simple and systematic work, most students are learning the subject as a set of rules and formulas, which is really unfortunate. I wish to discourage this trend.

Professor Ulrich L. Rohde, Faculty of Mechanical, Electrical, and Industrial Engineering (RF and Microwave Circuit Design & Techniques) Brandenburg University of Technology, Cottbus, Germany has optimized this book by expanding it, adding useful applications, and adapting it for today's needs. Parts of the mathematical approach from the Rohde, Poddar, and Böeck textbook on wireless oscillators (*The Design of Modern Microwave Oscillators for Wireless Applications: Theory and Optimization*, John Wiley & Sons, ISBN 0-471-72342-8, 2005) were used as they combine differentiation and integration to calculate the damped and starting oscillation condition using simple differential equations. This is a good transition for more challenging tasks for scientific studies with engineering applications for beginners who find difficulties in understanding the problem-solving power of Calculus.

Mr. Jain is not an educator by profession, but his curiosity to go to the roots of the subject to prepare the so-called *concept-oriented notes for systematic studies in Calculus* is his contribution toward creating interest among students for learning mathematics in general, and Calculus in particular. This book started with these concept-oriented notes prepared for teaching students to face real-life engineering problems. Most of the material pertaining to this manuscript on calculus was prepared by Mr. G. C. Jain in the process of teaching his kids and helping other students who needed help in learning the subject. Later on, his friends (including me) realized the beauty of his compilation and we wanted to see his useful work published.

I am also aware that Mr. Jain got his notes examined from some professors at the Department of Mathematics, Pune University, India. I know Mr. Jain right from his scientific career at Armament Research and Development Establishment (ARDE) at Pashan, Pune, India, where I was a Senior Scientist (1982–1998) and headed the Aerodynamic Group ARDE, Pune in DRDO (Defense Research and Development Organization), India. Coincidently, Dr. Ajay K. Poddar, Chief Scientist at Synergy Microwave Corp., NJ 07504, USA was also a Senior Scientist (1990–2001) in a very responsible position in the Fuze Division of ARDE and was aware of the aptitude of Mr. Jain.

Dr. Ajay K. Poddar has been the main driving force towards the realization of the conceptualized notes prepared by Mr. Jain in manuscript form and his sincere efforts made timely publications possible. Dr. Poddar has made tireless effort by extending all possible help to ensure that Mr. Jain's notes are published for the benefit of the students. His contributions include (but are not limited to) valuable inputs and suggestions throughout the preparation of this manuscript for its improvement, as well as many relevant literature acquisitions. I am sure, as a leading scientist, Dr. Poddar will have realized how important it is for the younger generation to avoid shortcomings in terms of basic understanding of the fundamental concepts of Calculus.

I have had a long time association with Mr. Jain and Dr. Poddar at ARDE, Pune. My objective has been to proofread the manuscript and highlight its salient features. However, only a personal examination of the book will convey to the reader the broad scope of its coverage and its contribution in addressing the proper way of learning Calculus. I hope this book will prove to be very useful to the students of Junior Colleges and to those in higher classes (of science and engineering streams) who might need it to get rid of confusions, if any.

My special thanks goes to Dr. Poddar, who is not only a gifted scientist but has also been a mentor. It was his suggestion to publish the manuscript in two parts (Part I: *Introduction to Differential Calculus: Systematic Studies with Engineering Applications for Beginners* and Part II: *Introduction to Integral Calculus: Systematic Studies with Engineering Applications for Beginners*) so that beginners could digest the concepts of Differential and Integral Calculus without confusion and misunderstanding. It is the purpose of this book to provide a clear understanding of the concepts needed by beginners and engineers who are interested in the application of Calculus of their field of study. This book has been designed as a supplement to all current standard textbooks on Calculus and each chapter begins with a clear statement of pertinent definitions, principles, and theorems together with illustrative and other descriptive material. Considerably more material has been included here than can be covered in most high schools and undergraduate study courses. This has been done to make the book more flexible; to provide concept-oriented notes and stimulate interest in the relevant topics. I believe that students learn best when procedural techniques are laid out as clearly and simply as possible. Consistent with the reader's needs and for completeness, there are a large number of examples for self-practice.

The authors are to be commended for their efforts in this endeavor, and I am sure that both Part I and Part II will be an asset to the beginner's handbook on the bookshelf. I hope that after reading this book, the students will begin to share the enthusiasm of the authors in understanding and applying the principles of Calculus and its usefulness. With all these changes, the authors have not compromised our belief that the fundamental goal of Calculus is to help prepare beginners enter the world of mathematics, science, and engineering.

Finally, I would like to thank Susanne Steitz-Filler, Editor (Mathematics and Statistics) at John Wiley & Sons, Inc., Danielle Lacourciere, Senior Production Editor at John Wiley & Sons, Inc., and Sanchari Sil at Thomson Digital for her patience and splendid cooperation throughout the journey of this publication.

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Preface

In general, there is a perception that Calculus is an extremely difficult subject, probably because the required number of good teachers and good books are not available. We know that books cannot replace teachers, but we are of the opinion that good books can definitely reduce dependence on teachers, and students can gain more confidence by learning most of the concepts on their own. In the process of helping students to learn Calculus, we have gone through many books on the subject and realized that whereas a large number of good books are available at the graduate level, there is hardly any book available for introducing the subject to beginners. The reason for such a situation can be easily understood by anyone who knows the subject of Calculus and hence the practical difficulties associated with the process of learning the subject. In the market hundreds of books are available on Calculus. All these books contain a large number of important solved problems. Besides, the rules for solving the problems and the list of necessary formulae are given in the books, without discussing anything about the basic concepts involved. Of course, such books are useful for passing the examination(s), but Calculus is hardly learnt from these books. Initially, the coauthors had compiled *concept-oriented notes for systematic studies in differential and integral Calculus,* intended for beginners. These notes were used by students in school- and undergraduate-level courses. The response and the appreciation experienced from the students and their parents encouraged us to make these notes available to the beginners. It is due to the efforts of our friends and well-wishers that our dream has now materialized in the form of two independent books: Part I for Differential Calculus and Part II for Integral Calculus. Of course there are some world class authors who have written useful books on the subject at introductory level, presuming that the reader has the necessary knowledge of prerequisites. Some such books are: ***What is calculus about?*** (By Professor WW Sawyer), ***Teach yourself calculus*** (By P. Abbott, B.A), ***Calculus Made Easy*** (By S.P. Thomson) and ***Calculus Explained*** (By W.J. Reichmann). Any person with some knowledge of Calculus will definitely appreciate the contents and the approach of the authors. However, a reader will be easily convinced that most of the beginners may not be able to get (from these books) the desired benefit, for various reasons. From this point of view, both Parts (Part-I & Part-II) of our book would prove to be unique since this provide comprehensive material on Calculus for the beginners. The first six chapters of Part-I would help the beginner to come up to the level, so that one can easily learn ***the concept of limit***, which is in the foundation of calculus. The purpose of these works is to provide *the basic (but solid) foundation of Calculus to beginners*. The books aim to show them the *enjoyment in the beauty and power of Calculus and develop the ability to select proper material needed for their studies in any technical and scientific field, involving Calculus*.

One reason for such a high dropout rate is that at beginner levels, Calculus is so poorly taught. Classes tend to be so boring that students sometimes fall asleep. Calculus textbooks get fatter and fatter every year, with more multicolor overlays, computer graphics, and photographs of eminent mathematicians (starting with Newton and Leibniz), yet they never seem easier to comprehend. We look through them in vain for simple, clear exposition, and for problems that will hook a student's interest. Recent years have seen a great hue and cry in mathematical circle over ways to improve teaching Calculus to beginner and high-school students. Endless conferences have been held, many